Computer algebra independent integration tests

4-Trig-functions/4.4-Cotangent/4.4.10-c+d-x-^m-a+b-cot-^n

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [61]. This is test number [109].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (61)	% 0.00 (0)
Mathematica	% 100.00 (61)	% 0.00 (0)
Maple	% 95.08 (58)	% 4.92 (3)
Maxima	% 75.41 (46)	% 24.59 (15)
Fricas	% 100.00 (61)	% 0.00 (0)
Sympy	% 45.90 (28)	% 54.10 (33)
Giac	% 57.38 (35)	% 42.62 (26)
Mupad	% 45.90 (28)	% 54.10 (33)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

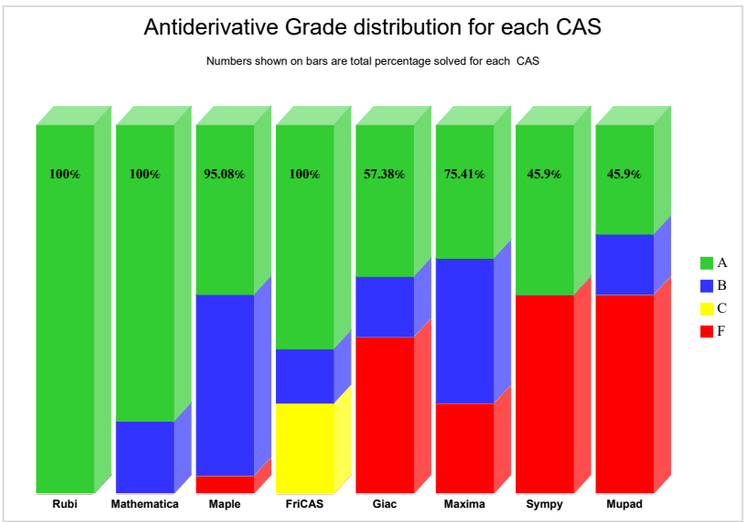
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

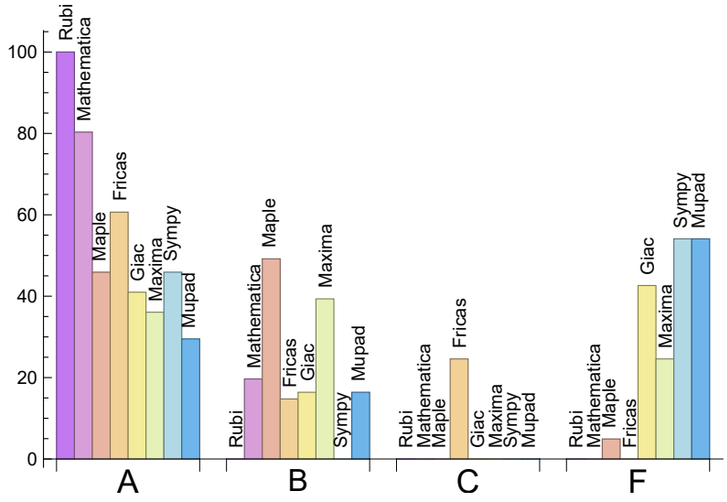
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	80.33	19.67	0.00	0.00
Maple	45.90	49.18	0.00	4.92
Maxima	36.07	39.34	0.00	24.59
Fricas	60.66	14.75	24.59	0.00
Sympy	45.90	0.00	0.00	54.10
Giac	40.98	16.39	0.00	42.62
Mupad	29.51	16.39	0.00	54.10

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	3	100.00 %	0.00 %	0.00 %
Maxima	15	20.00 %	20.00 %	60.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
Sympy	33	100.00 %	0.00 %	0.00 %
Giac	26	100.00 %	0.00 %	0.00 %
Mupad	33	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

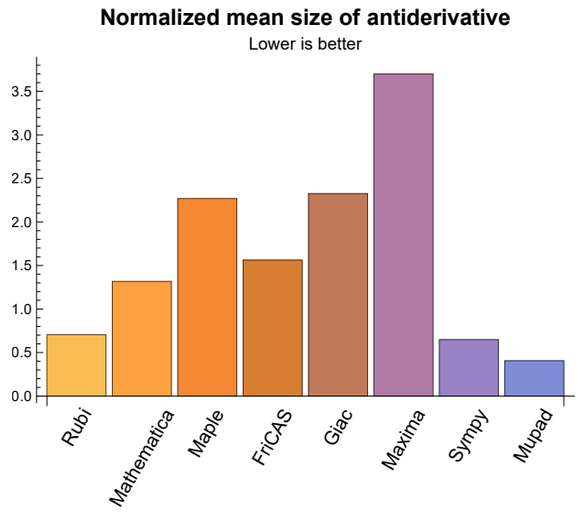
1.3 Performance

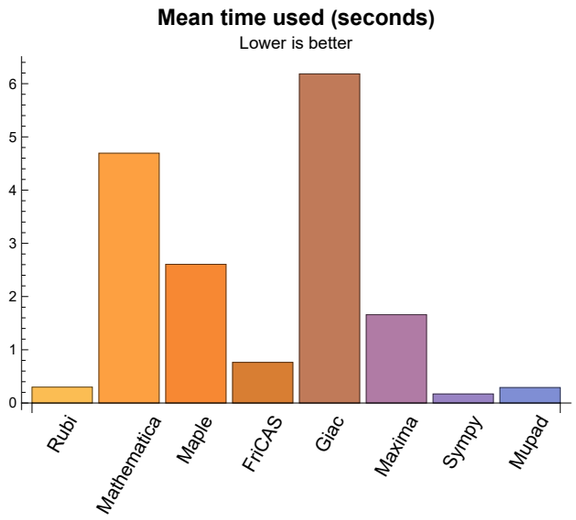
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.30	169.20	0.70	126.00	1.00
Mathematica	4.69	317.84	1.32	182.00	1.25
Maple	2.61	600.38	2.27	235.50	2.14
Maxima	1.66	982.39	3.70	263.00	2.57
Fricas	0.76	369.33	1.56	140.00	0.75
Sympy	0.17	134.79	0.65	0.00	0.00
Giac	6.18	449.91	2.33	0.00	0.00
Mupad	0.29	79.00	0.41	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{4, 5, 9, 10, 14, 15, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {3, 7, 11, 13, 20, 21, 37, 38, 39, 42, 43, 47, 48, 57, 59}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

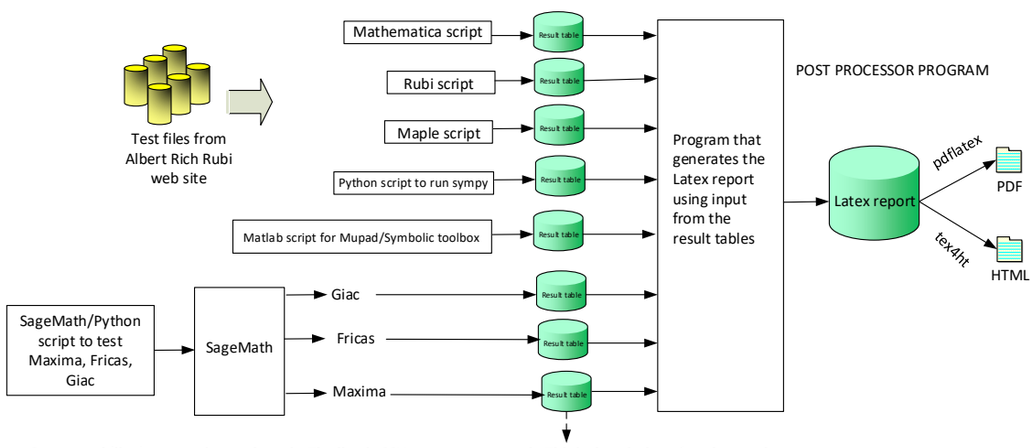
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 4, 5, 6, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 58, 60, 61 }

B grade: { 3, 7, 11, 37, 38, 39, 42, 43, 47, 48, 57, 59 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 4, 5, 8, 9, 10, 14, 15, 16, 17, 19, 20, 21, 25, 26, 30, 31, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 1, 2, 3, 6, 7, 11, 12, 13, 18, 22, 23, 24, 27, 28, 29, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

C grade: { }

F grade: { 34, 35, 36 }

2.1.4 Maxima

A grade: { 4, 5, 9, 10, 14, 15, 19, 20, 21, 25, 26, 30, 31, 32, 33, 40, 41, 46, 50, 51, 55, 56 }

B grade: { 1, 2, 3, 6, 7, 8, 11, 12, 13, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

C grade: { }

F grade: { 16, 17, 18, 22, 23, 24, 27, 28, 29, 34, 35, 36, 45, 60, 61 }

2.1.5 FriCAS

A grade: { 4, 5, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 3, 7, 8, 13, 39, 44, 49, 54, 59 }

C grade: { 1, 2, 6, 11, 12, 37, 38, 42, 43, 47, 48, 52, 53, 57, 58 }

F grade: { }

2.1.6 Sympy

A grade: { 4, 5, 8, 9, 10, 14, 15, 16, 17, 18, 22, 23, 24, 27, 28, 29, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 11, 12, 13, 19, 20, 21, 25, 26, 30, 31, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

2.1.7 Giac

A grade: { 4, 5, 9, 10, 14, 15, 16, 17, 18, 23, 24, 28, 29, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 8, 19, 20, 21, 22, 25, 26, 27, 30, 31 }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 11, 12, 13, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

2.1.8 Mupad

A grade: { 4, 5, 9, 10, 14, 15, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 8, 16, 17, 18, 22, 23, 24, 27, 28, 29 }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 11, 12, 13, 19, 20, 21, 25, 26, 30, 31, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	184	240	391	306	0	0	-1
normalized size	1	1.00	1.82	2.38	3.87	3.03	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.714	0.862	0.484	0.557	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	136	198	257	244	0	0	-1
normalized size	1	1.00	1.84	2.68	3.47	3.30	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.464	0.798	0.486	0.574	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	135	150	140	174	0	0	-1
normalized size	1	1.00	2.55	2.83	2.64	3.28	0.00	0.00	-0.02
time (sec)	N/A	0.085	3.728	0.823	0.456	0.526	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.016	1.898	0.896	0.000	0.544	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.017	3.325	1.083	0.000	0.624	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	171	231	953	372	0	0	-1
normalized size	1	1.00	1.76	2.38	9.82	3.84	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.918	0.952	0.594	0.688	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	153	183	384	281	0	0	-1
normalized size	1	1.00	2.07	2.47	5.19	3.80	0.00	0.00	-0.01
time (sec)	N/A	0.119	5.355	0.898	0.618	1.163	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	44	30	269	75	68	1250	54
normalized size	1	1.00	1.42	0.97	8.68	2.42	2.19	40.32	1.74
time (sec)	N/A	0.023	0.219	0.719	0.467	0.740	0.460	4.743	0.452

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	3.816	1.052	0.000	2.102	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	3.429	1.118	0.000	0.857	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	461	444	1970	565	0	0	-1
normalized size	1	1.00	2.28	2.20	9.75	2.80	0.00	0.00	-0.00
time (sec)	N/A	0.301	6.807	1.060	0.754	2.053	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	221	293	1216	423	0	0	-1
normalized size	1	1.00	1.75	2.33	9.65	3.36	0.00	0.00	-0.01
time (sec)	N/A	0.186	5.485	1.008	1.179	0.665	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	179	197	591	291	0	0	-1
normalized size	1	1.00	1.97	2.16	6.49	3.20	0.00	0.00	-0.01
time (sec)	N/A	0.107	4.295	0.803	0.882	0.657	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.028	6.506	2.916	0.000	0.552	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	5.270	3.788	0.000	0.920	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	246	170	0	153	318	243	423
normalized size	1	1.00	1.30	0.90	0.00	0.81	1.68	1.29	2.24
time (sec)	N/A	0.200	0.628	2.069	0.000	0.755	0.365	0.421	1.236

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	149	108	0	95	197	143	241
normalized size	1	1.00	1.09	0.79	0.00	0.69	1.44	1.04	1.76
time (sec)	N/A	0.123	0.366	1.864	0.000	0.499	0.287	0.418	0.769

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	107	177	0	48	104	67	105
normalized size	1	1.00	1.27	2.11	0.00	0.57	1.24	0.80	1.25
time (sec)	N/A	0.054	0.247	0.622	0.000	0.483	0.226	0.651	0.382

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	77	204	111	51	0	367	-1
normalized size	1	1.00	0.48	1.27	0.69	0.32	0.00	2.28	-0.01
time (sec)	N/A	0.278	0.284	0.597	0.449	0.565	0.000	0.552	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	215	271	122	72	0	367	-1
normalized size	1	1.00	1.30	1.63	0.73	0.43	0.00	2.21	-0.01
time (sec)	N/A	0.227	1.245	0.575	0.791	0.644	0.000	47.649	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	283	143	160	118	0	1630	-1
normalized size	1	1.00	1.25	0.63	0.70	0.52	0.00	7.18	-0.00
time (sec)	N/A	0.312	1.592	1.635	0.910	0.833	0.000	0.619	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	362	2261	0	253	653	433	294
normalized size	1	1.00	1.34	8.37	0.00	0.94	2.42	1.60	1.09
time (sec)	N/A	0.276	1.605	2.155	0.000	1.905	0.636	0.513	1.119

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	255	1073	0	151	406	247	186
normalized size	1	1.00	1.26	5.31	0.00	0.75	2.01	1.22	0.92
time (sec)	N/A	0.198	0.737	1.948	0.000	1.514	0.480	0.504	0.579

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	165	390	0	69	214	108	102
normalized size	1	1.00	1.09	2.58	0.00	0.46	1.42	0.72	0.68
time (sec)	N/A	0.138	0.613	1.662	0.000	0.758	0.356	1.734	0.338

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	136	382	194	80	0	987	-1
normalized size	1	1.00	0.45	1.25	0.64	0.26	0.00	3.24	-0.00
time (sec)	N/A	0.752	0.609	1.425	0.963	0.882	0.000	1.834	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	203	537	214	129	0	2369	-1
normalized size	1	1.00	0.47	1.24	0.49	0.30	0.00	5.46	-0.00
time (sec)	N/A	0.681	0.587	1.429	0.809	0.605	0.000	64.087	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	664	3997	0	353	935	623	418
normalized size	1	1.00	1.68	10.09	0.00	0.89	2.36	1.57	1.06
time (sec)	N/A	0.381	2.850	1.721	0.000	0.949	0.843	0.611	1.624

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	369	1843	0	207	578	351	263
normalized size	1	1.00	1.26	6.27	0.00	0.70	1.97	1.19	0.89
time (sec)	N/A	0.269	0.812	1.481	0.000	0.813	0.636	0.552	1.001

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	244	653	0	94	301	151	144
normalized size	1	1.00	1.17	3.12	0.00	0.45	1.44	0.72	0.69
time (sec)	N/A	0.218	0.663	1.061	0.000	0.658	0.458	2.084	0.625

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	197	560	275	113	0	1887	-1
normalized size	1	1.00	0.44	1.25	0.61	0.25	0.00	4.20	-0.00
time (sec)	N/A	1.732	0.578	0.641	0.902	0.833	0.000	1.535	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	292	756	300	180	0	4524	-1
normalized size	1	1.00	0.41	1.06	0.42	0.25	0.00	6.35	-0.00
time (sec)	N/A	1.643	0.643	0.645	1.652	0.764	0.000	87.906	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	10.378	0.706	0.000	0.714	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	5.738	1.006	0.000	0.633	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	190	0	0	84	0	0	-1
normalized size	1	1.00	1.94	0.00	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.124	1.292	1.287	0.000	0.542	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	152	0	0	140	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.185	2.832	2.102	0.000	0.681	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	238	0	0	192	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.242	4.099	2.342	0.000	0.551	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	730	857	966	626	0	0	-1
normalized size	1	1.00	4.97	5.83	6.57	4.26	0.00	0.00	-0.01
time (sec)	N/A	0.255	7.503	1.320	1.164	0.739	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	406	516	520	403	0	0	-1
normalized size	1	1.00	3.62	4.61	4.64	3.60	0.00	0.00	-0.01
time (sec)	N/A	0.210	2.571	1.221	1.753	0.607	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	204	240	208	224	0	0	-1
normalized size	1	1.00	2.46	2.89	2.51	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.123	5.036	1.113	0.930	0.520	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	2.227	1.779	0.000	0.648	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	8.192	1.799	0.000	0.503	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	1611	1571	4012	1154	0	0	-1
normalized size	1	1.00	5.46	5.33	13.60	3.91	0.00	0.00	-0.00
time (sec)	N/A	0.531	7.433	1.819	4.352	0.904	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	729	899	1945	715	0	0	-1
normalized size	1	1.00	3.21	3.96	8.57	3.15	0.00	0.00	-0.00
time (sec)	N/A	0.383	7.224	1.636	1.993	0.827	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	200	365	776	378	0	0	-1
normalized size	1	1.00	1.46	2.66	5.66	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.174	2.251	1.348	1.447	0.875	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	19.847	5.793	0.000	0.502	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	16.277	7.245	0.000	0.456	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	603	603	3045	3113	11204	2749	0	0	-1
normalized size	1	1.00	5.05	5.16	18.58	4.56	0.00	0.00	-0.00
time (sec)	N/A	0.981	8.544	2.398	35.800	0.706	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	2013	1740	5411	1564	0	0	-1
normalized size	1	1.00	4.65	4.02	12.50	3.61	0.00	0.00	-0.00
time (sec)	N/A	0.649	7.898	2.066	3.977	1.176	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	276	745	2017	721	0	0	-1
normalized size	1	1.00	0.99	2.68	7.26	2.59	0.00	0.00	-0.00
time (sec)	N/A	0.323	3.647	1.809	1.212	0.608	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	9.594	8.519	0.000	0.631	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	12.557	11.166	0.000	0.508	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	345	1386	987	1029	0	0	-1
normalized size	1	1.00	1.43	5.73	4.08	4.25	0.00	0.00	-0.00
time (sec)	N/A	0.341	2.135	2.556	0.846	0.594	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	289	881	719	730	0	0	-1
normalized size	1	1.00	1.60	4.87	3.97	4.03	0.00	0.00	-0.01
time (sec)	N/A	0.288	1.547	2.273	0.823	0.747	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	182	445	406	475	0	0	-1
normalized size	1	1.00	1.44	3.53	3.22	3.77	0.00	0.00	-0.01
time (sec)	N/A	0.162	1.857	2.251	0.665	0.656	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	1.967	3.290	0.000	0.534	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	3.944	5.052	0.000	0.640	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	839	839	1682	3446	4699	3359	0	0	-1
normalized size	1	1.00	2.00	4.11	5.60	4.00	0.00	0.00	-0.00
time (sec)	N/A	1.969	10.660	2.917	4.717	0.874	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	718	2137	2592	2036	0	0	-1
normalized size	1	1.00	1.10	3.29	3.99	3.13	0.00	0.00	-0.00
time (sec)	N/A	1.477	9.213	2.717	2.212	0.603	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	730	990	1181	1053	0	0	-1
normalized size	1	1.00	3.43	4.65	5.54	4.94	0.00	0.00	-0.00
time (sec)	N/A	0.292	7.100	2.563	2.027	0.650	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	19.281	13.352	0.000	0.673	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	17.173	21.116	0.000	0.598	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [11] had the largest ratio of [.8333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	6	1.00	10	0.600
2	A	5	5	1.00	10	0.500
3	A	4	4	1.00	8	0.500
4	A	0	0	0.00	0	0.000
5	A	0	0	0.00	0	0.000
6	A	7	7	1.00	12	0.583
7	A	6	6	1.00	12	0.500
8	A	3	3	1.00	10	0.300
9	A	0	0	0.00	0	0.000
10	A	0	0	0.00	0	0.000
11	A	13	10	1.00	12	0.833
12	A	9	8	1.00	12	0.667
13	A	7	7	1.00	10	0.700

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	0	0	0.00	0	0.000
15	A	0	0	0.00	0	0.000
16	A	5	3	1.00	23	0.130
17	A	4	3	1.00	23	0.130
18	A	3	3	1.00	21	0.143
19	A	7	4	1.00	23	0.174
20	A	7	4	1.00	23	0.174
21	A	8	5	1.00	23	0.217
22	A	10	3	1.00	23	0.130
23	A	8	3	1.00	23	0.130
24	A	7	3	1.00	21	0.143
25	A	21	5	1.00	23	0.217
26	A	24	7	1.00	23	0.304
27	A	14	3	1.00	23	0.130
28	A	11	3	1.00	23	0.130
29	A	11	3	1.00	21	0.143
30	A	53	7	1.00	23	0.304
31	A	60	9	1.00	23	0.391
32	A	0	0	0.00	0	0.000
33	A	0	0	0.00	0	0.000
34	A	2	2	1.00	23	0.087
35	A	4	2	1.00	23	0.087
36	A	5	2	1.00	23	0.087
37	A	8	7	1.00	18	0.389
38	A	7	6	1.00	18	0.333
39	A	6	5	1.00	16	0.312
40	A	0	0	0.00	0	0.000
41	A	0	0	0.00	0	0.000
42	A	15	9	1.00	20	0.450
43	A	13	10	1.00	20	0.500
44	A	9	7	1.00	18	0.389
45	A	0	0	0.00	0	0.000
46	A	0	0	0.00	0	0.000
47	A	28	11	1.00	20	0.550
48	A	22	11	1.00	20	0.550
49	A	16	9	1.00	18	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
50	A	0	0	0.00	0	0.000
51	A	0	0	0.00	0	0.000
52	A	6	6	1.00	20	0.300
53	A	5	5	1.00	20	0.250
54	A	4	4	1.00	18	0.222
55	A	0	0	0.00	0	0.000
56	A	0	0	0.00	0	0.000
57	A	21	9	1.00	20	0.450
58	A	18	10	1.00	20	0.500
59	A	5	5	1.00	18	0.278
60	A	0	0	0.00	0	0.000
61	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int x^3 \cot(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{3i\mathrm{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}+\frac{3x\mathrm{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{3ix^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{ix^4}{4}$$

```
[Out] -1/4*I*x^4+x^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*x^2*polylog(2,exp(2*I*(b*x+a)
))/b^2+3/2*x*polylog(3,exp(2*I*(b*x+a)))/b^3+3/4*I*polylog(4,exp(2*I*(b*x+a
)))/b^4
```

Rubi [A] time = 0.17, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3717, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3ix^2\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3x\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3i\mathrm{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}+\frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{ix^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cot[a + b*x],x]
```

```
[Out] (-I/4)*x^4 + (x^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*x^2*PolyLog[
2, E^((2*I)*(a + b*x))])/b^2 + (3*x*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3
) + (((3*I)/4)*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cot(a+bx)\,dx &= -\frac{ix^4}{4} - 2i\int \frac{e^{2i(a+bx)}x^3}{1-e^{2i(a+bx)}}\,dx \\
&= -\frac{ix^4}{4} + \frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3\int x^2\log\left(1-e^{2i(a+bx)}\right)\,dx}{b} \\
&= -\frac{ix^4}{4} + \frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3ix^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{(3i)\int x\mathrm{Li}_2\left(e^{2i(a+bx)}\right)\,dx}{b^2} \\
&= -\frac{ix^4}{4} + \frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3ix^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3x\mathrm{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3} - \frac{3\int \mathrm{Li}_3\left(e^{2i(a+bx)}\right)\,dx}{2b^3} \\
&= -\frac{ix^4}{4} + \frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3ix^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3x\mathrm{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{\mathrm{Li}_3(x)}{x}\right.}{4b^4} \\
&= -\frac{ix^4}{4} + \frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3ix^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3x\mathrm{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3} + \frac{3i\mathrm{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}
\end{aligned}$$

Mathematica [A] time = 0.71, size = 184, normalized size = 1.82

$$\frac{4b^3x^3\log\left(1-e^{-i(a+bx)}\right) + 4b^3x^3\log\left(1+e^{-i(a+bx)}\right) + 12ib^2x^2\mathrm{Li}_2\left(-e^{-i(a+bx)}\right) + 12ib^2x^2\mathrm{Li}_2\left(e^{-i(a+bx)}\right) + 24bx\mathrm{Li}_3\left(-\right.}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cot[a + b*x],x]
```

```
[Out] (I*b^4*x^4 + 4*b^3*x^3*Log[1 - E^((-I)*(a + b*x))] + 4*b^3*x^3*Log[1 + E^((
-I)*(a + b*x))] + (12*I)*b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] + (12*I)*b
^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] + 24*b*x*PolyLog[3, -E^((-I)*(a + b*x
))] + 24*b*x*PolyLog[3, E^((-I)*(a + b*x))] - (24*I)*PolyLog[4, -E^((-I)*(a
 + b*x))] - (24*I)*PolyLog[4, E^((-I)*(a + b*x))])/(4*b^4)
```

fricas [C] time = 0.56, size = 306, normalized size = 3.03

$$\frac{-6i\,b^2x^2\mathrm{Li}_2\left(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)\right)+6i\,b^2x^2\mathrm{Li}_2\left(\cos\left(2\,bx+2\,a\right)-i\,\sin\left(2\,bx+2\,a\right)\right)-4\,a^3\log\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(-6*I*b^2*x^2*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 6*I*b^2*x^
2*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) - 4*a^3*log(-1/2*cos(2*b*x +
 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) - 4*a^3*log(-1/2*cos(2*b*x + 2*a) - 1
/2*I*sin(2*b*x + 2*a) + 1/2) + 6*b*x*polylog(3, cos(2*b*x + 2*a) + I*sin(2*
b*x + 2*a)) + 6*b*x*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(
b^3*x^3 + a^3)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 4*(b^3*x^3
 + a^3)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 3*I*polylog(4, co
s(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 3*I*polylog(4, cos(2*b*x + 2*a) - I*
sin(2*b*x + 2*a)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cot\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cot(b*x + a), x)
```

maple [B] time = 0.86, size = 240, normalized size = 2.38

$$-\frac{ix^4}{4}-\frac{3i\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right)x^2}{b^2}-\frac{3i\,\mathrm{polylog}\left(2,\mathrm{e}^{i(bx+a)}\right)x^2}{b^2}-\frac{2ia^3x}{b^3}+\frac{\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a^3}{b^4}+\frac{\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x^3}{b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(b*x+a),x)
```

```
[Out] -1/4*I*x^4-3*I/b^2*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*polylog(2,exp(I*(
b*x+a)))*x^2-2*I/b^3*a^3*x+1/b^4*ln(1-exp(I*(b*x+a)))*a^3+1/b*ln(1-exp(I*(b
*x+a)))*x^3+1/b*ln(exp(I*(b*x+a))+1)*x^3+6*I/b^4*polylog(4,-exp(I*(b*x+a)))
+6*I/b^4*polylog(4,exp(I*(b*x+a)))-3/2*I/b^4*a^4+2/b^4*a^3*ln(exp(I*(b*x+a)
))-1/b^4*a^3*ln(exp(I*(b*x+a))-1)+6/b^3*polylog(3,exp(I*(b*x+a)))*x+6/b^3*p
olylog(3,-exp(I*(b*x+a)))*x
```

maxima [B] time = 0.48, size = 391, normalized size = 3.87

$$-\frac{i\left(bx+a\right)^4-4i\left(bx+a\right)^3a+6i\left(bx+a\right)^2a^2+4\,a^3\log\left(\sin\left(bx+a\right)\right)-24\,bx\mathrm{Li}_3(-e^{(i\,bx+i\,a)})-24\,bx\mathrm{Li}_3(e^{(i\,bx+i\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*(I*(b*x + a)^4 - 4*I*(b*x + a)^3*a + 6*I*(b*x + a)^2*a^2 + 4*a^3*log(s
in(b*x + a)) - 24*b*x*polylog(3, -e^(I*b*x + I*a)) - 24*b*x*polylog(3, e^(I
*b*x + I*a)) - (4*I*(b*x + a)^3 - 12*I*(b*x + a)^2*a + 12*I*(b*x + a)*a^2)*
arctan2(sin(b*x + a), cos(b*x + a) + 1) - (-4*I*(b*x + a)^3 + 12*I*(b*x + a
)^2*a - 12*I*(b*x + a)*a^2)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (-12
*I*(b*x + a)^2 + 24*I*(b*x + a)*a - 12*I*a^2)*dilog(-e^(I*b*x + I*a)) - (-1
2*I*(b*x + a)^2 + 24*I*(b*x + a)*a - 12*I*a^2)*dilog(e^(I*b*x + I*a)) - 2*(
```

```
(b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*log(cos(b*x + a)^2 + sin(b
*x + a)^2 + 2*cos(b*x + a) + 1) - 2*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x
 + a)*a^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 24*I
*polylog(4, -e^(I*b*x + I*a)) - 24*I*polylog(4, e^(I*b*x + I*a)))/b^4
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \cot(a + b\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(a + b*x),x)
```

```
[Out] int(x^3*cot(a + b*x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cot(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cot(b*x+a),x)
```

```
[Out] Integral(x**3*cot(a + b*x), x)
```

3.2 $\int x^2 \cot(a+bx)\,dx$

Optimal. Leaf size=74

$$\frac{\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3} - \frac{ix\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{ix^3}{3}$$

```
[Out] -1/3*I*x^3+x^2*ln(1-exp(2*I*(b*x+a)))/b-I*x*polylog(2,exp(2*I*(b*x+a)))/b^2
+1/2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

Rubi [A] time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3717, 2190, 2531, 2282, 6589}

$$-\frac{ix\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} + \frac{\text{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} + \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{ix^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cot[a + b*x],x]
```

```
[Out] (-I/3)*x^3 + (x^2*Log[1 - E^((2*I)*(a + b*x))])/b - (I*x*PolyLog[2, E^((2*I
)*(a + b*x))])/b^2 + PolyLog[3, E^((2*I)*(a + b*x))]/(2*b^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x^2 \cot(a+bx)\,dx &= -\frac{ix^3}{3} - 2i\int \frac{e^{2i(a+bx)}x^2}{1-e^{2i(a+bx)}}\,dx \\ &= -\frac{ix^3}{3} + \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2\int x\log\left(1-e^{2i(a+bx)}\right)\,dx}{b} \\ &= -\frac{ix^3}{3} + \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{ix\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{i\int \mathrm{Li}_2\left(e^{2i(a+bx)}\right)\,dx}{b^2} \\ &= -\frac{ix^3}{3} + \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{ix\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2(x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^3} \\ &= -\frac{ix^3}{3} + \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{ix\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{\mathrm{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.46, size = 136, normalized size = 1.84

$$\frac{3b^2x^2\log\left(1-e^{-i(a+bx)}\right) + 3b^2x^2\log\left(1+e^{-i(a+bx)}\right) + 6ibx\mathrm{Li}_2\left(-e^{-i(a+bx)}\right) + 6ibx\mathrm{Li}_2\left(e^{-i(a+bx)}\right) + 6\mathrm{Li}_3\left(-e^{-i(a+bx)}\right) +}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cot[a + b*x],x]
```

```
[Out] (I*b^3*x^3 + 3*b^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 3*b^2*x^2*Log[1 + E^((
-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b*x*Pol
yLog[2, E^((-I)*(a + b*x))] + 6*PolyLog[3, -E^((-I)*(a + b*x))] + 6*PolyLog
[3, E^((-I)*(a + b*x))])/(3*b^3)
```

fricas [C] time = 0.57, size = 244, normalized size = 3.30

$$\frac{-2i\,bx\mathrm{Li}_2\left(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)\right)+2i\,bx\mathrm{Li}_2\left(\cos\left(2\,bx+2\,a\right)-i\,\sin\left(2\,bx+2\,a\right)\right)+2\,a^2\log\left(-\frac{1}{2}\,\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(-2*I*b*x*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 2*I*b*x*dilog(
cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*a^2*log(-1/2*cos(2*b*x + 2*a) +
1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*a^2*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin
(2*b*x + 2*a) + 1/2) + 2*(b^2*x^2 - a^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*
x + 2*a) + 1) + 2*(b^2*x^2 - a^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a
) + 1) + polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + polylog(3, cos
(2*b*x + 2*a) - I*sin(2*b*x + 2*a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cot\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cot(b*x + a), x)
```

maple [B] time = 0.80, size = 198, normalized size = 2.68

$$-\frac{ix^3}{3}+\frac{2ia^2x}{b^2}-\frac{\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a^2}{b^3}+\frac{4ia^3}{3b^3}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x^2}{b}-\frac{2i\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right)x}{b^2}+\frac{2\,\mathrm{polylog}\left(3,-\mathrm{e}^{i(bx+a)}\right)}{b^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(b*x+a),x)
```

```
[Out] -1/3*I*x^3+2*I/b^2*a^2*x-1/b^3*ln(1-exp(I*(b*x+a)))*a^2+4/3*I/b^3*a^3+1/b*l
n(exp(I*(b*x+a))+1)*x^2-2*I/b^2*polylog(2,-exp(I*(b*x+a)))*x+2/b^3*polylog(
3,-exp(I*(b*x+a)))+1/b*ln(1-exp(I*(b*x+a)))*x^2-2*I/b^2*polylog(2,exp(I*(b*
x+a)))*x+2/b^3*polylog(3,exp(I*(b*x+a)))+1/b^3*a^2*ln(exp(I*(b*x+a))-1)-2/b
^3*a^2*ln(exp(I*(b*x+a)))
```

maxima [B] time = 0.49, size = 257, normalized size = 3.47

$$-\frac{2i\,(bx+a)^3 - 6i\,(bx+a)^2 a + 12i\,bx\mathrm{Li}_2\left(-e^{(i\,bx+i\,a)}\right) + 12i\,bx\mathrm{Li}_2\left(e^{(i\,bx+i\,a)}\right) - 6\,a^2\log\left(\sin\left(bx+a\right)\right) - \left(6i\,(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(2*I*(b*x + a)^3 - 6*I*(b*x + a)^2*a + 12*I*b*x*dilog(-e^(I*b*x + I*a)
) + 12*I*b*x*dilog(e^(I*b*x + I*a)) - 6*a^2*log(sin(b*x + a)) - (6*I*(b*x +
 a)^2 - 12*I*(b*x + a)*a)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (-6*I*(
b*x + a)^2 + 12*I*(b*x + a)*a)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 3
*((b*x + a)^2 - 2*(b*x + a)*a)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(
b*x + a) + 1) - 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*cos(b*x + a) + 1) - 12*polylog(3, -e^(I*b*x + I*a)) - 12*polyl
og(3, e^(I*b*x + I*a)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cot\left(a + b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(a + b*x),x)
```

```
[Out] int(x^2*cot(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cot\left(a + bx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cot(b*x+a),x)
```

```
[Out] Integral(x**2*cot(a + b*x), x)
```

3.3 $\int x \cot(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{i\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{x\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{ix^2}{2}$$

```
[Out] -1/2*I*x^2+x*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3717, 2190, 2279, 2391}

$$-\frac{i\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{x\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[a + b*x],x]
```

```
[Out] (-I/2)*x^2 + (x*Log[1 - E^((2*I)*(a + b*x))])/b - ((I/2)*PolyLog[2, E^((2*I
)*(a + b*x))])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cot(a+bx)\,dx &= -\frac{ix^2}{2} - 2i\int \frac{e^{2i(a+bx)}x}{1-e^{2i(a+bx)}}\,dx \\ &= -\frac{ix^2}{2} + \frac{x\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{\int \log\left(1-e^{2i(a+bx)}\right)\,dx}{b} \\ &= -\frac{ix^2}{2} + \frac{x\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^2} \\ &= -\frac{ix^2}{2} + \frac{x\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{i\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}\end{aligned}$$

Mathematica [B] time = 3.73, size = 135, normalized size = 2.55

$$\frac{1}{2}\left(-\frac{i\text{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right) - ibx\left(\pi - 2\tan^{-1}(\tan(a))\right) - 2\left(\tan^{-1}(\tan(a)) + bx\right)\log\left(1-e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right) -}{b^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Cot[a + b*x],x]
```

```
[Out] (x^2*Cot[a] - ((-I)*b*x*(Pi - 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)
] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi
*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLo
g[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b^2 - E^(I*ArcTan[Tan[a]])*x^2*Cot[
a]*Sqrt[Sec[a]^2])/2
```

fricas [B] time = 0.53, size = 174, normalized size = 3.28

$$-\frac{2\,a\log\left(-\frac{1}{2}\cos\left(2\,bx+2\,a\right)+\frac{1}{2}i\sin\left(2\,bx+2\,a\right)+\frac{1}{2}\right)+2\,a\log\left(-\frac{1}{2}\cos\left(2\,bx+2\,a\right)-\frac{1}{2}i\sin\left(2\,bx+2\,a\right)+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*a*l
og(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) - 2*(b*x + a)*log(
-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-cos(2*b*x +
2*a) - I*sin(2*b*x + 2*a) + 1) + I*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2
*a)) - I*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cot\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cot(b*x + a), x)
```

maple [B] time = 0.82, size = 150, normalized size = 2.83

$$-\frac{ix^2}{2} - \frac{2iax}{b} - \frac{ia^2}{b^2} + \frac{\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x}{b} - \frac{i\,\text{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right)}{b^2} + \frac{\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x}{b} + \frac{\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a}{b^2} - \frac{i\,\text{polylog}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(b*x+a),x)
```

```
[Out] -1/2*I*x^2-2*I/b*a*x-I/b^2*a^2+1/b*ln(exp(I*(b*x+a))+1)*x-I/b^2*polylog(2,-
exp(I*(b*x+a)))+1/b*ln(1-exp(I*(b*x+a)))*x+1/b^2*ln(1-exp(I*(b*x+a)))*a-I/b
^2*polylog(2,exp(I*(b*x+a)))+2/b^2*a*ln(exp(I*(b*x+a)))-1/b^2*a*ln(exp(I*(b
*x+a))-1)
```

maxima [B] time = 0.46, size = 140, normalized size = 2.64

$$\frac{-i\,b^2x^2 + 2i\,bx\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)+1\right) - 2i\,bx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right) + bx\log\left(\text{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*b^2*x^2 + 2*I*b*x*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*I*b*x
*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + b*x*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) + b*x*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
2*cos(b*x + a) + 1) - 2*I*dilog(-e^(I*b*x + I*a)) - 2*I*dilog(e^(I*b*x + I*
a)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\cot\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(a + b*x),x)
```

```
[Out] int(x*cot(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cot\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a),x)
```

```
[Out] Integral(x*cot(a + b*x), x)
```

3.4 $\int \frac{\cot(a+bx)}{x}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\cot(a+bx)}{x}, x\right)$$

```
[Out] Unintegrable(cot(b*x+a)/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]/x,x]
```

```
[Out] Defer[Int][Cot[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)}{x}\,dx = \int \frac{\cot(a+bx)}{x}\,dx$$

Mathematica [A] time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]/x,x]
```

```
[Out] Integrate[Cot[a + b*x]/x, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)/x, x)
```

maple [A] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)/x,x)
[Out] int(cot(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(cot(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\cot(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)/x,x)
[Out] int(cot(a + b*x)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x,x)
[Out] Integral(cot(a + b*x)/x, x)
```

3.5 $\int \frac{\cot(a+bx)}{x^2}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\cot(a+bx)}{x^2}, x\right)$$

```
[Out] Unintegrable(cot(b*x+a)/x^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)}{x^2}\,dx = \int \frac{\cot(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 3.32, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]/x^2,x]
```

```
[Out] Integrate[Cot[a + b*x]/x^2, x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)/x^2, x)
```

maple [A] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)/x^2,x)
[Out] int(cot(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x^2,x, algorithm="maxima")
[Out] integrate(cot(b*x + a)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\cot(a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)/x^2,x)
[Out] int(cot(a + b*x)/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)/x**2,x)
[Out] Integral(cot(a + b*x)/x**2, x)
```

3.6 $\int x^3 \cot^2(a+bx)\,dx$

Optimal. Leaf size=97

$$\frac{3\mathrm{Li}_3\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3ix\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{x^3\cot(a+bx)}{b}-\frac{ix^3}{b}-\frac{x^4}{4}$$

```
[Out] -I*x^3/b-1/4*x^4-x^3*cot(b*x+a)/b+3*x^2*ln(1-exp(2*I*(b*x+a)))/b^2-3*I*x*po
lylog(2,exp(2*I*(b*x+a)))/b^3+3/2*polylog(3,exp(2*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.18, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3720, 3717, 2190, 2531, 2282, 6589, 30}

$$-\frac{3ix\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{3\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}+\frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{x^3\cot(a+bx)}{b}-\frac{ix^3}{b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*x^3)/b - x^4/4 - (x^3*Cot[a + b*x])/b + (3*x^2*Log[1 - E^((2*I)*(a +
b*x))])/b^2 - ((3*I)*x*PolyLog[2, E^((2*I)*(a + b*x))])/b^3 + (3*PolyLog[3,
 E^((2*I)*(a + b*x))])/(2*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cot^2(a+bx)\,dx &= -\frac{x^3\cot(a+bx)}{b} + \frac{3\int x^2\cot(a+bx)\,dx}{b} - \int x^3\,dx \\
&= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3\cot(a+bx)}{b} - \frac{(6i)\int \frac{e^{2i(a+bx)}x^2}{1-e^{2i(a+bx)}}\,dx}{b} \\
&= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3\cot(a+bx)}{b} + \frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{6\int x\log\left(1-e^{2i(a+bx)}\right)\,dx}{b^2} \\
&= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3\cot(a+bx)}{b} + \frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3ix\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^3} + \frac{(3i)\int \mathrm{Li}_2\left(e^{2i(a} \\
&= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3\cot(a+bx)}{b} + \frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3ix\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^3} + \frac{3\,\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2(} \\
&= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3\cot(a+bx)}{b} + \frac{3x^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3ix\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^3} + \frac{3\mathrm{Li}_3\left(e^{2i(a+bx)}\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.92, size = 171, normalized size = 1.76

$$\frac{-\frac{2ib^3x^3}{-1+e^{2ia}} + 3b^2x^2\log\left(1-e^{-i(a+bx)}\right) + 3b^2x^2\log\left(1+e^{-i(a+bx)}\right) + 6ibx\mathrm{Li}_2\left(-e^{-i(a+bx)}\right) + 6ibx\mathrm{Li}_2\left(e^{-i(a+bx)}\right) + 6\mathrm{Li}_3\left(-\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cot[a + b*x]^2,x]
```

```
[Out] -1/4*x^4 + (((-2*I)*b^3*x^3)/(-1 + E^((2*I)*a)) + 3*b^2*x^2*Log[1 - E^((-I)
*(a + b*x))] + 3*b^2*x^2*Log[1 + E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2,
 -E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2, E^((-I)*(a + b*x))] + 6*PolyLo
g[3, -E^((-I)*(a + b*x))] + 6*PolyLog[3, E^((-I)*(a + b*x))])/b^4 + (x^3*Cs
c[a]*Csc[a + b*x]*Sin[b*x])/b
```

fricas [C] time = 0.69, size = 372, normalized size = 3.84

$$-\frac{b^4x^4\sin(2bx+2a) + 4b^3x^3\cos(2bx+2a) + 4b^3x^3 + 6ibx\mathrm{Li}_2\left(\cos(2bx+2a) + i\sin(2bx+2a)\right)\sin(2bx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4*sin(2*b*x + 2*a) + 4*b^3*x^3*cos(2*b*x + 2*a) + 4*b^3*x^3 + 6
*I*b*x*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 6*I*
```

```
b*x*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 6*a^2*l
og(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) -
 6*a^2*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x
+ 2*a) - 6*(b^2*x^2 - a^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*
sin(2*b*x + 2*a) - 6*(b^2*x^2 - a^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x +
2*a) + 1)*sin(2*b*x + 2*a) - 3*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x +
2*a))*sin(2*b*x + 2*a) - 3*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)
)*sin(2*b*x + 2*a))/(b^4*sin(2*b*x + 2*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cot(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cot(b*x + a)^2, x)
```

maple [B] time = 0.95, size = 231, normalized size = 2.38

$$-\frac{x^4}{4}-\frac{2ix^3}{b\left(e^{2i(bx+a)}-1\right)}+\frac{6ia^2x}{b^3}+\frac{3\ln\left(e^{i(bx+a)}+1\right)x^2}{b^2}-\frac{6i\,\mathrm{polylog}\left(2,-e^{i(bx+a)}\right)x}{b^3}+\frac{6\,\mathrm{polylog}\left(3,-e^{i(bx+a)}\right)}{b^4}+\frac{3\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(b*x+a)^2,x)
```

```
[Out] -1/4*x^4-2*I*x^3/b/(exp(2*I*(b*x+a))-1)+6*I/b^3*a^2*x+3/b^2*ln(exp(I*(b*x+a
))+1)*x^2-6*I/b^3*polylog(2,-exp(I*(b*x+a)))*x+6/b^4*polylog(3,-exp(I*(b*x+
a)))+3/b^2*ln(1-exp(I*(b*x+a)))*x^2-3/b^4*ln(1-exp(I*(b*x+a)))*a^2-6*I/b^3*
polylog(2,exp(I*(b*x+a)))*x+6/b^4*polylog(3,exp(I*(b*x+a)))+3/b^4*a^2*ln(ex
p(I*(b*x+a))-1)-6/b^4*a^2*ln(exp(I*(b*x+a)))-2*I/b*x^3+4*I/b^4*a^3
```

maxima [B] time = 0.59, size = 953, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a + 1/tan(b*x + a))*a^3 - 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 +
 (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a
)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(
b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2/(cos(2
*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1) + 2*(-I*(b*x +
 a)^4 + 4*I*(b*x + a)^3*a - (12*(b*x + a)^2 - 24*(b*x + a)*a - 12*((b*x + a
)^2 - 2*(b*x + a)*a)*cos(2*b*x + 2*a) - (12*I*(b*x + a)^2 - 24*I*(b*x + a)*
a)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (12*(b*x + a
)^2 - 24*(b*x + a)*a - 12*((b*x + a)^2 - 2*(b*x + a)*a)*cos(2*b*x + 2*a) +
(-12*I*(b*x + a)^2 + 24*I*(b*x + a)*a)*sin(2*b*x + 2*a))*arctan2(sin(b*x +
a), -cos(b*x + a) + 1) + (I*(b*x + a)^4 - 4*(b*x + a)^3*(I*a + 2) + 24*(b*x
 + a)^2*a)*cos(2*b*x + 2*a) - 24*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*b*x +
2*a) - b*x)*dilog(-e^(I*b*x + I*a)) - 24*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(
2*b*x + 2*a) - b*x)*dilog(e^(I*b*x + I*a)) + (6*I*(b*x + a)^2 - 12*I*(b*x +
 a)*a + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*cos(2*b*x + 2*a) + 6*((b*x +
a)^2 - 2*(b*x + a)*a)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*cos(b*x + a) + 1) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a + (-6*I*(b*x +
```

```
 a)^2 + 12*I*(b*x + a)*a)*cos(2*b*x + 2*a) + 6*((b*x + a)^2 - 2*(b*x + a)*a
)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) +
1) + (-24*I*cos(2*b*x + 2*a) + 24*sin(2*b*x + 2*a) + 24*I)*polylog(3, -e^(I
*b*x + I*a)) + (-24*I*cos(2*b*x + 2*a) + 24*sin(2*b*x + 2*a) + 24*I)*polylo
g(3, e^(I*b*x + I*a)) - ((b*x + a)^4 - (b*x + a)^3*(4*a - 8*I) - 24*I*(b*x
+ a)^2*a)*sin(2*b*x + 2*a))/(-4*I*cos(2*b*x + 2*a) + 4*sin(2*b*x + 2*a) + 4
*I))/b^4
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \cot(a + b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(a + b*x)^2,x)
```

```
[Out] int(x^3*cot(a + b*x)^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cot^2(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cot(b*x+a)**2,x)
```

```
[Out] Integral(x**3*cot(a + b*x)**2, x)
```

3.7 $\int x^2 \cot^2(a+bx)\,dx$

Optimal. Leaf size=74

$$-\frac{i\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{2x\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{x^2\cot(a+bx)}{b}-\frac{ix^2}{b}-\frac{x^3}{3}$$

```
[Out] -I*x^2/b-1/3*x^3-x^2*cot(b*x+a)/b+2*x*ln(1-exp(2*I*(b*x+a)))/b^2-I*polylog(
2,exp(2*I*(b*x+a)))/b^3
```

Rubi [A] time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3720, 3717, 2190, 2279, 2391, 30}

$$-\frac{i\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{2x\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{x^2\cot(a+bx)}{b}-\frac{ix^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*x^2)/b - x^3/3 - (x^2*Cot[a + b*x])/b + (2*x*Log[1 - E^((2*I)*(a + b*
x))])/b^2 - (I*PolyLog[2, E^((2*I)*(a + b*x))])/b^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
```

```
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \cot^2(a+bx)\,dx &= -\frac{x^2\cot(a+bx)}{b} + \frac{2\int x\cot(a+bx)\,dx}{b} - \int x^2\,dx \\ &= -\frac{ix^2}{b} - \frac{x^3}{3} - \frac{x^2\cot(a+bx)}{b} - \frac{(4i)\int \frac{e^{2i(a+bx)}x}{1-e^{2i(a+bx)}}\,dx}{b} \\ &= -\frac{ix^2}{b} - \frac{x^3}{3} - \frac{x^2\cot(a+bx)}{b} + \frac{2x\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{2\int \log\left(1-e^{2i(a+bx)}\right)\,dx}{b^2} \\ &= -\frac{ix^2}{b} - \frac{x^3}{3} - \frac{x^2\cot(a+bx)}{b} + \frac{2x\log\left(1-e^{2i(a+bx)}\right)}{b^2} + \frac{i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{b^3} \\ &= -\frac{ix^2}{b} - \frac{x^3}{3} - \frac{x^2\cot(a+bx)}{b} + \frac{2x\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{i\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}\end{aligned}$$

Mathematica [B] time = 5.35, size = 153, normalized size = 2.07

$$\frac{-b^2x^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)} - i\text{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right) + ibx\left(\pi - 2\tan^{-1}(\tan(a))\right) + 2\left(\tan^{-1}(\tan(a)) + bx\right.}{i}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Cot[a + b*x]^2,x]
```

```
[Out] -1/3*x^3 + (I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi*Log[1 + E^((-2*I)*b*x)] + 2*
(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - Pi*Log[C
os[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] - I*PolyLog[2, E
^((2*I)*(b*x + ArcTan[Tan[a]]))] - b^2*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt
[Sec[a]^2])/b^3 + (x^2*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

fricas [B] time = 1.16, size = 281, normalized size = 3.80

$$-\frac{2\,b^3x^3\sin(2\,bx+2\,a) + 6\,b^2x^2\cos(2\,bx+2\,a) + 6\,b^2x^2 + 6\,a\log\left(-\frac{1}{2}\cos(2\,bx+2\,a) + \frac{1}{2}i\sin(2\,bx+2\,a) + \frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*x^3*sin(2*b*x + 2*a) + 6*b^2*x^2*cos(2*b*x + 2*a) + 6*b^2*x^2 +
 6*a*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x +
2*a) + 6*a*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*
b*x + 2*a) - 6*(b*x + a)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*si
n(2*b*x + 2*a) - 6*(b*x + a)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1
)*sin(2*b*x + 2*a) + 3*I*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2
*b*x + 2*a) - 3*I*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x +
2*a))/(b^3*sin(2*b*x + 2*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cot\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cot(b*x + a)^2, x)
```

maple [B] time = 0.90, size = 183, normalized size = 2.47

$$-\frac{x^3}{3}-\frac{2ix^2}{b\left(e^{2i(bx+a)}-1\right)}-\frac{2ix^2}{b}-\frac{4iax}{b^2}-\frac{2ia^2}{b^3}+\frac{2\ln\left(e^{i(bx+a)}+1\right)x}{b^2}-\frac{2i\,\mathrm{polylog}\left(2,-e^{i(bx+a)}\right)}{b^3}+\frac{2\ln\left(1-e^{i(bx+a)}\right)x}{b^2}+\frac{2}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(b*x+a)^2,x)
```

```
[Out] -1/3*x^3-2*I*x^2/b/(exp(2*I*(b*x+a))-1)-2*I/b*x^2-4*I/b^2*a*x-2*I/b^3*a^2+2
/b^2*ln(exp(I*(b*x+a))+1)*x-2*I/b^3*polylog(2,-exp(I*(b*x+a)))+2/b^2*ln(1-e
xp(I*(b*x+a)))*x+2/b^3*ln(1-exp(I*(b*x+a)))*a-2*I/b^3*polylog(2,exp(I*(b*x+
a)))+4/b^3*a*ln(exp(I*(b*x+a)))-2/b^3*a*ln(exp(I*(b*x+a))-1)
```

maxima [B] time = 0.62, size = 384, normalized size = 5.19

$$\frac{-i\,b^3x^3+6\,(bx\cos(2\,bx+2\,a)+i\,bx\sin(2\,bx+2\,a)-bx)\arctan(\sin(bx+a),\cos(bx+a)+1)-6\,(bx\cos(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-I*b^3*x^3 + 6*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*b*x + 2*a) - b*x)*arcta
n2(sin(b*x + a), cos(b*x + a) + 1) - 6*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*
b*x + 2*a) - b*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*b^3*x^3 - 6
*b^2*x^2)*cos(2*b*x + 2*a) - 6*(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) - 1)*
dilog(-e^(I*b*x + I*a)) - 6*(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) - 1)*dil
og(e^(I*b*x + I*a)) + (-3*I*b*x*cos(2*b*x + 2*a) + 3*b*x*sin(2*b*x + 2*a) +
 3*I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (-3*I
*b*x*cos(2*b*x + 2*a) + 3*b*x*sin(2*b*x + 2*a) + 3*I*b*x)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (b^3*x^3 + 6*I*b^2*x^2)*sin(2*b*
x + 2*a))/(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) + 3*I*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\cot(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(a + b*x)^2,x)
```

```
[Out] int(x^2*cot(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cot(b*x+a)**2,x)
```

```
[Out] Integral(x**2*cot(a + b*x)**2, x)
```

3.8 $\int x \cot^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\log(\sin(a+bx))}{b^2} - \frac{x\cot(a+bx)}{b} - \frac{x^2}{2}$$

```
[Out] -1/2*x^2-x*cot(b*x+a)/b+ln(sin(b*x+a))/b^2
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3720, 3475, 30}

$$\frac{\log(\sin(a+bx))}{b^2} - \frac{x\cot(a+bx)}{b} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[a + b*x]^2,x]
```

```
[Out] -x^2/2 - (x*Cot[a + b*x])/b + Log[Sin[a + b*x]]/b^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cot^2(a+bx)\,dx &= -\frac{x\cot(a+bx)}{b} + \frac{\int \cot(a+bx)\,dx}{b} - \int x\,dx \\ &= -\frac{x^2}{2} - \frac{x\cot(a+bx)}{b} + \frac{\log(\sin(a+bx))}{b^2}\end{aligned}$$

Mathematica [A] time = 0.22, size = 44, normalized size = 1.42

$$\frac{\log(\sin(a+bx))}{b^2} - \frac{x\cot(a)}{b} + \frac{x\csc(a)\sin(bx)\csc(a+bx)}{b} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cot[a + b*x]^2,x]
```

```
[Out] -1/2*x^2 - (x*Cot[a])/b + Log[Sin[a + b*x]]/b^2 + (x*Csc[a]*Csc[a + b*x]*Si
n[b*x])/b
```

fricas [B] time = 0.74, size = 75, normalized size = 2.42

$$-\frac{b^2x^2\sin(2bx+2a)+2bx\cos(2bx+2a)+2bx-\log\left(-\frac{1}{2}\cos(2bx+2a)+\frac{1}{2}\right)\sin(2bx+2a)}{2b^2\sin(2bx+2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2*sin(2*b*x + 2*a) + 2*b*x*cos(2*b*x + 2*a) + 2*b*x - log(-1/2*
cos(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a))/(b^2*sin(2*b*x + 2*a))
```

giac [B] time = 4.74, size = 1250, normalized size = 40.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*x^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*x^2*tan(1/2*b*x)*tan(1/2*a)^2
 - b*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*x^2*tan(1/2*b*x) - b^2*x^2*tan(1/2
*a) + b*x*tan(1/2*b*x)^2 + 4*b*x*tan(1/2*b*x)*tan(1/2*a) - log(16*(tan(1/2*
b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/
2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6
 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b
*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan
(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan
(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan
(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + b*x*tan(1/2*a)
^2 - log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 +
tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^
6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a
)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1
/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2
 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a)
 + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*
a)^2 - b*x + log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2
*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1
/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*t
an(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)
^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)
^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(
1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*ta
n(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)
+ log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan
(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*t
an(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4
 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*
a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(
1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 -
2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) +
tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*a))/(b^2*tan(1/2
*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2*a)^2 - b^2*tan(1/2*b*x) - b^2
*tan(1/2*a))
```

maple [A] time = 0.72, size = 30, normalized size = 0.97

$$-\frac{x^2}{2}-\frac{x\cot(bx+a)}{b}+\frac{\ln(\sin(bx+a))}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(b*x+a)^2,x)
```

```
[Out] -1/2*x^2-x*cot(b*x+a)/b+ln(sin(b*x+a))/b^2
```

maxima [B] time = 0.47, size = 269, normalized size = 8.68

$$2\left(bx+a+\frac{1}{\tan(bx+a)}\right)a-\frac{(bx+a)^2\cos(2\,bx+2\,a)^2+(bx+a)^2\sin(2\,bx+2\,a)^2-2\,(bx+a)^2\cos(2\,bx+2\,a)+(bx+a)^2-\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a + 1/tan(b*x + a))*a - ((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*
x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2
- (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(co
s(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 +
 sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))/(cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1))/b^2
```

mupad [B] time = 0.45, size = 54, normalized size = 1.74

$$\frac{\ln\left(e^{a\,2i}\,e^{b\,x\,2i}-1\right)}{b^2}-\frac{x\,2i}{b}-\frac{x^2}{2}-\frac{x\,2i}{b\left(e^{a\,2i+b\,x\,2i}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(a + b*x)^2,x)
```

```
[Out] log(exp(a*2i)*exp(b*x*2i) - 1)/b^2 - (x*2i)/b - x^2/2 - (x*2i)/(b*(exp(a*2i
 + b*x*2i) - 1))
```

sympy [A] time = 0.46, size = 68, normalized size = 2.19

$$\begin{cases}\tilde{\infty}x^2 & \text{for } (a=0\vee a=-bx)\wedge(a=-bx\vee b=0)\\ \frac{x^2\cot^2(a)}{2} & \text{for } b=0\\ -\frac{x^2}{2}-\frac{x}{b\tan(a+bx)}-\frac{\log\left(\tan^2(a+bx)+1\right)}{2b^2}+\frac{\log(\tan(a+bx))}{b^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**2, (Eq(a, 0) | Eq(a, -b*x)) & (Eq(b, 0) | Eq(a, -b*x))),
(x**2*cot(a)**2/2, Eq(b, 0)), (-x**2/2 - x/(b*tan(a + b*x)) - log(tan(a + b
*x)**2 + 1)/(2*b**2) + log(tan(a + b*x))/b**2, True))
```

3.9 $\int \frac{\cot^2(a+bx)}{x}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cot^2(a+bx)}{x}, x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^2/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][Cot[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{x}\,dx = \int \frac{\cot^2(a+bx)}{x}\,dx$$

Mathematica [A] time = 3.82, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/x,x]
```

```
[Out] Integrate[Cot[a + b*x]^2/x, x]
```

fricas [A] time = 2.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2/x, x)
```

maple [A] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/x,x)
```

```
[Out] int(cot(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bx\cos(2bx+2a)^2\log(x)+bx\log(x)\sin(2bx+2a)^2-2bx\cos(2bx+2a)\log(x)+bx\log(x)-\frac{\left(b^2x\cos(2bx+2a\right.}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -(b*x*cos(2*b*x + 2*a)^2*log(x) + b*x*log(x)*sin(2*b*x + 2*a)^2 - 2*b*x*cos
(2*b*x + 2*a)*log(x) + b*x*log(x) - (b^2*x*cos(2*b*x + 2*a)^2 + b^2*x*sin(2
*b*x + 2*a)^2 - 2*b^2*x*cos(2*b*x + 2*a) + b^2*x)*integrate(sin(b*x + a)/(b
^2*x^2*cos(b*x + a)^2 + b^2*x^2*sin(b*x + a)^2 + 2*b^2*x^2*cos(b*x + a) + b
^2*x^2), x) + (b^2*x*cos(2*b*x + 2*a)^2 + b^2*x*sin(2*b*x + 2*a)^2 - 2*b^2*
x*cos(2*b*x + 2*a) + b^2*x)*integrate(sin(b*x + a)/(b^2*x^2*cos(b*x + a)^2
+ b^2*x^2*sin(b*x + a)^2 - 2*b^2*x^2*cos(b*x + a) + b^2*x^2), x) + 2*sin(2*
b*x + 2*a))/(b*x*cos(2*b*x + 2*a)^2 + b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*
b*x + 2*a) + b*x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cot(a+b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/x,x)
```

```
[Out] int(cot(a + b*x)^2/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/x,x)
```

```
[Out] Integral(cot(a + b*x)**2/x, x)
```

3.10 $\int \frac{\cot^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cot^2(a+bx)}{x^2}, x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^2/x^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{x^2}\,dx = \int \frac{\cot^2(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 3.43, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[Cot[a + b*x]^2/x^2, x]
```

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2/x^2, x)
```

maple [A] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/x^2,x)
```

```
[Out] int(cot(b*x+a)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$bx\cos(2bx+2a)^2 + bx\sin(2bx+2a)^2 - 2bx\cos(2bx+2a) + bx + \frac{2\left(b^2x^2\cos(2bx+2a)^2+b^2x^2\sin(2bx+2a)^2-2b^2x^2\cos(\right.}{bx^2\cos(2bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] (b*x*cos(2*b*x + 2*a)^2 + b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*b*x + 2*a) +
 b*x + 2*(b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x
^2*cos(2*b*x + 2*a) + b^2*x^2)*integrate(sin(b*x + a)/(b^2*x^3*cos(b*x + a)
^2 + b^2*x^3*sin(b*x + a)^2 + 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) - 2*(b^
2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x^2*cos(2*b*x
 + 2*a) + b^2*x^2)*integrate(sin(b*x + a)/(b^2*x^3*cos(b*x + a)^2 + b^2*x^3
*sin(b*x + a)^2 - 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) - 2*sin(2*b*x + 2*a
))/(b*x^2*cos(2*b*x + 2*a)^2 + b*x^2*sin(2*b*x + 2*a)^2 - 2*b*x^2*cos(2*b*x
 + 2*a) + b*x^2)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cot(a+bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/x^2,x)
```

```
[Out] int(cot(a + b*x)^2/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/x**2,x)
```

```
[Out] Integral(cot(a + b*x)**2/x**2, x)
```

3.11 $\int x^3 \cot^3(a+bx)\,dx$

Optimal. Leaf size=202

$$-\frac{3i\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3i\mathrm{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3x\mathrm{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3x\log\left(1-e^{2i(a+bx)}\right)}{b^3}+\frac{3ix^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{3x^2\cot(a+bx}{2b^2}$$

```
[Out] -3/2*I*x^2/b^2-1/2*x^3/b+1/4*I*x^4-3/2*x^2*cot(b*x+a)/b^2-1/2*x^3*cot(b*x+a
)^2/b+3*x*ln(1-exp(2*I*(b*x+a)))/b^3-x^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*pol
ylog(2,exp(2*I*(b*x+a)))/b^4+3/2*I*x^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*
x*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.30, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3720, 3717, 2190, 2279, 2391, 30, 2531, 6609, 2282, 6589}

$$\frac{3ix^2\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{3x\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{3i\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4}-\frac{3i\mathrm{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}-\frac{3x^2\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cot[a + b*x]^3,x]
```

```
[Out] (((-3*I)/2)*x^2)/b^2 - x^3/(2*b) + (I/4)*x^4 - (3*x^2*Cot[a + b*x])/(2*b^2)
 - (x^3*Cot[a + b*x]^2)/(2*b) + (3*x*Log[1 - E^((2*I)*(a + b*x))])/b^3 - (x
^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*PolyLog[2, E^((2*I)*(a + b*
x))])/b^4 + (((3*I)/2)*x^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*x*Poly
Log[3, E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*PolyLog[4, E^((2*I)*(a +
b*x))])/b^4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \cot^3(a+bx)\,dx = -\frac{x^3\cot^2(a+bx)}{2b} + \frac{3\int x^2\cot^2(a+bx)\,dx}{2b} - \int x^3\cot(a+bx)\,dx$$

$$= \frac{ix^4}{4} - \frac{3x^2\cot(a+bx)}{2b^2} - \frac{x^3\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}x^3}{1-e^{2i(a+bx)}}\,dx + \frac{3\int x\cot(a+bx)\,dx}{b^2}$$

$$= -\frac{3ix^2}{2b^2} - \frac{x^3}{2b} + \frac{ix^4}{4} - \frac{3x^2\cot(a+bx)}{2b^2} - \frac{x^3\cot^2(a+bx)}{2b} - \frac{x^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{(6i)\int$$

$$= -\frac{3ix^2}{2b^2} - \frac{x^3}{2b} + \frac{ix^4}{4} - \frac{3x^2\cot(a+bx)}{2b^2} - \frac{x^3\cot^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2i(a+bx)}\right)}{b^3} - \frac{x^3\log$$

$$= -\frac{3ix^2}{2b^2} - \frac{x^3}{2b} + \frac{ix^4}{4} - \frac{3x^2\cot(a+bx)}{2b^2} - \frac{x^3\cot^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2i(a+bx)}\right)}{b^3} - \frac{x^3\log$$

$$= -\frac{3ix^2}{2b^2} - \frac{x^3}{2b} + \frac{ix^4}{4} - \frac{3x^2\cot(a+bx)}{2b^2} - \frac{x^3\cot^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2i(a+bx)}\right)}{b^3} - \frac{x^3\log$$

Mathematica [B] time = 6.81, size = 461, normalized size = 2.28

$$\frac{3x^2\csc(a)\sin(bx)\csc(a+bx)}{2b^2} - \frac{3\csc(a)\sec(a)\left(b^2x^2e^{i\tan^{-1}(\tan(a))} + \frac{\tan(a)\left(i\mathrm{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)+ibx\left(2\tan^{-1}(\tan(a))-\pi\right)\right.}{2b^4\sqrt{s}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Cot[a + b*x]^3,x]
```

```
[Out] -1/4*(x^4*Cot[a]) - (x^3*Csc[a + b*x]^2)/(2*b) + (E^(I*a)*Csc[a]*((b^4*x^4)
/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))]
 + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 +
E^((2*I)*a))*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3
, -E^((-I)*(a + b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) -
(6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*P
olyLog[3, E^((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)
*a)))/(4*b^4) + (3*x^2*Csc[a]*Csc[a + b*x]*Sin[b*x])/(2*b^2) - (3*Csc[a]*Se
c[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*
Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x +
ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTa
n[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[
1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

fricas [C] time = 2.05, size = 565, normalized size = 2.80

$$\frac{8\,b^3x^3 + 12\,b^2x^2\sin(2\,bx+2\,a) + \left(-6i\,b^2x^2 + \left(6i\,b^2x^2 - 6i\right)\cos(2\,bx+2\,a) + 6i\right)\mathrm{Li}_2\left(\cos(2\,bx+2\,a) + i\,\sin(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^3*x^3 + 12*b^2*x^2*sin(2*b*x + 2*a) + (-6*I*b^2*x^2 + (6*I*b^2*x^2
 - 6*I)*cos(2*b*x + 2*a) + 6*I)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)
) + (6*I*b^2*x^2 + (-6*I*b^2*x^2 + 6*I)*cos(2*b*x + 2*a) - 6*I)*dilog(cos(2
```

```
*b*x + 2*a) - I*sin(2*b*x + 2*a)) - 4*(a^3 - (a^3 - 3*a)*cos(2*b*x + 2*a) -
 3*a)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) - 4*(a^3 -
(a^3 - 3*a)*cos(2*b*x + 2*a) - 3*a)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2
*b*x + 2*a) + 1/2) + 4*(b^3*x^3 + a^3 - 3*b*x - (b^3*x^3 + a^3 - 3*b*x - 3*
a)*cos(2*b*x + 2*a) - 3*a)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)
+ 4*(b^3*x^3 + a^3 - 3*b*x - (b^3*x^3 + a^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a)
 - 3*a)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + (-3*I*cos(2*b*x +
 2*a) + 3*I)*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (3*I*cos(2
*b*x + 2*a) - 3*I)*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) - 6*(b
*x*cos(2*b*x + 2*a) - b*x)*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)
) - 6*(b*x*cos(2*b*x + 2*a) - b*x)*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*
x + 2*a)))/(b^4*cos(2*b*x + 2*a) - b^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cot(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cot(b*x + a)^3, x)
```

maple [B] time = 1.06, size = 444, normalized size = 2.20

$$\frac{3i\,\mathrm{polylog}\left(2, e^{i(bx+a)}\right)x^2}{b^2}+\frac{x^2\left(2bx\,e^{2i(bx+a)}-3ie^{2i(bx+a)}+3i\right)}{b^2\left(e^{2i(bx+a)}-1\right)^2}+\frac{2ia^3x}{b^3}+\frac{3i\,\mathrm{polylog}\left(2,-e^{i(bx+a)}\right)x^2}{b^2}-\frac{6iax}{b^3}-\frac{3ix^2}{b^2}+\frac{3}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(b*x+a)^3,x)
```

```
[Out] -3*I/b^2*x^2-6*I/b^3*a*x+3*I/b^2*polylog(2,-exp(I*(b*x+a)))*x^2+3*I/b^2*pol
ylog(2,exp(I*(b*x+a)))*x^2+2*I/b^3*a^3*x+1/4*I*x^4-1/b*ln(1-exp(I*(b*x+a)))
*x^3-1/b*ln(exp(I*(b*x+a))+1)*x^3+3/b^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*ln(1-e
xp(I*(b*x+a)))*x-6/b^3*polylog(3,exp(I*(b*x+a)))*x-6/b^3*polylog(3,-exp(I*(
b*x+a)))*x+6/b^4*a*ln(exp(I*(b*x+a)))-1/b^4*ln(1-exp(I*(b*x+a)))*a^3+x^2*(2
*b*x*exp(2*I*(b*x+a))-3*I*exp(2*I*(b*x+a))+3*I)/b^2/(exp(2*I*(b*x+a))-1)^2-
3/b^4*a*ln(exp(I*(b*x+a))-1)-2/b^4*a^3*ln(exp(I*(b*x+a)))+3/b^4*a*ln(1-exp(
I*(b*x+a)))-3*I/b^4*a^2+3/2*I/b^4*a^4-3*I/b^4*polylog(2,exp(I*(b*x+a)))-3*I
/b^4*polylog(2,-exp(I*(b*x+a)))-6*I/b^4*polylog(4,exp(I*(b*x+a)))-6*I/b^4*p
olylog(4,-exp(I*(b*x+a)))+1/b^4*a^3*ln(exp(I*(b*x+a))-1)
```

maxima [B] time = 0.75, size = 1970, normalized size = 9.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) + 2*((b*x + a)^4 - 4*(b*x
 + a)^3*a + 6*(b*x + a)^2*a^2 + 12*a^2 - (4*(b*x + a)^3 - 12*(b*x + a)^2*a
+ 12*(a^2 - 1)*(b*x + a) + 4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(
b*x + a) + 3*a)*cos(4*b*x + 4*a) - 8*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^
2 - 1)*(b*x + a) + 3*a)*cos(2*b*x + 2*a) + (4*I*(b*x + a)^3 - 12*I*(b*x + a
)^2*a + (12*I*a^2 - 12*I)*(b*x + a) + 12*I*a)*sin(4*b*x + 4*a) + (-8*I*(b*x
 + a)^3 + 24*I*(b*x + a)^2*a + (-24*I*a^2 + 24*I)*(b*x + a) - 24*I*a)*sin(2
*b*x + 2*a) + 12*a)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (12*a*cos(4*b
*x + 4*a) - 24*a*cos(2*b*x + 2*a) + 12*I*a*sin(4*b*x + 4*a) - 24*I*a*sin(2*
b*x + 2*a) + 12*a)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (4*(b*x + a)^3
```

```
 - 12*(b*x + a)^2*a + 12*(a^2 - 1)*(b*x + a) + 4*((b*x + a)^3 - 3*(b*x + a)
^2*a + 3*(a^2 - 1)*(b*x + a))*cos(4*b*x + 4*a) - 8*((b*x + a)^3 - 3*(b*x +
a)^2*a + 3*(a^2 - 1)*(b*x + a))*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3 + 12*I
*(b*x + a)^2*a + (-12*I*a^2 + 12*I)*(b*x + a))*sin(4*b*x + 4*a) - (8*I*(b*x
 + a)^3 - 24*I*(b*x + a)^2*a + (24*I*a^2 - 24*I)*(b*x + a))*sin(2*b*x + 2*a
))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + ((b*x + a)^4 - 4*(b*x + a)^3*
a + 6*(a^2 - 2)*(b*x + a)^2 + 24*(b*x + a)*a)*cos(4*b*x + 4*a) - (2*(b*x +
a)^4 - (b*x + a)^3*(8*a - 8*I) + 12*(a^2 - 2*I*a - 1)*(b*x + a)^2 + (24*I*a
^2 + 24*a)*(b*x + a) + 12*a^2)*cos(2*b*x + 2*a) + (12*(b*x + a)^2 - 24*(b*x
 + a)*a + 12*a^2 + 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2 - 1)*cos(4*b*x + 4
*a) - 24*((b*x + a)^2 - 2*(b*x + a)*a + a^2 - 1)*cos(2*b*x + 2*a) - (-12*I*
(b*x + a)^2 + 24*I*(b*x + a)*a - 12*I*a^2 + 12*I)*sin(4*b*x + 4*a) - (24*I*
(b*x + a)^2 - 48*I*(b*x + a)*a + 24*I*a^2 - 24*I)*sin(2*b*x + 2*a) - 12)*di
log(-e^(I*b*x + I*a)) + (12*(b*x + a)^2 - 24*(b*x + a)*a + 12*a^2 + 12*((b*
x + a)^2 - 2*(b*x + a)*a + a^2 - 1)*cos(4*b*x + 4*a) - 24*((b*x + a)^2 - 2*
(b*x + a)*a + a^2 - 1)*cos(2*b*x + 2*a) - (-12*I*(b*x + a)^2 + 24*I*(b*x +
a)*a - 12*I*a^2 + 12*I)*sin(4*b*x + 4*a) - (24*I*(b*x + a)^2 - 48*I*(b*x +
a)*a + 24*I*a^2 - 24*I)*sin(2*b*x + 2*a) - 12)*dilog(e^(I*b*x + I*a)) - (-2
*I*(b*x + a)^3 + 6*I*(b*x + a)^2*a + (-6*I*a^2 + 6*I)*(b*x + a) + (-2*I*(b*
x + a)^3 + 6*I*(b*x + a)^2*a + (-6*I*a^2 + 6*I)*(b*x + a) - 6*I*a)*cos(4*b*
x + 4*a) + (4*I*(b*x + a)^3 - 12*I*(b*x + a)^2*a + (12*I*a^2 - 12*I)*(b*x +
 a) + 12*I*a)*cos(2*b*x + 2*a) + 2*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2
 - 1)*(b*x + a) + 3*a)*sin(4*b*x + 4*a) - 4*((b*x + a)^3 - 3*(b*x + a)^2*a +
 3*(a^2 - 1)*(b*x + a) + 3*a)*sin(2*b*x + 2*a) - 6*I*a)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-2*I*(b*x + a)^3 + 6*I*(b*x + a)^
2*a + (-6*I*a^2 + 6*I)*(b*x + a) + (-2*I*(b*x + a)^3 + 6*I*(b*x + a)^2*a +
(-6*I*a^2 + 6*I)*(b*x + a) - 6*I*a)*cos(4*b*x + 4*a) + (4*I*(b*x + a)^3 - 1
2*I*(b*x + a)^2*a + (12*I*a^2 - 12*I)*(b*x + a) + 12*I*a)*cos(2*b*x + 2*a)
+ 2*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(b*x + a) + 3*a)*sin(4*b*x
 + 4*a) - 4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(b*x + a) + 3*a)*s
in(2*b*x + 2*a) - 6*I*a)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x +
a) + 1) - (24*cos(4*b*x + 4*a) - 48*cos(2*b*x + 2*a) + 24*I*sin(4*b*x + 4*a
) - 48*I*sin(2*b*x + 2*a) + 24)*polylog(4, -e^(I*b*x + I*a)) - (24*cos(4*b*
x + 4*a) - 48*cos(2*b*x + 2*a) + 24*I*sin(4*b*x + 4*a) - 48*I*sin(2*b*x + 2
*a) + 24)*polylog(4, e^(I*b*x + I*a)) - (-24*I*b*x*cos(4*b*x + 4*a) + 48*I*
b*x*cos(2*b*x + 2*a) + 24*b*x*sin(4*b*x + 4*a) - 48*b*x*sin(2*b*x + 2*a) -
24*I*b*x)*polylog(3, -e^(I*b*x + I*a)) - (-24*I*b*x*cos(4*b*x + 4*a) + 48*I
*b*x*cos(2*b*x + 2*a) + 24*b*x*sin(4*b*x + 4*a) - 48*b*x*sin(2*b*x + 2*a) -
 24*I*b*x)*polylog(3, e^(I*b*x + I*a)) - (-I*(b*x + a)^4 + 4*I*(b*x + a)^3*
a + (-6*I*a^2 + 12*I)*(b*x + a)^2 - 24*I*(b*x + a)*a)*sin(4*b*x + 4*a) - (2
*I*(b*x + a)^4 - 8*(b*x + a)^3*(I*a + 1) + (12*I*a^2 + 24*a - 12*I)*(b*x +
a)^2 - 24*(a^2 - I*a)*(b*x + a) + 12*I*a^2)*sin(2*b*x + 2*a))/(-4*I*cos(4*b
*x + 4*a) + 8*I*cos(2*b*x + 2*a) + 4*sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a)
- 4*I))/b^4
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \cot(a + b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cot(a + b*x)^3,x)
```

```
[Out] int(x^3*cot(a + b*x)^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cot^3(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cot(b*x+a)**3,x)
[Out] Integral(x**3*cot(a + b*x)**3, x)
```

3.12 $\int x^2 \cot^3(a+bx)\,dx$

Optimal. Leaf size=126

$$-\frac{\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{\log(\sin(a+bx))}{b^3}+\frac{ix\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{x\cot(a+bx)}{b^2}-\frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{x^2\cot^2(a+bx)}{2b}-\frac{x^2}{2b}$$

```
[Out] -1/2*x^2/b+1/3*I*x^3-x*cot(b*x+a)/b^2-1/2*x^2*cot(b*x+a)^2/b-x^2*ln(1-exp(2
*I*(b*x+a)))/b+ln(sin(b*x+a))/b^3+I*x*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*p
olylog(3,exp(2*I*(b*x+a)))/b^3
```

Rubi [A] time = 0.19, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3720, 3475, 30, 3717, 2190, 2531, 2282, 6589}

$$\frac{ix\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{x\cot(a+bx)}{b^2}+\frac{\log(\sin(a+bx))}{b^3}-\frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{x^2\cot}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cot[a + b*x]^3,x]
```

```
[Out] -x^2/(2*b) + (I/3)*x^3 - (x*Cot[a + b*x])/b^2 - (x^2*Cot[a + b*x]^2)/(2*b)
 - (x^2*Log[1 - E^((2*I)*(a + b*x))])/b + Log[Sin[a + b*x]]/b^3 + (I*x*PolyL
og[2, E^((2*I)*(a + b*x))])/b^2 - PolyLog[3, E^((2*I)*(a + b*x))]/(2*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \cot^3(a+bx)\,dx = -\frac{x^2\cot^2(a+bx)}{2b} + \frac{\int x\cot^2(a+bx)\,dx}{b} - \int x^2\cot(a+bx)\,dx$$

$$= \frac{ix^3}{3} - \frac{x\cot(a+bx)}{b^2} - \frac{x^2\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}x^2}{1-e^{2i(a+bx)}}\,dx + \frac{\int \cot(a+bx)\,dx}{b^2} - \frac{\int x\,dx}{b}$$

$$= -\frac{x^2}{2b} + \frac{ix^3}{3} - \frac{x\cot(a+bx)}{b^2} - \frac{x^2\cot^2(a+bx)}{2b} - \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{\log(\sin(a+bx))}{b^3} +$$

$$= -\frac{x^2}{2b} + \frac{ix^3}{3} - \frac{x\cot(a+bx)}{b^2} - \frac{x^2\cot^2(a+bx)}{2b} - \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{\log(\sin(a+bx))}{b^3} +$$

$$= -\frac{x^2}{2b} + \frac{ix^3}{3} - \frac{x\cot(a+bx)}{b^2} - \frac{x^2\cot^2(a+bx)}{2b} - \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{\log(\sin(a+bx))}{b^3} +$$

$$= -\frac{x^2}{2b} + \frac{ix^3}{3} - \frac{x\cot(a+bx)}{b^2} - \frac{x^2\cot^2(a+bx)}{2b} - \frac{x^2\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{\log(\sin(a+bx))}{b^3} +$$

Mathematica [A] time = 5.49, size = 221, normalized size = 1.75

$$-\frac{2b^3x^3\cot(a) + 3b^2x^2\csc^2(a+bx) + 2e^{-ia}\sin(a)(\cot(a)+i)\left(-b^3x^3\cot(a) + 3b^2x^2\log\left(1-e^{-i(a+bx)}\right) + 3b^2x^2\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cot[a + b*x]^3,x]
```

```
[Out] -1/6*(6*b*x*Cot[a] + 2*b^3*x^3*Cot[a] + 3*b^2*x^2*Csc[a + b*x]^2 - 6*Log[Si
n[a + b*x]] + (2*(I + Cot[a])*(I*b^3*x^3 - b^3*x^3*Cot[a] + 3*b^2*x^2*Log[1
 - E^((-I)*(a + b*x))] + 3*b^2*x^2*Log[1 + E^((-I)*(a + b*x))] + (6*I)*b*x*
PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2, E^((-I)*(a + b*x))]
+ 6*PolyLog[3, -E^((-I)*(a + b*x))] + 6*PolyLog[3, E^((-I)*(a + b*x))])*Sin
[a])/E^(I*a) - 6*b*x*Csc[a]*Csc[a + b*x]*Sin[b*x])/b^3
```

fricas [C] time = 0.66, size = 423, normalized size = 3.36

$$\frac{4\,b^2x^2 + 4\,bx\sin(2\,bx+2\,a) + (2i\,bx\cos(2\,bx+2\,a) - 2i\,bx)\mathrm{Li}_2\left(\cos(2\,bx+2\,a) + i\,\sin(2\,bx+2\,a)\right) + (-2i\,bx\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^2 + 4*b*x*sin(2*b*x + 2*a) + (2*I*b*x*cos(2*b*x + 2*a) - 2*I*b
*x)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (-2*I*b*x*cos(2*b*x + 2*
a) + 2*I*b*x)*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*(a^2 - (a^2
 - 1)*cos(2*b*x + 2*a) - 1)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*
a) + 1/2) + 2*(a^2 - (a^2 - 1)*cos(2*b*x + 2*a) - 1)*log(-1/2*cos(2*b*x + 2
*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^2*x^2 - a^2 - (b^2*x^2 - a^2)*co
s(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b^2*x^
2 - a^2 - (b^2*x^2 - a^2)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2
*b*x + 2*a) + 1) - (cos(2*b*x + 2*a) - 1)*polylog(3, cos(2*b*x + 2*a) + I*s
in(2*b*x + 2*a)) - (cos(2*b*x + 2*a) - 1)*polylog(3, cos(2*b*x + 2*a) - I*s
in(2*b*x + 2*a)))/(b^3*cos(2*b*x + 2*a) - b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cot(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cot(b*x + a)^3, x)
```

maple [B] time = 1.01, size = 293, normalized size = 2.33

$$\frac{2i\,\mathrm{polylog}\left(2, e^{i(bx+a)}\right)x}{b^2}+\frac{2x\left(bx\,e^{2i(bx+a)}-ie^{2i(bx+a)}+i\right)}{b^2\left(e^{2i(bx+a)}-1\right)^2}-\frac{2\,\mathrm{polylog}\left(3, e^{i(bx+a)}\right)}{b^3}-\frac{2\,\mathrm{polylog}\left(3, -e^{i(bx+a)}\right)}{b^3}-\frac{2ia}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(b*x+a)^3,x)
```

```
[Out] -2*I/b^2*a^2*x+2*x*(b*x*exp(2*I*(b*x+a))-I*exp(2*I*(b*x+a))+I)/b^2/(exp(2*I
*(b*x+a))-1)^2-2/b^3*polylog(3,exp(I*(b*x+a)))-2/b^3*polylog(3,-exp(I*(b*x+
a)))-4/3*I/b^3*a^3+1/b^3*ln(1-exp(I*(b*x+a)))*a^2+1/b^3*ln(exp(I*(b*x+a))-1
)+2*I/b^2*polylog(2,-exp(I*(b*x+a)))*x-1/b*ln(exp(I*(b*x+a))+1)*x^2+1/3*I*x
^3-1/b*ln(1-exp(I*(b*x+a)))*x^2+2*I/b^2*polylog(2,exp(I*(b*x+a)))*x-2/b^3*l
n(exp(I*(b*x+a)))+1/b^3*ln(exp(I*(b*x+a))+1)-1/b^3*a^2*ln(exp(I*(b*x+a))-1)
+2/b^3*a^2*ln(exp(I*(b*x+a)))
```

maxima [B] time = 1.18, size = 1216, normalized size = 9.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 2*(2*(b*x + a)^3 - 6*(
b*x + a)^2*a - (6*(b*x + a)^2 - 12*(b*x + a)*a + 6*((b*x + a)^2 - 2*(b*x +
a)*a - 1)*cos(4*b*x + 4*a) - 12*((b*x + a)^2 - 2*(b*x + a)*a - 1)*cos(2*b*x
 + 2*a) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a - 6*I)*sin(4*b*x + 4*a) + (-1
2*I*(b*x + a)^2 + 24*I*(b*x + a)*a + 12*I)*sin(2*b*x + 2*a) - 6)*arctan2(si
n(b*x + a), cos(b*x + a) + 1) + (6*cos(4*b*x + 4*a) - 12*cos(2*b*x + 2*a) +
 6*I*sin(4*b*x + 4*a) - 12*I*sin(2*b*x + 2*a) + 6)*arctan2(sin(b*x + a), co
s(b*x + a) - 1) + (6*(b*x + a)^2 - 12*(b*x + a)*a + 6*((b*x + a)^2 - 2*(b*x
 + a)*a)*cos(4*b*x + 4*a) - 12*((b*x + a)^2 - 2*(b*x + a)*a)*cos(2*b*x + 2*
a) - (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*sin(4*b*x + 4*a) - (12*I*(b*x +
```

```
a)^2 - 24*I*(b*x + a)*a)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x +
 a) + 1) + 2*((b*x + a)^3 - 3*(b*x + a)^2*a - 6*b*x - 6*a)*cos(4*b*x + 4*a)
 - (4*(b*x + a)^3 - (b*x + a)^2*(12*a - 12*I) - 12*(b*x + a)*(2*I*a + 1) -
12*a)*cos(2*b*x + 2*a) + (12*b*x*cos(4*b*x + 4*a) - 24*b*x*cos(2*b*x + 2*a)
 + 12*I*b*x*sin(4*b*x + 4*a) - 24*I*b*x*sin(2*b*x + 2*a) + 12*b*x)*dilog(-e
^(I*b*x + I*a)) + (12*b*x*cos(4*b*x + 4*a) - 24*b*x*cos(2*b*x + 2*a) + 12*I
*b*x*sin(4*b*x + 4*a) - 24*I*b*x*sin(2*b*x + 2*a) + 12*b*x)*dilog(e^(I*b*x
+ I*a)) - (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a + (-3*I*(b*x + a)^2 + 6*I*(b*
x + a)*a + 3*I)*cos(4*b*x + 4*a) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a - 6*
I)*cos(2*b*x + 2*a) + 3*((b*x + a)^2 - 2*(b*x + a)*a - 1)*sin(4*b*x + 4*a)
- 6*((b*x + a)^2 - 2*(b*x + a)*a - 1)*sin(2*b*x + 2*a) + 3*I)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-3*I*(b*x + a)^2 + 6*I*(b*x
 + a)*a + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a + 3*I)*cos(4*b*x + 4*a) + (6*
I*(b*x + a)^2 - 12*I*(b*x + a)*a - 6*I)*cos(2*b*x + 2*a) + 3*((b*x + a)^2 -
 2*(b*x + a)*a - 1)*sin(4*b*x + 4*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a - 1)*
sin(2*b*x + 2*a) + 3*I)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a
) + 1) - (-12*I*cos(4*b*x + 4*a) + 24*I*cos(2*b*x + 2*a) + 12*sin(4*b*x + 4
*a) - 24*sin(2*b*x + 2*a) - 12*I)*polylog(3, -e^(I*b*x + I*a)) - (-12*I*cos
(4*b*x + 4*a) + 24*I*cos(2*b*x + 2*a) + 12*sin(4*b*x + 4*a) - 24*sin(2*b*x
+ 2*a) - 12*I)*polylog(3, e^(I*b*x + I*a)) - (-2*I*(b*x + a)^3 + 6*I*(b*x +
 a)^2*a + 12*I*b*x + 12*I*a)*sin(4*b*x + 4*a) - (4*I*(b*x + a)^3 - 12*(b*x
+ a)^2*(I*a + 1) + (b*x + a)*(24*a - 12*I) - 12*I*a)*sin(2*b*x + 2*a) - 12*
a)/(-6*I*cos(4*b*x + 4*a) + 12*I*cos(2*b*x + 2*a) + 6*sin(4*b*x + 4*a) - 12
*sin(2*b*x + 2*a) - 6*I))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cot(a + b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cot(a + b*x)^3,x)
```

```
[Out] int(x^2*cot(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cot^3(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cot(b*x+a)**3,x)
```

```
[Out] Integral(x**2*cot(a + b*x)**3, x)
```

3.13 $\int x \cot^3(a+bx)\,dx$

Optimal. Leaf size=91

$$\frac{i\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} - \frac{\cot(a+bx)}{2b^2} - \frac{x\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{x\cot^2(a+bx)}{2b} - \frac{x}{2b} + \frac{ix^2}{2}$$

```
[Out] -1/2*x/b+1/2*I*x^2-1/2*cot(b*x+a)/b^2-1/2*x*cot(b*x+a)^2/b-x*ln(1-exp(2*I*(
b*x+a)))/b+1/2*I*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3720, 3473, 8, 3717, 2190, 2279, 2391}

$$\frac{i\mathrm{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} - \frac{\cot(a+bx)}{2b^2} - \frac{x\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{x\cot^2(a+bx)}{2b} - \frac{x}{2b} + \frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[a + b*x]^3,x]
```

```
[Out] -x/(2*b) + (I/2)*x^2 - Cot[a + b*x]/(2*b^2) - (x*Cot[a + b*x]^2)/(2*b) - (x
*Log[1 - E^((2*I)*(a + b*x))])/b + ((I/2)*PolyLog[2, E^((2*I)*(a + b*x))])/
b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cot^3(a+bx)\,dx &= -\frac{x\cot^2(a+bx)}{2b} + \frac{\int \cot^2(a+bx)\,dx}{2b} - \int x\cot(a+bx)\,dx \\ &= \frac{ix^2}{2} - \frac{\cot(a+bx)}{2b^2} - \frac{x\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}x}{1-e^{2i(a+bx)}}\,dx - \frac{\int 1\,dx}{2b} \\ &= -\frac{x}{2b} + \frac{ix^2}{2} - \frac{\cot(a+bx)}{2b^2} - \frac{x\cot^2(a+bx)}{2b} - \frac{x\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{\int \log\left(1-e^{2i(a+bx)}\right)\,dx}{b} \\ &= -\frac{x}{2b} + \frac{ix^2}{2} - \frac{\cot(a+bx)}{2b^2} - \frac{x\cot^2(a+bx)}{2b} - \frac{x\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x\right.}{2b^2} \\ &= -\frac{x}{2b} + \frac{ix^2}{2} - \frac{\cot(a+bx)}{2b^2} - \frac{x\cot^2(a+bx)}{2b} - \frac{x\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{i\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 4.30, size = 179, normalized size = 1.97

$$\frac{-b^2x^2\cot(a) + b^2x^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)} + i\text{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right) - bx\csc^2(a+bx) - 2bx\log\left(1-e^{2i(\tan}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Cot[a + b*x]^3,x]
```

```
[Out] ((-I)*b*Pi*x - b^2*x^2*Cot[a] - b*x*Csc[a + b*x]^2 - Pi*Log[1 + E^((-2*I)*b
*x)] - 2*b*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] +
 2*ArcTan[Tan[a]]*(I*b*x - Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Log[
Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))
] + b^2*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2] + Csc[a]*Csc[a + b*x
]*Sin[b*x])/(2*b^2)
```

fricas [B] time = 0.66, size = 291, normalized size = 3.20

$$\frac{4\,bx + (i\cos(2\,bx+2\,a) - i)\text{Li}_2(\cos(2\,bx+2\,a) + i\sin(2\,bx+2\,a)) + (-i\cos(2\,bx+2\,a) + i)\text{Li}_2(\cos(2\,bx+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*x + (I*cos(2*b*x + 2*a) - I)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x
+ 2*a)) + (-I*cos(2*b*x + 2*a) + I)*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x +
2*a)) + 2*(a*cos(2*b*x + 2*a) - a)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*
b*x + 2*a) + 1/2) + 2*(a*cos(2*b*x + 2*a) - a)*log(-1/2*cos(2*b*x + 2*a) -
1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*lo
g(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b*x - (b*x + a)*cos(2*b*
x + 2*a) + a)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 2*sin(2*b*x
 + 2*a))/(b^2*cos(2*b*x + 2*a) - b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cot(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cot(b*x + a)^3, x)
```

maple [B] time = 0.80, size = 197, normalized size = 2.16

$$\frac{ix^2}{2}+\frac{2bx\,e^{2i(bx+a)}-ie^{2i(bx+a)}+i}{b^2\left(e^{2i(bx+a)}-1\right)^2}+\frac{2iax}{b}+\frac{ia^2}{b^2}-\frac{\ln\left(e^{i(bx+a)}+1\right)x}{b}+\frac{i\,\mathrm{polylog}\left(2,-e^{i(bx+a)}\right)}{b^2}-\frac{\ln\left(1-e^{i(bx+a)}\right)x}{b}-\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(b*x+a)^3,x)
```

```
[Out] 1/2*I*x^2+(2*b*x*exp(2*I*(b*x+a))-I*exp(2*I*(b*x+a))+I)/b^2/(exp(2*I*(b*x+a
))-1)^2+2*I/b*a*x+I/b^2*a^2-1/b*ln(exp(I*(b*x+a))+1)*x+I/b^2*polylog(2,-exp
(I*(b*x+a)))-1/b*ln(1-exp(I*(b*x+a)))*x-1/b^2*ln(1-exp(I*(b*x+a)))*a+I/b^2*
polylog(2,exp(I*(b*x+a)))-2/b^2*a*ln(exp(I*(b*x+a)))+1/b^2*a*ln(exp(I*(b*x+
a))-1)
```

maxima [B] time = 0.88, size = 591, normalized size = 6.49

$$\frac{b^2x^2\cos(4\,bx+4\,a)+i\,b^2x^2\sin(4\,bx+4\,a)+b^2x^2-(2\,bx\cos(4\,bx+4\,a)-4\,bx\cos(2\,bx+2\,a)+2i\,bx\sin(4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (b^2*x^2*cos(4*b*x + 4*a) + I*b^2*x^2*sin(4*b*x + 4*a) + b^2*x^2 - (2*b*x*c
os(4*b*x + 4*a) - 4*b*x*cos(2*b*x + 2*a) + 2*I*b*x*sin(4*b*x + 4*a) - 4*I*b
*x*sin(2*b*x + 2*a) + 2*b*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (2*b
*x*cos(4*b*x + 4*a) - 4*b*x*cos(2*b*x + 2*a) + 2*I*b*x*sin(4*b*x + 4*a) - 4
*I*b*x*sin(2*b*x + 2*a) + 2*b*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) -
 2*(b^2*x^2 + 2*I*b*x + 1)*cos(2*b*x + 2*a) + (2*cos(4*b*x + 4*a) - 4*cos(2
*b*x + 2*a) + 2*I*sin(4*b*x + 4*a) - 4*I*sin(2*b*x + 2*a) + 2)*dilog(-e^(I*
b*x + I*a)) + (2*cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a) + 2*I*sin(4*b*x + 4*
a) - 4*I*sin(2*b*x + 2*a) + 2)*dilog(e^(I*b*x + I*a)) - (-I*b*x*cos(4*b*x +
 4*a) + 2*I*b*x*cos(2*b*x + 2*a) + b*x*sin(4*b*x + 4*a) - 2*b*x*sin(2*b*x +
 2*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
(-I*b*x*cos(4*b*x + 4*a) + 2*I*b*x*cos(2*b*x + 2*a) + b*x*sin(4*b*x + 4*a)
- 2*b*x*sin(2*b*x + 2*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*c
os(b*x + a) + 1) - (2*I*b^2*x^2 - 4*b*x + 2*I)*sin(2*b*x + 2*a) + 2)/(-2*I*
b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) -
4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \cot(a+b\,x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(a + b*x)^3,x)
```

```
[Out] int(x*cot(a + b*x)^3, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cot^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(b*x+a)**3,x)
[Out] Integral(x*cot(a + b*x)**3, x)
```

3.14 $\int \frac{\cot^3(a+bx)}{x}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cot^3(a+bx)}{x}, x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^3/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/x,x]
```

```
[Out] Defer[Int][Cot[a + b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{x}\,dx = \int \frac{\cot^3(a+bx)}{x}\,dx$$

Mathematica [A] time = 6.51, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/x,x]
```

```
[Out] Integrate[Cot[a + b*x]^3/x, x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^3/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^3/x, x)
```

maple [A] time = 2.92, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/x,x)
```

```
[Out] int(cot(b*x+a)^3/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\,bx\cos(2\,bx+2\,a)^2+4\,bx\sin(2\,bx+2\,a)^2-2\,bx\cos(2\,bx+2\,a)-(2\,bx\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -(4*b*x*cos(2*b*x + 2*a)^2 + 4*b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*b*x + 2
*a) - (2*b*x*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (b^2*x
^2*cos(4*b*x + 4*a)^2 + 4*b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(4*b*x +
4*a)^2 - 4*b^2*x^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^2*sin(2*b*x
+ 2*a)^2 - 4*b^2*x^2*cos(2*b*x + 2*a) + b^2*x^2 - 2*(2*b^2*x^2*cos(2*b*x +
2*a) - b^2*x^2)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 1)*sin(b*x + a)/(b^2
*x^3*cos(b*x + a)^2 + b^2*x^3*sin(b*x + a)^2 + 2*b^2*x^3*cos(b*x + a) + b^2
*x^3), x) + (b^2*x^2*cos(4*b*x + 4*a)^2 + 4*b^2*x^2*cos(2*b*x + 2*a)^2 + b^
2*x^2*sin(4*b*x + 4*a)^2 - 4*b^2*x^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*
b^2*x^2*sin(2*b*x + 2*a)^2 - 4*b^2*x^2*cos(2*b*x + 2*a) + b^2*x^2 - 2*(2*b^
2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 1)
*sin(b*x + a)/(b^2*x^3*cos(b*x + a)^2 + b^2*x^3*sin(b*x + a)^2 - 2*b^2*x^3*
cos(b*x + a) + b^2*x^3), x) - (2*b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) -
1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))/(b^2*x^2*cos(4*b*x + 4*a)^2 + 4*b^2
*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 - 4*b^2*x^2*sin(4*b*x
+ 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^2*sin(2*b*x + 2*a)^2 - 4*b^2*x^2*cos(2*b*
x + 2*a) + b^2*x^2 - 2*(2*b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x + 4
*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cot(a+b\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3/x,x)
```

```
[Out] int(cot(a + b*x)^3/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/x,x)
```

```
[Out] Integral(cot(a + b*x)**3/x, x)
```

3.15 $\int \frac{\cot^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cot^3(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^3/x^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/x^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{x^2}\,dx = \int \frac{\cot^3(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 5.27, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/x^2,x]
```

```
[Out] Integrate[Cot[a + b*x]^3/x^2, x]
```

fricas [A] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^3/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^3/x^2, x)
```

maple [A] time = 3.79, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/x^2,x)
```

```
[Out] int(cot(b*x+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\,bx\cos(2\,bx+2\,a)^2+4\,bx\sin(2\,bx+2\,a)^2-2\,bx\cos(2\,bx+2\,a)-2\,(bx\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -(4*b*x*cos(2*b*x + 2*a)^2 + 4*b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*b*x + 2
*a) - 2*(b*x*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (b^2*x
^3*cos(4*b*x + 4*a)^2 + 4*b^2*x^3*cos(2*b*x + 2*a)^2 + b^2*x^3*sin(4*b*x +
4*a)^2 - 4*b^2*x^3*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^3*sin(2*b*x
+ 2*a)^2 - 4*b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3 - 2*(2*b^2*x^3*cos(2*b*x +
2*a) - b^2*x^3)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 3)*sin(b*x + a)/(b^2
*x^4*cos(b*x + a)^2 + b^2*x^4*sin(b*x + a)^2 + 2*b^2*x^4*cos(b*x + a) + b^2
*x^4), x) + (b^2*x^3*cos(4*b*x + 4*a)^2 + 4*b^2*x^3*cos(2*b*x + 2*a)^2 + b^
2*x^3*sin(4*b*x + 4*a)^2 - 4*b^2*x^3*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*
b^2*x^3*sin(2*b*x + 2*a)^2 - 4*b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3 - 2*(2*b^
2*x^3*cos(2*b*x + 2*a) - b^2*x^3)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 3)
*sin(b*x + a)/(b^2*x^4*cos(b*x + a)^2 + b^2*x^4*sin(b*x + a)^2 - 2*b^2*x^4*
cos(b*x + a) + b^2*x^4), x) - 2*(b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) -
1)*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))/(b^2*x^3*cos(4*b*x + 4*a)^2 + 4*b
^2*x^3*cos(2*b*x + 2*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 - 4*b^2*x^3*sin(4*b*
x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^3*sin(2*b*x + 2*a)^2 - 4*b^2*x^3*cos(2*
b*x + 2*a) + b^2*x^3 - 2*(2*b^2*x^3*cos(2*b*x + 2*a) - b^2*x^3)*cos(4*b*x +
 4*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cot(a+b\,x)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3/x^2,x)
```

```
[Out] int(cot(a + b*x)^3/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/x**2,x)
```

```
[Out] Integral(cot(a + b*x)**3/x**2, x)
```

3.16 $\int \frac{(c+dx)^3}{a+ia\cot(e+fx)}\,dx$

Optimal. Leaf size=189

$$\frac{3id^2(c+dx)}{4f^3(a+ia\cot(e+fx))}+\frac{3d(c+dx)^2}{4f^2(a+ia\cot(e+fx))}-\frac{i(c+dx)^3}{2f(a+ia\cot(e+fx))}-\frac{3d(c+dx)^2}{8af^2}+\frac{i(c+dx)^3}{4af}+\frac{(c+dx)^4}{8ad}-$$

```
[Out] -3/8*I*d^3*x/a/f^3-3/8*d*(d*x+c)^2/a/f^2+1/4*I*(d*x+c)^3/a/f+1/8*(d*x+c)^4/
a/d-3/8*d^3/f^4/(a+I*a*cot(f*x+e))+3/4*I*d^2*(d*x+c)/f^3/(a+I*a*cot(f*x+e))
+3/4*d*(d*x+c)^2/f^2/(a+I*a*cot(f*x+e))-1/2*I*(d*x+c)^3/f/(a+I*a*cot(f*x+e)
)
```

Rubi [A] time = 0.20, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3723, 3479, 8}

$$\frac{3id^2(c+dx)}{4f^3(a+ia\cot(e+fx))}+\frac{3d(c+dx)^2}{4f^2(a+ia\cot(e+fx))}-\frac{i(c+dx)^3}{2f(a+ia\cot(e+fx))}-\frac{3d(c+dx)^2}{8af^2}+\frac{i(c+dx)^3}{4af}+\frac{(c+dx)^4}{8ad}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] (((-3*I)/8)*d^3*x)/(a*f^3) - (3*d*(c + d*x)^2)/(8*a*f^2) + ((I/4)*(c + d*x)
^3)/(a*f) + (c + d*x)^4/(8*a*d) - (3*d^3)/(8*f^4*(a + I*a*Cot[e + f*x])) +
(((3*I)/4)*d^2*(c + d*x))/(f^3*(a + I*a*Cot[e + f*x])) + (3*d*(c + d*x)^2)/
(4*f^2*(a + I*a*Cot[e + f*x])) - ((I/2)*(c + d*x)^3)/(f*(a + I*a*Cot[e + f*
x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3723

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[(a*d*m)/(2*b*f)
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[(a*(c + d*x)^m)
/(2*b*f*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+ia\cot(e+fx)}\,dx = \frac{(c+dx)^4}{8ad} - \frac{i(c+dx)^3}{2f(a+ia\cot(e+fx))} + \frac{(3id)\int \frac{(c+dx)^2}{a+ia\cot(e+fx)}\,dx}{2f}$$

$$= \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} + \frac{3d(c+dx)^2}{4f^2(a+ia\cot(e+fx))} - \frac{i(c+dx)^3}{2f(a+ia\cot(e+fx))} - \frac{\left(3d^2\right)\int \frac{}{a}}{}$$

$$= -\frac{3d(c+dx)^2}{8af^2} + \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} + \frac{3id^2(c+dx)}{4f^3(a+ia\cot(e+fx))} + \frac{3d(c+dx)^2}{4f^2(a+ia\cot(e+}$$

$$= -\frac{3d(c+dx)^2}{8af^2} + \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+ia\cot(e+fx))} + \frac{3id^2(c+dx)}{4f^3(a+ia\cot(e+}$$

$$= -\frac{3id^3x}{8af^3} - \frac{3d(c+dx)^2}{8af^2} + \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+ia\cot(e+fx))} + \frac{3id^2}{4f^3(a+i}$$

Mathematica [A] time = 0.63, size = 246, normalized size = 1.30

$$\frac{i(\cos(2e)+i\sin(2e))\cos(2fx)\left(4c^3f^3+6c^2df^2(2fx+i)+6cd^2f\left(2f^2x^2+2ifx-1\right)+d^3\left(4f^3x^3+6if^2x^2-6fx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] (2*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + I*(4*c^3*f^3 + 6*c^2
*d*f^2*(I + 2*f*x) + 6*c*d^2*f*(-1 + (2*I)*f*x + 2*f^2*x^2) + d^3*(-3*I - 6
*f*x + (6*I)*f^2*x^2 + 4*f^3*x^3))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e]) - (4*
c^3*f^3 + 6*c^2*d*f^2*(I + 2*f*x) + 6*c*d^2*f*(-1 + (2*I)*f*x + 2*f^2*x^2)
+ d^3*(-3*I - 6*f*x + (6*I)*f^2*x^2 + 4*f^3*x^3))*(Cos[2*e] + I*Sin[2*e])*S
in[2*f*x])/(16*a*f^4)
```

fricas [A] time = 0.76, size = 153, normalized size = 0.81

$$\frac{2\,d^3f^4x^4+8\,cd^2f^4x^3+12\,c^2df^4x^2+8\,c^3f^4x+\left(4i\,d^3f^3x^3+4i\,c^3f^3-6\,c^2df^2-6i\,cd^2f+3\,d^3+\left(12i\,cd^2f^3-6\,d^3\right.\right.}{16\,af^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*d^3*f^4*x^4 + 8*c*d^2*f^4*x^3 + 12*c^2*d*f^4*x^2 + 8*c^3*f^4*x + (4
*I*d^3*f^3*x^3 + 4*I*c^3*f^3 - 6*c^2*d*f^2 - 6*I*c*d^2*f + 3*d^3 + (12*I*c*
d^2*f^3 - 6*d^3*f^2)*x^2 + (12*I*c^2*d*f^3 - 12*c*d^2*f^2 - 6*I*d^3*f)*x)*e
^(2*I*f*x + 2*I*e))/(a*f^4)
```

giac [A] time = 0.42, size = 243, normalized size = 1.29

$$\frac{2\,d^3f^4x^4+8\,cd^2f^4x^3+4i\,d^3f^3x^3e^{\left(2if x+2ie\right)}+12\,c^2df^4x^2+12i\,cd^2f^3x^2e^{\left(2if x+2ie\right)}+8\,c^3f^4x+12i\,c^2df^3xe^{\left(2if x+2ie\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/16*(2*d^3*f^4*x^4 + 8*c*d^2*f^4*x^3 + 4*I*d^3*f^3*x^3*e^(2*I*f*x + 2*I*e)
 + 12*c^2*d*f^4*x^2 + 12*I*c*d^2*f^3*x^2*e^(2*I*f*x + 2*I*e) + 8*c^3*f^4*x
+ 12*I*c^2*d*f^3*x*e^(2*I*f*x + 2*I*e) - 6*d^3*f^2*x^2*e^(2*I*f*x + 2*I*e)
+ 4*I*c^3*f^3*e^(2*I*f*x + 2*I*e) - 12*c*d^2*f^2*x*e^(2*I*f*x + 2*I*e) - 6*
```

```
c^2*d*f^2*e^(2*I*f*x + 2*I*e) - 6*I*d^3*f*x*e^(2*I*f*x + 2*I*e) - 6*I*c*d^2
*f*e^(2*I*f*x + 2*I*e) + 3*d^3*e^(2*I*f*x + 2*I*e))/(a*f^4)
```

maple [A] time = 2.07, size = 170, normalized size = 0.90

$$\frac{d^3x^4}{8a}+\frac{d^2c\,x^3}{2a}+\frac{3d\,c^2x^2}{4a}+\frac{c^3x}{2a}+\frac{c^4}{8da}+\frac{i\left(4d^3x^3f^3+12c\,d^2f^3x^2+6id^3f^2x^2+12c^2d\,f^3x+12ic\,d^2f^2x+4c^3f^3+6i\right.}{16a\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*cot(f*x+e)),x)
```

```
[Out] 1/8*d^3/a*x^4+1/2*d^2/a*c*x^3+3/4*d/a*c^2*x^2+1/2*c^3*x/a+1/8/d/a*c^4+1/16*
I*(4*d^3*x^3*f^3+6*I*d^3*f^2*x^2+12*c*d^2*f^3*x^2+12*I*c*d^2*f^2*x+12*c^2*d
*f^3*x+6*I*c^2*d*f^2+4*c^3*f^3-6*d^3*f*x-3*I*d^3-6*c*d^2*f)/a/f^4*exp(2*I*(
f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 1.24, size = 423, normalized size = 2.24

$$\frac{3\,d^3\,\cos\left(2\,e+2\,f\,x\right)+8\,c^3\,f^4\,x-4\,c^3\,f^3\,\sin\left(2\,e+2\,f\,x\right)+2\,d^3\,f^4\,x^4-6\,c^2\,d\,f^2\,\cos\left(2\,e+2\,f\,x\right)+12\,c^2\,d\,f^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*cot(e + f*x)*1i),x)
```

```
[Out] (3*d^3*cos(2*e + 2*f*x) + d^3*sin(2*e + 2*f*x)*3i + 8*c^3*f^4*x + c^3*f^3*c
os(2*e + 2*f*x)*4i - 4*c^3*f^3*sin(2*e + 2*f*x) + 2*d^3*f^4*x^4 - 6*c^2*d*f
^2*cos(2*e + 2*f*x) - c^2*d*f^2*sin(2*e + 2*f*x)*6i + 12*c^2*d*f^4*x^2 + 8*
c*d^2*f^4*x^3 - 6*d^3*f^2*x^2*cos(2*e + 2*f*x) + d^3*f^3*x^3*cos(2*e + 2*f*
x)*4i - d^3*f^2*x^2*sin(2*e + 2*f*x)*6i - 4*d^3*f^3*x^3*sin(2*e + 2*f*x) -
c*d^2*f*cos(2*e + 2*f*x)*6i + 6*c*d^2*f*sin(2*e + 2*f*x) - d^3*f*x*cos(2*e
+ 2*f*x)*6i + 6*d^3*f*x*sin(2*e + 2*f*x) - 12*c*d^2*f^2*x*cos(2*e + 2*f*x)
+ c^2*d*f^3*x*cos(2*e + 2*f*x)*12i - c*d^2*f^2*x*sin(2*e + 2*f*x)*12i - 12*
c^2*d*f^3*x*sin(2*e + 2*f*x) + c*d^2*f^3*x^2*cos(2*e + 2*f*x)*12i - 12*c*d^
2*f^3*x^2*sin(2*e + 2*f*x))/(16*a*f^4)
```

sympy [A] time = 0.37, size = 318, normalized size = 1.68

$$\begin{cases} -\frac{\left(-4ic^3f^3e^{2ie}-12ic^2df^3xe^{2ie}+6c^2df^2e^{2ie}-12icd^2f^3x^2e^{2ie}+12cd^2f^2xe^{2ie}+6icd^2fe^{2ie}-4id^3f^3x^3e^{2ie}+6d^3f^2x^2e^{2ie}+6id^3fxe^{2ie}-3d^3e^{2ie}\right)e^{2ifx}}{16af^4} & \text{for} \\ -\frac{c^3xe^{2ie}}{2a}-\frac{3c^2dx^2e^{2ie}}{4a}-\frac{cd^2x^3e^{2ie}}{2a}-\frac{d^3x^4e^{2ie}}{8a} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*cot(f*x+e)),x)
```

```
[Out] Piecewise((-(-4*I*c**3*f**3*exp(2*I*e) - 12*I*c**2*d*f**3*x*exp(2*I*e) + 6*
c**2*d*f**2*exp(2*I*e) - 12*I*c*d**2*f**3*x**2*exp(2*I*e) + 12*c*d**2*f**2*
```

```
x*exp(2*I*e) + 6*I*c*d**2*f*exp(2*I*e) - 4*I*d**3*f**3*x**3*exp(2*I*e) + 6*
d**3*f**2*x**2*exp(2*I*e) + 6*I*d**3*f*x*exp(2*I*e) - 3*d**3*exp(2*I*e))*ex
p(2*I*f*x)/(16*a*f**4), Ne(16*a*f**4, 0)), (-c**3*x*exp(2*I*e)/(2*a) - 3*c*
*2*d*x**2*exp(2*I*e)/(4*a) - c*d**2*x**3*exp(2*I*e)/(2*a) - d**3*x**4*exp(2
*I*e)/(8*a), True)) + c**3*x/(2*a) + 3*c**2*d*x**2/(4*a) + c*d**2*x**3/(2*a
) + d**3*x**4/(8*a)
```

3.17 $\int \frac{(c+dx)^2}{a+ia\cot(e+fx)}\,dx$

Optimal. Leaf size=137

$$\frac{d(c+dx)}{2f^2(a+ia\cot(e+fx))}-\frac{i(c+dx)^2}{2f(a+ia\cot(e+fx))}+\frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}+\frac{id^2}{4f^3(a+ia\cot(e+fx))}-\frac{d^2x}{4af^2}$$

```
[Out] -1/4*d^2*x/a/f^2+1/4*I*(d*x+c)^2/a/f+1/6*(d*x+c)^3/a/d+1/4*I*d^2/f^3/(a+I*a
*cot(f*x+e))+1/2*d*(d*x+c)/f^2/(a+I*a*cot(f*x+e))-1/2*I*(d*x+c)^2/f/(a+I*a*
cot(f*x+e))
```

Rubi [A] time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3723, 3479, 8}

$$\frac{d(c+dx)}{2f^2(a+ia\cot(e+fx))}-\frac{i(c+dx)^2}{2f(a+ia\cot(e+fx))}+\frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}+\frac{id^2}{4f^3(a+ia\cot(e+fx))}-\frac{d^2x}{4af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] -(d^2*x)/(4*a*f^2) + ((I/4)*(c + d*x)^2)/(a*f) + (c + d*x)^3/(6*a*d) + ((I/
4)*d^2)/(f^3*(a + I*a*Cot[e + f*x])) + (d*(c + d*x))/(2*f^2*(a + I*a*Cot[e
+ f*x])) - ((I/2)*(c + d*x)^2)/(f*(a + I*a*Cot[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3723

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[(a*d*m)/(2*b*f)
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[(a*(c + d*x)^m)
/(2*b*f*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+ia\cot(e+fx)}\,dx = \frac{(c+dx)^3}{6ad}-\frac{i(c+dx)^2}{2f(a+ia\cot(e+fx))}+\frac{(id)\int \frac{c+dx}{a+ia\cot(e+fx)}\,dx}{f}$$

$$= \frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}+\frac{d(c+dx)}{2f^2(a+ia\cot(e+fx))}-\frac{i(c+dx)^2}{2f(a+ia\cot(e+fx))}-\frac{d^2\int \frac{}{a}}{}$$

$$= \frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}+\frac{id^2}{4f^3(a+ia\cot(e+fx))}+\frac{d(c+dx)}{2f^2(a+ia\cot(e+fx))}-\frac{}{2f(a}$$

$$= -\frac{d^2x}{4af^2}+\frac{i(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}+\frac{id^2}{4f^3(a+ia\cot(e+fx))}+\frac{d(c+dx)}{2f^2(a+ia\cot(e+fx)}$$

Mathematica [A] time = 0.37, size = 149, normalized size = 1.09

$$\frac{4f^3x\left(3c^2+3cdx+d^2x^2\right)+3(\cos(2e)+i\sin(2e))\cos(2fx)((1+i)cf+d(-1+(1+i)fx))((1+i)cf+d((1+i)fx}{24af^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] (4*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 3*((1 + I)*c*f + d*(-1 + (1 + I)*f*x
))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e]) +
 (3*I)*((1 + I)*c*f + d*(-1 + (1 + I)*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f
*x))*(Cos[2*e] + I*Sin[2*e])*Sin[2*f*x])/(24*a*f^3)
```

fricas [A] time = 0.50, size = 95, normalized size = 0.69

$$\frac{4\,d^2f^3x^3+12\,cdf^3x^2+12\,c^2f^3x+\left(6i\,d^2f^2x^2+6i\,c^2f^2-6\,cdf-3i\,d^2+\left(12i\,cdf^2-6\,d^2f\right)x\right)e^{\left(2i\,fx+2i\,e\right)}}{24\,af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*d^2*f^3*x^3 + 12*c*d*f^3*x^2 + 12*c^2*f^3*x + (6*I*d^2*f^2*x^2 + 6*
I*c^2*f^2 - 6*c*d*f - 3*I*d^2 + (12*I*c*d*f^2 - 6*d^2*f)*x)*e^(2*I*f*x + 2*
I*e))/(a*f^3)
```

giac [A] time = 0.42, size = 143, normalized size = 1.04

$$\frac{4\,d^2f^3x^3+12\,cdf^3x^2+6i\,d^2f^2x^2e^{\left(2i\,fx+2i\,e\right)}+12\,c^2f^3x+12i\,cdf^2xe^{\left(2i\,fx+2i\,e\right)}+6i\,c^2f^2e^{\left(2i\,fx+2i\,e\right)}-6\,d^2fxe^{\left(2i\,fx+2\right.}}{24\,af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/24*(4*d^2*f^3*x^3 + 12*c*d*f^3*x^2 + 6*I*d^2*f^2*x^2*e^(2*I*f*x + 2*I*e)
 + 12*c^2*f^3*x + 12*I*c*d*f^2*x*e^(2*I*f*x + 2*I*e) + 6*I*c^2*f^2*e^(2*I*f*
x + 2*I*e) - 6*d^2*f*x*e^(2*I*f*x + 2*I*e) - 6*c*d*f*e^(2*I*f*x + 2*I*e) -
3*I*d^2*e^(2*I*f*x + 2*I*e))/(a*f^3)
```

maple [A] time = 1.86, size = 108, normalized size = 0.79

$$\frac{d^2x^3}{6a}+\frac{dc\,x^2}{2a}+\frac{c^2x}{2a}+\frac{c^3}{6da}+\frac{i\left(2d^2f^2x^2+4cd\,f^2x+2id^2fx+2c^2f^2+2icdf-d^2\right)\mathrm{e}^{2i\left(fx+e\right)}}{8a\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*cot(f*x+e)),x)
```

```
[Out] 1/6*d^2/a*x^3+1/2*d/a*c*x^2+1/2*c^2*x/a+1/6/d/a*c^3+1/8*I*(2*d^2*f^2*x^2+2*
I*d^2*f*x+4*c*d*f^2*x+2*I*c*d*f+2*c^2*f^2-d^2)/a/f^3*exp(2*I*(f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.77, size = 241, normalized size = 1.76

$$-\frac{6c^2f^2\sin\left(2e+2fx\right)-12c^2f^3x-3d^2\sin\left(2e+2fx\right)-4d^2f^3x^3+6cdf\cos\left(2e+2fx\right)+6d^2f^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*cot(e + f*x)*1i),x)
```

```
[Out] -(d^2*cos(2*e + 2*f*x)*3i - 3*d^2*sin(2*e + 2*f*x) - 12*c^2*f^3*x - c^2*f^2
*cos(2*e + 2*f*x)*6i + 6*c^2*f^2*sin(2*e + 2*f*x) - 4*d^2*f^3*x^3 + 6*c*d*f
*cos(2*e + 2*f*x) + c*d*f*sin(2*e + 2*f*x)*6i - d^2*f^2*x^2*cos(2*e + 2*f*x
)*6i + 6*d^2*f^2*x^2*sin(2*e + 2*f*x) - 12*c*d*f^3*x^2 + 6*d^2*f*x*cos(2*e
+ 2*f*x) + d^2*f*x*sin(2*e + 2*f*x)*6i - c*d*f^2*x*cos(2*e + 2*f*x)*12i + 1
2*c*d*f^2*x*sin(2*e + 2*f*x))/(24*a*f^3)
```

sympy [A] time = 0.29, size = 197, normalized size = 1.44

$$\begin{cases} -\frac{\left(-2ic^2f^2e^{2ie}-4icdf^2xe^{2ie}+2cdfe^{2ie}-2id^2f^2x^2e^{2ie}+2d^2fxe^{2ie}+id^2e^{2ie}\right)e^{2ifx}}{8af^3} & \text{for } 8af^3 \neq 0 \\ -\frac{c^2xe^{2ie}}{2a}-\frac{cdx^2e^{2ie}}{2a}-\frac{d^2x^3e^{2ie}}{6a} & \text{otherwise} \end{cases} + \frac{c^2x}{2a} + \frac{cdx^2}{2a} + \frac{d^2x^3}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*cot(f*x+e)),x)
```

```
[Out] Piecewise((-(-2*I*c**2*f**2*exp(2*I*e) - 4*I*c*d*f**2*x*exp(2*I*e) + 2*c*d*
f*exp(2*I*e) - 2*I*d**2*f**2*x**2*exp(2*I*e) + 2*d**2*f*x*exp(2*I*e) + I*d*
*2*exp(2*I*e))*exp(2*I*f*x)/(8*a*f**3), Ne(8*a*f**3, 0)), (-c**2*x*exp(2*I*
e)/(2*a) - c*d*x**2*exp(2*I*e)/(2*a) - d**2*x**3*exp(2*I*e)/(6*a), True)) +
 c**2*x/(2*a) + c*d*x**2/(2*a) + d**2*x**3/(6*a)
```

3.18 $\int \frac{c+dx}{a+ia\cot(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{i(c+dx)}{2f(a+ia\cot(e+fx))}+\frac{(c+dx)^2}{4ad}+\frac{d}{4f^2(a+ia\cot(e+fx))}+\frac{idx}{4af}$$

```
[Out] 1/4*I*d*x/a/f+1/4*(d*x+c)^2/a/d+1/4*d/f^2/(a+I*a*cot(f*x+e))-1/2*I*(d*x+c)/
f/(a+I*a*cot(f*x+e))
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3723, 3479, 8}

$$-\frac{i(c+dx)}{2f(a+ia\cot(e+fx))}+\frac{(c+dx)^2}{4ad}+\frac{d}{4f^2(a+ia\cot(e+fx))}+\frac{idx}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] ((I/4)*d*x)/(a*f) + (c + d*x)^2/(4*a*d) + d/(4*f^2*(a + I*a*Cot[e + f*x]))
- ((I/2)*(c + d*x))/(f*(a + I*a*Cot[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3723

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[(a*d*m)/(2*b*f)
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[(a*(c + d*x)^m)
/(2*b*f*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a+ia\cot(e+fx)}\,dx &= \frac{(c+dx)^2}{4ad}-\frac{i(c+dx)}{2f(a+ia\cot(e+fx))}+\frac{(id)\int \frac{1}{a+ia\cot(e+fx)}\,dx}{2f}\\ &= \frac{(c+dx)^2}{4ad}+\frac{d}{4f^2(a+ia\cot(e+fx))}-\frac{i(c+dx)}{2f(a+ia\cot(e+fx))}+\frac{(id)\int 1\,dx}{4af}\\ &= \frac{idx}{4af}+\frac{(c+dx)^2}{4ad}+\frac{d}{4f^2(a+ia\cot(e+fx))}-\frac{i(c+dx)}{2f(a+ia\cot(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.25, size = 107, normalized size = 1.27

$$\frac{(\cos(e+fx)+i\sin(e+fx))\left(\left(2cf(2fx+i)+d\left(2f^2x^2+2ifx-1\right)\right)\cos(e+fx)-i\left(2cf(2fx-i)+d\left(2f^2x^2-2\right.\right.\right.}{8af^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] ((Cos[e + f*x] + I*Sin[e + f*x])*((2*c*f*(I + 2*f*x) + d*(-1 + (2*I)*f*x +
2*f^2*x^2))*Cos[e + f*x] - I*(2*c*f*(-I + 2*f*x) + d*(1 - (2*I)*f*x + 2*f^2
*x^2))*Sin[e + f*x]))/(8*a*f^2)
```

fricas [A] time = 0.48, size = 48, normalized size = 0.57

$$\frac{2\,df^2x^2 + 4\,cf^2x + \left(2i\,dfx + 2i\,cf - d\right)e^{\left(2i\,fx+2i\,e\right)}}{8\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*d*f^2*x^2 + 4*c*f^2*x + (2*I*d*f*x + 2*I*c*f - d)*e^(2*I*f*x + 2*I*e
))/(a*f^2)
```

giac [A] time = 0.65, size = 67, normalized size = 0.80

$$\frac{2\,df^2x^2 + 4\,cf^2x + 2i\,dfxe^{\left(2i\,fx+2i\,e\right)} + 2i\,cfe^{\left(2i\,fx+2i\,e\right)} - de^{\left(2i\,fx+2i\,e\right)}}{8\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/8*(2*d*f^2*x^2 + 4*c*f^2*x + 2*I*d*f*x*e^(2*I*f*x + 2*I*e) + 2*I*c*f*e^(2
*I*f*x + 2*I*e) - d*e^(2*I*f*x + 2*I*e))/(a*f^2)
```

maple [B] time = 0.62, size = 177, normalized size = 2.11

$$\frac{d\left(\left(fx+e\right)\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{\left(fx+e\right)^2}{4}+\frac{\left(\sin^2\left(fx+e\right)\right)}{4}\right)+cf\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-de\left(-\frac{\sin}{f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*cot(f*x+e)),x)
```

```
[Out] 1/f^2/a*(d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^
2+1/4*sin(f*x+e)^2)+c*f*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-d*e*(-1/
2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-I*d*(-1/2*(f*x+e)*cos(f*x+e)^2+1/4*s
in(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*e)+1/2*I*c*f*cos(f*x+e)^2-1/2*I*d*e*cos(f*
x+e)^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.38, size = 105, normalized size = 1.25

$$-\frac{d\,\cos\left(2e+2fx\right)-2\,df^2x^2+2\,cf\,\sin\left(2e+2fx\right)-4\,cf^2x+2\,dfx\,\sin\left(2e+2fx\right)+d\,\sin\left(2e+2fx\right.}{8\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*cot(e + f*x)*1i),x)
```

```
[Out] -(d*cos(2*e + 2*f*x) + d*sin(2*e + 2*f*x)*1i - 2*d*f^2*x^2 - c*f*cos(2*e +
2*f*x)*2i + 2*c*f*sin(2*e + 2*f*x) - 4*c*f^2*x - d*f*x*cos(2*e + 2*f*x)*2i
+ 2*d*f*x*sin(2*e + 2*f*x))/(8*a*f^2)
```

sympy [A] time = 0.23, size = 104, normalized size = 1.24

$$\begin{cases} -\frac{\left(-2icfe^{2ie}-2idfxe^{2ie}+de^{2ie}\right)e^{2ifx}}{8af^2} & \text{for } 8af^2 \neq 0 \\ -\frac{cxe^{2ie}}{2a} - \frac{dx^2e^{2ie}}{4a} & \text{otherwise} \end{cases} + \frac{cx}{2a} + \frac{dx^2}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x)
```

```
[Out] Piecewise((-(-2*I*c*f*exp(2*I*e) - 2*I*d*f*x*exp(2*I*e) + d*exp(2*I*e))*exp
(2*I*f*x)/(8*a*f**2), Ne(8*a*f**2, 0)), (-c*x*exp(2*I*e)/(2*a) - d*x**2*exp
(2*I*e)/(4*a), True)) + c*x/(2*a) + d*x**2/(4*a)
```

3.19 $\int \frac{1}{(c+dx)(a+ia\cot(e+fx))}\,dx$

Optimal. Leaf size=161

$$-\frac{i\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)}{2ad}-\frac{\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\cos\left(2e-\frac{2cf}{d}\right)}{2ad}+\frac{\sin\left(2e-\frac{2cf}{d}\right)\mathrm{Si}\left(2xf+\frac{2cf}{d}\right)}{2ad}-\frac{i\cos\left(2e-\frac{2cf}{d}\right.}{2}$$

```
[Out] -1/2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a/d+1/2*ln(d*x+c)/a/d-1/2*I*cos(-2
*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a/d+1/2*I*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)
/a/d-1/2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a/d
```

Rubi [A] time = 0.28, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3726, 3303, 3299, 3302}

$$-\frac{i\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{2ad}-\frac{\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{2ad}+\frac{\sin\left(2e-\frac{2cf}{d}\right)\mathrm{Si}\left(2xf+\right.}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + I*a*Cot[e + f*x])),x]
```

```
[Out] -(Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*a*d) + Log[c + d*
x]/(2*a*d) - ((I/2)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a
*d) - ((I/2)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a*d) + (
Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*a*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3726

```
Int[1/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Symb
ol] :> Simp[Log[c + d*x]/(2*a*d), x] + (Dist[1/(2*a), Int[Cos[2*e + 2*f*x]/
(c + d*x), x], x] + Dist[1/(2*b), Int[Sin[2*e + 2*f*x]/(c + d*x), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+ia\cot(e+fx))}\,dx = \frac{\log(c+dx)}{2ad} + \frac{i\int \frac{\sin\left(2\left(e+\frac{\pi}{2}\right)+2fx\right)}{c+dx}\,dx}{2a} + \frac{\int \frac{\cos\left(2\left(e+\frac{\pi}{2}\right)+2fx\right)}{c+dx}\,dx}{2a}$$

$$= \frac{\log(c+dx)}{2ad} - \frac{\left(i\cos\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\sin\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{2a} - \frac{\cos\left(2e-\frac{2cf}{d}\right)\int \frac{\cos\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{2a}$$

$$= -\frac{\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{2ad} + \frac{\log(c+dx)}{2ad} - \frac{i\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{2ad}$$

Mathematica [A] time = 0.28, size = 77, normalized size = 0.48

$$\frac{\log(c+dx) - \left(\mathrm{Ci}\left(\frac{2f(c+dx)}{d}\right) + i\mathrm{Si}\left(\frac{2f(c+dx)}{d}\right)\right)\left(\cos\left(2e-\frac{2cf}{d}\right) + i\sin\left(2e-\frac{2cf}{d}\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Cot[e + f*x])),x]
```

```
[Out] (Log[c + d*x] - (Cos[2*e - (2*c*f)/d] + I*Sin[2*e - (2*c*f)/d])*(CosIntegra
l[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]))/(2*a*d)
```

fricas [A] time = 0.57, size = 51, normalized size = 0.32

$$-\frac{\mathrm{Ei}\left(\frac{2i\,df x+2i\,cf}{d}\right)e^{\left(\frac{2i\,de-2i\,cf}{d}\right)} - \log\left(\frac{dx+c}{d}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(Ei((2*I*d*f*x + 2*I*c*f)/d)*e^((2*I*d*e - 2*I*c*f)/d) - log((d*x + c)
/d))/(a*d)
```

giac [B] time = 0.55, size = 367, normalized size = 2.28

$$-\frac{\cos\left(\frac{2cf}{d}\right)\cos(e)^2\,\mathrm{Ci}\left(\frac{2\left(dfx+cf\right)}{d}\right) - i\,\cos(e)^2\,\mathrm{Ci}\left(\frac{2\left(dfx+cf\right)}{d}\right)\sin\left(\frac{2cf}{d}\right) + 2i\,\cos\left(\frac{2cf}{d}\right)\cos(e)\,\mathrm{Ci}\left(\frac{2\left(dfx+cf\right)}{d}\right)\sin(e) +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*(cos(2*c*f/d)*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d) - I*cos(e)^2*co
s_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 2*I*cos(2*c*f/d)*cos(e)*cos_in
tegral(2*(d*f*x + c*f)/d)*sin(e) + 2*cos(e)*cos_integral(2*(d*f*x + c*f)/d)
*sin(2*c*f/d)*sin(e) - cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^
2 + I*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e)^2 + I*cos(2*c*f/d
)*cos(e)^2*sin_integral(2*(d*f*x + c*f)/d) + cos(e)^2*sin(2*c*f/d)*sin_inte
gral(2*(d*f*x + c*f)/d) - 2*cos(2*c*f/d)*cos(e)*sin(e)*sin_integral(2*(d*f*
x + c*f)/d) + 2*I*cos(e)*sin(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)/d
) - I*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) - sin(2*c*f/d)*
sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) - log(d*x + c))/(a*d)
```

maple [A] time = 0.60, size = 204, normalized size = 1.27

$$\frac{\ln\left(\left(fx+e\right)d+cf-de\right)}{2ad} - \frac{\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{2ad} - \frac{\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{2ad} - \frac{i\,\mathrm{Si}\left(2fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*cot(f*x+e)),x)
```

```
[Out] 1/2/a*ln((f*x+e)*d+c*f-d*e)/d-1/2/a*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-
d*e)/d)/d-1/2/a*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-1/2*I/a*Si
(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d+1/2*I/a*Ci(2*f*x+2*e+2*(c*f-
d*e)/d)*sin(2*(c*f-d*e)/d)/d
```

maxima [A] time = 0.45, size = 111, normalized size = 0.69

$$\frac{f\cos\left(-\frac{2\left(de-cf\right)}{d}\right)E_1\left(-\frac{2i\left(fx+e\right)d-2i\,de+2i\,cf}{d}\right)-i\,fE_1\left(-\frac{2i\left(fx+e\right)d-2i\,de+2i\,cf}{d}\right)\sin\left(-\frac{2\left(de-cf\right)}{d}\right)+f\log\left(\left(fx+e\right)d-d\right.}{2\,adf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*cos(-2*(d*e - c*f)/d)*exp_integral_e(1, -(2*I*(f*x + e)*d - 2*I*d*e
+ 2*I*c*f)/d) - I*f*exp_integral_e(1, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f
)/d)*sin(-2*(d*e - c*f)/d) + f*log((f*x + e)*d - d*e + c*f))/(a*d*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\cot\left(e+f\,x\right)1\mathrm{i}\right)\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{c\cot\left(e+fx\right)-ic+dx\cot\left(e+fx\right)-idx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x)
```

```
[Out] -I*Integral(1/(c*cot(e + f*x) - I*c + d*x*cot(e + f*x) - I*d*x), x)/a
```

3.20 $\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))}\,dx$

Optimal. Leaf size=166

$$\frac{f\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)}{ad^2}-\frac{if\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\cos\left(2e-\frac{2cf}{d}\right)}{ad^2}+\frac{if\sin\left(2e-\frac{2cf}{d}\right)\mathrm{Si}\left(2xf+\frac{2cf}{d}\right)}{ad^2}+\frac{f\cos\left(2e-\frac{2c}{d}\right.}{}$$

```
[Out] -I*f*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a/d^2-1/d/(d*x+c)/(a+I*a*cot(f*x+e
))+f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a/d^2-f*Ci(2*c*f/d+2*f*x)*sin(-2*e
+2*c*f/d)/a/d^2-I*f*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a/d^2
```

Rubi [A] time = 0.23, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3724, 3303, 3299, 3302}

$$\frac{f\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{ad^2}-\frac{if\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{ad^2}+\frac{if\sin\left(2e-\frac{2cf}{d}\right)\mathrm{Si}\left(2xf+\right.}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])),x]
```

```
[Out] ((-I)*f*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) - 1/(d
*(c + d*x)*(a + I*a*Cot[e + f*x])) + (f*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[
2*e - (2*c*f)/d])/(a*d^2) + (f*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d +
 2*f*x])/(a*d^2) + (I*f*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x]
)/(a*d^2)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3724

```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Dist[f/(a*d)
, Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*
x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))}\,dx = -\frac{1}{d(c+dx)(a+ia\cot(e+fx))} + \frac{(if)\int \frac{\cos\left(2\left(e+\frac{\pi}{2}\right)+2fx\right)}{c+dx}\,dx}{ad} - \frac{f\int \frac{\sin\left(2\left(e+\frac{\pi}{2}\right)+2fx\right)}{c+dx}\,dx}{ad}$$

$$= -\frac{1}{d(c+dx)(a+ia\cot(e+fx))} - \frac{\left(if\cos\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\cos\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{ad} + \frac{}{}$$

$$= -\frac{if\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{ad^2} - \frac{1}{d(c+dx)(a+ia\cot(e+fx))} + \frac{f\mathrm{Ci}}{}$$

Mathematica [A] time = 1.24, size = 215, normalized size = 1.30

$$\frac{\left(\cos\left(f\left(x-\frac{c}{d}\right)+e\right)+i\sin\left(f\left(x-\frac{c}{d}\right)+e\right)\right)\left(2f(c+dx)\mathrm{Ci}\left(\frac{2f(c+dx)}{d}\right)\left(\sin\left(e-\frac{f(c+dx)}{d}\right)-i\cos\left(e-\frac{f(c+dx)}{d}\right)\right)+2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])),x]
```

```
[Out] ((Cos[e + f*(-(c/d) + x)] + I*Sin[e + f*(-(c/d) + x)])*(d*(-Cos[e + f*(-(c/
d) + x)] + Cos[e + f*(c/d + x)] + I*(Sin[e + f*(-(c/d) + x)] + Sin[e + f*(c
/d + x)])) + 2*f*(c + d*x)*CosIntegral[(2*f*(c + d*x))/d]*((-I)*Cos[e - (f*
(c + d*x))/d] + Sin[e - (f*(c + d*x))/d]) + 2*f*(c + d*x)*(Cos[e - (f*(c +
d*x))/d] + I*Sin[e - (f*(c + d*x))/d])*SinIntegral[(2*f*(c + d*x))/d]))/(2*
a*d^2*(c + d*x))
```

fricas [A] time = 0.64, size = 72, normalized size = 0.43

$$\frac{\left(-2i\,dfx-2i\,cf\right)\mathrm{Ei}\left(\frac{2i\,dfx+2i\,cf}{d}\right)e^{\left(\frac{2i\,de-2i\,cf}{d}\right)}+de^{\left(2i\,fx+2i\,e\right)}-d}{2\left(ad^3x+acd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((-2*I*d*f*x - 2*I*c*f)*Ei((2*I*d*f*x + 2*I*c*f)/d)*e^((2*I*d*e - 2*I*c
*f)/d) + d*e^(2*I*f*x + 2*I*e) - d)/(a*d^3*x + a*c*d^2)
```

giac [B] time = 47.65, size = 367, normalized size = 2.21

$$-\frac{i\left(-2i\,(dx+c)\left(-\frac{i\,cf}{dx+c}+i\,f+\frac{i\,de}{dx+c}\right)f^2\mathrm{Ei}\left(\frac{2\,(dx+c)\left(-\frac{i\,cf}{dx+c}+i\,f+\frac{i\,de}{dx+c}\right)+2i\,cf-2i\,de}{d}\right)e^{\left(\frac{-2i\,cf+2i\,de}{d}\right)}+2\,cf^3\mathrm{Ei}\left(\frac{2\,(dx+c)\left(-\frac{i\,cf}{dx+c}+i\right.\right.\right.}{2\left(-i\,(dx+c)d^4\left(-\frac{i\,cf}{dx+c}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*I*(-2*I*(d*x + c)*(-I*c*f/(d*x + c) + I*f + I*d*e/(d*x + c))*f^2*Ei((2
*(d*x + c)*(-I*c*f/(d*x + c) + I*f + I*d*e/(d*x + c)) + 2*I*c*f - 2*I*d*e)/
d)*e^((-2*I*c*f + 2*I*d*e)/d) + 2*c*f^3*Ei((2*(d*x + c)*(-I*c*f/(d*x + c) +
 I*f + I*d*e/(d*x + c)) + 2*I*c*f - 2*I*d*e)/d)*e^((-2*I*c*f + 2*I*d*e)/d)
```

```
- 2*d*f^2*Ei((2*(d*x + c)*(-I*c*f/(d*x + c) + I*f + I*d*e/(d*x + c)) + 2*I*
c*f - 2*I*d*e)/d)*e^((-2*I*c*f + 2*I*d*e)/d + 1) + I*d*f^2*e^((d*x + c)*(-2
*I*c*f/(d*x + c) + 2*I*f + 2*I*d*e/(d*x + c))/d))*d^2/((-I*(d*x + c)*d^4*(-
I*c*f/(d*x + c) + I*f + I*d*e/(d*x + c)) + c*d^4*f - d^5*e)*a*f) - 1/2/((d*
x + c)*a*d)
```

maple [A] time = 0.58, size = 271, normalized size = 1.63

$$\frac{f\left(-\frac{1}{2\left(\left(fx+e\right)d+cf-de\right)d}+\frac{\cos\left(2fx+2e\right)}{2\left(\left(fx+e\right)d+cf-de\right)d}+\frac{\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}}{2d}-\frac{i\left(-\frac{2\sin\left(2fx+2e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}+\frac{4}{-}\right.}{}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x)
```

```
[Out] f/a*(-1/2/((f*x+e)*d+c*f-d*e)/d+1/2*cos(2*f*x+2*e)/((f*x+e)*d+c*f-d*e)/d+1/
2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f
-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d-1/4*I*(-2*sin(2*f*x+2*e)/((f*x+e)*d+c*f-d*
e)/d+2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2
*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d)/d))
```

maxima [A] time = 0.79, size = 122, normalized size = 0.73

$$\frac{8\,f^2\cos\left(-\frac{2\left(de-cf\right)}{d}\right)E_2\left(-\frac{2i\left(fx+e\right)d-2i\,de+2i\,cf}{d}\right)-8i\,f^2E_2\left(-\frac{2i\left(fx+e\right)d-2i\,de+2i\,cf}{d}\right)\sin\left(-\frac{2\left(de-cf\right)}{d}\right)-8\,f^2}{16\left(\left(fx+e\right)ad^2-ad^2e+acdf\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/16*(8*f^2*cos(-2*(d*e - c*f)/d)*exp_integral_e(2, -(2*I*(f*x + e)*d - 2*I
*d*e + 2*I*c*f)/d) - 8*I*f^2*exp_integral_e(2, -(2*I*(f*x + e)*d - 2*I*d*e
+ 2*I*c*f)/d)*sin(-2*(d*e - c*f)/d) - 8*f^2)/(((f*x + e)*a*d^2 - a*d^2*e +
a*c*d*f)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\cot\left(e+f\,x\right)1\mathrm{i}\right)\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{1}{c^2\cot\left(e+fx\right)-ic^2+2cdx\cot\left(e+fx\right)-2icdx+d^2x^2\cot\left(e+fx\right)-id^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*cot(f*x+e)),x)
```

```
[Out] -I*Integral(1/(c**2*cot(e + f*x) - I*c**2 + 2*c*d*x*cot(e + f*x) - 2*I*c*d*
x + d**2*x**2*cot(e + f*x) - I*d**2*x**2), x)/a
```

3.21 $\int \frac{1}{(c+dx)^3(a+ia\cot(e+fx))}\,dx$

Optimal. Leaf size=227

$$\frac{if^2\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)}{ad^3}+\frac{f^2\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\cos\left(2e-\frac{2cf}{d}\right)}{ad^3}-\frac{f^2\sin\left(2e-\frac{2cf}{d}\right)\mathrm{Si}\left(2xf+\frac{2cf}{d}\right)}{ad^3}+\frac{if^2\cos}{}$$

```
[Out] 1/2*I*f/a/d^2/(d*x+c)+f^2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a/d^3-1/2/d/(
d*x+c)^2/(a+I*a*cot(f*x+e))-I*f/d^2/(d*x+c)/(a+I*a*cot(f*x+e))+I*f^2*cos(-2
*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a/d^3-I*f^2*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/
d)/a/d^3+f^2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a/d^3
```

Rubi [A] time = 0.31, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3725, 3724, 3303, 3299, 3302}

$$\frac{if^2\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{ad^3}+\frac{f^2\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\cos\left(2e-\frac{2cf}{d}\right)}{ad^3}-\frac{f^2\sin\left(2e-\frac{2cf}{d}\right)\mathrm{Si}\left(\right.}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^3*(a + I*a*Cot[e + f*x])),x]
```

```
[Out] ((I/2)*f)/(a*d^2*(c + d*x)) + (f^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)
/d + 2*f*x])/(a*d^3) - 1/(2*d*(c + d*x)^2*(a + I*a*Cot[e + f*x])) - (I*f)/(
d^2*(c + d*x)*(a + I*a*Cot[e + f*x])) + (I*f^2*CosIntegral[(2*c*f)/d + 2*f*
x]*Sin[2*e - (2*c*f)/d])/(a*d^3) + (I*f^2*Cos[2*e - (2*c*f)/d]*SinIntegral[
(2*c*f)/d + 2*f*x])/(a*d^3) - (f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)
/d + 2*f*x])/(a*d^3)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3724

```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Dist[f/(a*d)
, Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*
x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3725

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[(f*(c + d*x)^(m + 2))/(b*d^2*(m + 1)*(m + 2)), x] + (Dist[(2*b
```

```
*f)/(a*d*(m + 1)), Int[(c + d*x)^(m + 1)/(a + b*Tan[e + f*x]), x], x] + Sim
p[(c + d*x)^(m + 1)/(d*(m + 1)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3(a+ia\cot(e+fx))}\,dx = \frac{if}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))} + \frac{(if)\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))}}{d}$$

$$= \frac{if}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))} - \frac{if}{d^2(c+dx)(a+ia\cot(e+$$

$$= \frac{if}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))} - \frac{if}{d^2(c+dx)(a+ia\cot(e+$$

$$= \frac{if}{2ad^2(c+dx)} + \frac{f^2\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{ad^3} - \frac{1}{2d(c+dx)^2(a+ia\cot(e$$

Mathematica [A] time = 1.59, size = 283, normalized size = 1.25

$$\frac{\left(\cos\left(f\left(x-\frac{c}{d}\right)+e\right)+i\sin\left(f\left(x-\frac{c}{d}\right)+e\right)\right)\left(4f^2(c+dx)^2\mathrm{Ci}\left(\frac{2f(c+dx)}{d}\right)\left(\cos\left(e-\frac{f(c+dx)}{d}\right)+i\sin\left(e-\frac{f(c+dx)}{d}\right)\right)+i\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + d*x)^3*(a + I*a*Cot[e + f*x])),x]
```

```
[Out] ((Cos[e + f*(-(c/d) + x)] + I*Sin[e + f*(-(c/d) + x)])*(4*f^2*(c + d*x)^2*C
osIntegral[(2*f*(c + d*x))/d]*(Cos[e - (f*(c + d*x))/d] + I*Sin[e - (f*(c +
 d*x))/d]) + I*(d*(I*d*Cos[e + f*(-(c/d) + x)] + ((-I)*d + 2*c*f + 2*d*f*x)
*Cos[e + f*(c/d + x)] + d*Sin[e + f*(-(c/d) + x)] + d*Sin[e + f*(c/d + x)]
+ (2*I)*c*f*Sin[e + f*(c/d + x)] + (2*I)*d*f*x*Sin[e + f*(c/d + x)]) + 4*f^
2*(c + d*x)^2*(Cos[e - (f*(c + d*x))/d] + I*Sin[e - (f*(c + d*x))/d])*SinIn
tegral[(2*f*(c + d*x))/d])))/(4*a*d^3*(c + d*x)^2)
```

fricas [A] time = 0.83, size = 118, normalized size = 0.52

$$\frac{4\left(d^2f^2x^2+2cdf^2x+c^2f^2\right)\mathrm{Ei}\left(\frac{2idfx+2icf}{d}\right)e^{\left(\frac{2ide-2icf}{d}\right)}-d^2+\left(2id^2fx+2icdf+d^2\right)e^{(2ifx+2ie)}}{4\left(ad^5x^2+2acd^4x+ac^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei((2*I*d*f*x + 2*I*c*f)/d)*e^
((2*I*d*e - 2*I*c*f)/d) - d^2 + (2*I*d^2*f*x + 2*I*c*d*f + d^2)*e^(2*I*f*x
+ 2*I*e))/(a*d^5*x^2 + 2*a*c*d^4*x + a*c^2*d^3)
```

giac [B] time = 0.62, size = 1630, normalized size = 7.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*(4*d^2*f^2*x^2*cos(2*c*f/d)*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d) -
4*I*d^2*f^2*x^2*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 8*I
*d^2*f^2*x^2*cos(2*c*f/d)*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) + 8
*d^2*f^2*x^2*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e) - 4
*d^2*f^2*x^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 + 4*I*d^
2*f^2*x^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e)^2 + 4*I*d^2*f
^2*x^2*cos(2*c*f/d)*cos(e)^2*sin_integral(2*(d*f*x + c*f)/d) + 4*d^2*f^2*x^
2*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - 8*d^2*f^2*x^2*cos
(2*c*f/d)*cos(e)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) + 8*I*d^2*f^2*x^2*c
os(e)*sin(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) - 4*I*d^2*f^2*x^2
*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) - 4*d^2*f^2*x^2*sin(
2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) + 8*c*d*f^2*x*cos(2*c*f/d
)*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d) - 8*I*c*d*f^2*x*cos(e)^2*cos_int
egral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 16*I*c*d*f^2*x*cos(2*c*f/d)*cos(e)*
cos_integral(2*(d*f*x + c*f)/d)*sin(e) + 16*c*d*f^2*x*cos(e)*cos_integral(2
*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e) - 8*c*d*f^2*x*cos(2*c*f/d)*cos_integr
al(2*(d*f*x + c*f)/d)*sin(e)^2 + 8*I*c*d*f^2*x*cos_integral(2*(d*f*x + c*f)
/d)*sin(2*c*f/d)*sin(e)^2 + 8*I*c*d*f^2*x*cos(2*c*f/d)*cos(e)^2*sin_integra
l(2*(d*f*x + c*f)/d) + 8*c*d*f^2*x*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*
f*x + c*f)/d) - 16*c*d*f^2*x*cos(2*c*f/d)*cos(e)*sin(e)*sin_integral(2*(d*f
*x + c*f)/d) + 16*I*c*d*f^2*x*cos(e)*sin(2*c*f/d)*sin(e)*sin_integral(2*(d*
f*x + c*f)/d) - 8*I*c*d*f^2*x*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x +
 c*f)/d) - 8*c*d*f^2*x*sin(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d
) + 4*c^2*f^2*cos(2*c*f/d)*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d) - 4*I*c
^2*f^2*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 8*I*c^2*f^2*
cos(2*c*f/d)*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) + 8*c^2*f^2*cos(
e)*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e) - 4*c^2*f^2*cos(2*c*
f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 + 4*I*c^2*f^2*cos_integral(2*
(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e)^2 + 4*I*c^2*f^2*cos(2*c*f/d)*cos(e)^2*
sin_integral(2*(d*f*x + c*f)/d) + 4*c^2*f^2*cos(e)^2*sin(2*c*f/d)*sin_integ
ral(2*(d*f*x + c*f)/d) - 8*c^2*f^2*cos(2*c*f/d)*cos(e)*sin(e)*sin_integral(
2*(d*f*x + c*f)/d) + 8*I*c^2*f^2*cos(e)*sin(2*c*f/d)*sin(e)*sin_integral(2*
(d*f*x + c*f)/d) - 4*I*c^2*f^2*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x
+ c*f)/d) - 4*c^2*f^2*sin(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d)
 + 2*I*d^2*f*x*cos(2*f*x)*cos(e)^2 - 2*d^2*f*x*cos(e)^2*sin(2*f*x) - 4*d^2*
f*x*cos(2*f*x)*cos(e)*sin(e) - 4*I*d^2*f*x*cos(e)*sin(2*f*x)*sin(e) - 2*I*d
^2*f*x*cos(2*f*x)*sin(e)^2 + 2*d^2*f*x*sin(2*f*x)*sin(e)^2 + 2*I*c*d*f*cos(
2*f*x)*cos(e)^2 - 2*c*d*f*cos(e)^2*sin(2*f*x) - 4*c*d*f*cos(2*f*x)*cos(e)*s
in(e) - 4*I*c*d*f*cos(e)*sin(2*f*x)*sin(e) - 2*I*c*d*f*cos(2*f*x)*sin(e)^2
+ 2*c*d*f*sin(2*f*x)*sin(e)^2 + d^2*cos(2*f*x)*cos(e)^2 + I*d^2*cos(e)^2*si
n(2*f*x) + 2*I*d^2*cos(2*f*x)*cos(e)*sin(e) - 2*d^2*cos(e)*sin(2*f*x)*sin(e
) - d^2*cos(2*f*x)*sin(e)^2 - I*d^2*sin(2*f*x)*sin(e)^2 - d^2)/(a*d^5*x^2 +
 2*a*c*d^4*x + a*c^2*d^3)
```

maple [A] time = 1.64, size = 143, normalized size = 0.63

$$-\frac{1}{4d\,(dx+c)^2\,a}-\frac{f^2 e^{2i\left(fx+e\right)}}{4a\,d^3\left(ifx+\frac{icf}{d}\right)^2}-\frac{f^2 e^{2i\left(fx+e\right)}}{2a\,d^3\left(ifx+\frac{icf}{d}\right)}-\frac{f^2 e^{-\frac{2i\left(cf-de\right)}{d}}\,\mathrm{Ei}\left(1,-2ifx-2ie-\frac{2\left(icf-ide\right)}{d}\right)}{a\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x)
```

```
[Out] -1/4/d/(d*x+c)^2/a-1/4*f^2/a/d^3*exp(2*I*(f*x+e))/(I*f*x+I/d*c*f)^2-1/2*f^2
/a/d^3*exp(2*I*(f*x+e))/(I*f*x+I/d*c*f)-f^2/a/d^3*exp(-2*I*(c*f-d*e)/d)*Ei(
1,-2*I*f*x-2*I*e-2*(I*c*f-I*d*e)/d)
```

maxima [A] time = 0.91, size = 160, normalized size = 0.70

$$\frac{2f^3\cos\left(-\frac{2(de-cf)}{d}\right)E_3\left(-\frac{2i(fx+e)d-2ide+2icf}{d}\right)-2if^3E_3\left(-\frac{2i(fx+e)d-2ide+2icf}{d}\right)\sin\left(-\frac{2(de-cf)}{d}\right)-f^3}{4\left(\left(fx+e\right)^2ad^3+ad^3e^2-2acd^2ef+ac^2df^2-2\left(ad^3e-acd^2f\right)\left(fx+e\right)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*f^3*cos(-2*(d*e - c*f)/d)*exp_integral_e(3, -(2*I*(f*x + e)*d - 2*I*
d*e + 2*I*c*f)/d) - 2*I*f^3*exp_integral_e(3, -(2*I*(f*x + e)*d - 2*I*d*e +
 2*I*c*f)/d)*sin(-2*(d*e - c*f)/d) - f^3)/(((f*x + e)^2*a*d^3 + a*d^3*e^2 -
 2*a*c*d^2*e*f + a*c^2*d*f^2 - 2*(a*d^3*e - a*c*d^2*f)*(f*x + e))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\cot\left(e+f\,x\right)1\mathrm{i}\right)\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^3),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{c^3\cot\left(e+fx\right)-ic^3+3c^2dx\cot\left(e+fx\right)-3ic^2dx+3cd^2x^2\cot\left(e+fx\right)-3icd^2x^2+d^3x^3\cot\left(e+fx\right)-id^3x^3}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(a+I*a*cot(f*x+e)),x)
```

```
[Out] -I*Integral(1/(c**3*cot(e + f*x) - I*c**3 + 3*c**2*d*x*cot(e + f*x) - 3*I*c
**2*d*x + 3*c*d**2*x**2*cot(e + f*x) - 3*I*c*d**2*x**2 + d**3*x**3*cot(e +
f*x) - I*d**3*x**3), x)/a
```

3.22 $\int \frac{(c+dx)^3}{(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=270

$$-\frac{3id^2(c+dx)e^{2ie+2ifx}}{8a^2f^3}+\frac{3id^2(c+dx)e^{4ie+4ifx}}{128a^2f^3}-\frac{3d(c+dx)^2e^{2ie+2ifx}}{8a^2f^2}+\frac{3d(c+dx)^2e^{4ie+4ifx}}{64a^2f^2}+\frac{i(c+dx)^3e^{2ie+2ifx}}{4a^2f}-\frac{i(}{}$$

```
[Out] 3/16*d^3*exp(2*I*e+2*I*f*x)/a^2/f^4-3/512*d^3*exp(4*I*e+4*I*f*x)/a^2/f^4-3/
8*I*d^2*exp(2*I*e+2*I*f*x)*(d*x+c)/a^2/f^3+3/128*I*d^2*exp(4*I*e+4*I*f*x)*(
d*x+c)/a^2/f^3-3/8*d*exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^2/f^2+3/64*d*exp(4*I*e+
4*I*f*x)*(d*x+c)^2/a^2/f^2+1/4*I*exp(2*I*e+2*I*f*x)*(d*x+c)^3/a^2/f-1/16*I*
exp(4*I*e+4*I*f*x)*(d*x+c)^3/a^2/f+1/16*(d*x+c)^4/a^2/d
```

Rubi [A] time = 0.28, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3729, 2176, 2194}

$$-\frac{3id^2(c+dx)e^{2ie+2ifx}}{8a^2f^3}+\frac{3id^2(c+dx)e^{4ie+4ifx}}{128a^2f^3}-\frac{3d(c+dx)^2e^{2ie+2ifx}}{8a^2f^2}+\frac{3d(c+dx)^2e^{4ie+4ifx}}{64a^2f^2}+\frac{i(c+dx)^3e^{2ie+2ifx}}{4a^2f}-\frac{i(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] (3*d^3*E^((2*I)*e + (2*I)*f*x))/(16*a^2*f^4) - (3*d^3*E^((4*I)*e + (4*I)*f*
x))/(512*a^2*f^4) - (((3*I)/8)*d^2*E^((2*I)*e + (2*I)*f*x)*(c + d*x))/(a^2*
f^3) + (((3*I)/128)*d^2*E^((4*I)*e + (4*I)*f*x)*(c + d*x))/(a^2*f^3) - (3*d
*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^2)/(8*a^2*f^2) + (3*d*E^((4*I)*e + (4*I)
*f*x)*(c + d*x)^2)/(64*a^2*f^2) + ((I/4)*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^
3)/(a^2*f) - ((I/16)*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^3)/(a^2*f) + (c + d*
x)^4/(16*a^2*d)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+ia\cot(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^3}{4a^2} - \frac{e^{2ie+2ifx}(c+dx)^3}{2a^2} + \frac{e^{4ie+4ifx}(c+dx)^3}{4a^2}\right) dx$$
$$= \frac{(c+dx)^4}{16a^2d} + \frac{\int e^{4ie+4ifx}(c+dx)^3\,dx}{4a^2} - \frac{\int e^{2ie+2ifx}(c+dx)^3\,dx}{2a^2}$$
$$= \frac{ie^{2ie+2ifx}(c+dx)^3}{4a^2f} - \frac{ie^{4ie+4ifx}(c+dx)^3}{16a^2f} + \frac{(c+dx)^4}{16a^2d} + \frac{(3id)\int e^{4ie+4ifx}(c+dx)^2\,dx}{16a^2f} -$$
$$= -\frac{3de^{2ie+2ifx}(c+dx)^2}{8a^2f^2} + \frac{3de^{4ie+4ifx}(c+dx)^2}{64a^2f^2} + \frac{ie^{2ie+2ifx}(c+dx)^3}{4a^2f} - \frac{ie^{4ie+4ifx}(c+dx}{16a^2f}$$
$$= -\frac{3id^2e^{2ie+2ifx}(c+dx)}{8a^2f^3} + \frac{3id^2e^{4ie+4ifx}(c+dx)}{128a^2f^3} - \frac{3de^{2ie+2ifx}(c+dx)^2}{8a^2f^2} + \frac{3de^{4ie+4ifx}(c}{64a^2f^2}$$
$$= \frac{3d^3e^{2ie+2ifx}}{16a^2f^4} - \frac{3d^3e^{4ie+4ifx}}{512a^2f^4} - \frac{3id^2e^{2ie+2ifx}(c+dx)}{8a^2f^3} + \frac{3id^2e^{4ie+4ifx}(c+dx)}{128a^2f^3} - \frac{3de^{2ie+2}}{8}$$

Mathematica [A] time = 1.61, size = 362, normalized size = 1.34

$$(\cos(2(e+fx)) + i\sin(2(e+fx)))\left(\left(32c^3f^3(4fx-i) + 24c^2df^2\left(8f^2x^2 - 4ifx + 1\right) + 4cd^2f\left(32f^3x^3 - 24if^2x^2 +\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] ((Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*((32*c^3*f^3*(-I + 4*f*x) + 24*c^2
*d*f^2*(1 - (4*I)*f*x + 8*f^2*x^2) + 4*c*d^2*f*(3*I + 12*f*x - (24*I)*f^2*x
^2 + 32*f^3*x^3) + d^3*(-3 + (12*I)*f*x + 24*f^2*x^2 - (32*I)*f^3*x^3 + 32*
f^4*x^4))*Cos[2*(e + f*x)] - I*(-32*(4*c^3*f^3 + 6*c^2*d*f^2*(I + 2*f*x) +
6*c*d^2*f*(-1 + (2*I)*f*x + 2*f^2*x^2) + d^3*(-3*I - 6*f*x + (6*I)*f^2*x^2
+ 4*f^3*x^3)) + (32*c^3*f^3*(I + 4*f*x) + 24*c^2*d*f^2*(-1 + (4*I)*f*x + 8*
f^2*x^2) + 4*c*d^2*f*(-3*I - 12*f*x + (24*I)*f^2*x^2 + 32*f^3*x^3) + d^3*(3
 - (12*I)*f*x - 24*f^2*x^2 + (32*I)*f^3*x^3 + 32*f^4*x^4))*Sin[2*(e + f*x)]
)))/(512*a^2*f^4)
```

fricas [A] time = 1.90, size = 253, normalized size = 0.94

$$32\,d^3f^4x^4 + 128\,cd^2f^4x^3 + 192\,c^2df^4x^2 + 128\,c^3f^4x + \left(-32i\,d^3f^3x^3 - 32i\,c^3f^3 + 24\,c^2df^2 + 12i\,cd^2f - 3\,d^3 + \left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/512*(32*d^3*f^4*x^4 + 128*c*d^2*f^4*x^3 + 192*c^2*d*f^4*x^2 + 128*c^3*f^4
*x + (-32*I*d^3*f^3*x^3 - 32*I*c^3*f^3 + 24*c^2*d*f^2 + 12*I*c*d^2*f - 3*d^
3 + (-96*I*c*d^2*f^3 + 24*d^3*f^2)*x^2 + (-96*I*c^2*d*f^3 + 48*c*d^2*f^2 +
12*I*d^3*f)*x)*e^(4*I*f*x + 4*I*e) + (128*I*d^3*f^3*x^3 + 128*I*c^3*f^3 - 1
92*c^2*d*f^2 - 192*I*c*d^2*f + 96*d^3 + (384*I*c*d^2*f^3 - 192*d^3*f^2)*x^2
 + (384*I*c^2*d*f^3 - 384*c*d^2*f^2 - 192*I*d^3*f)*x)*e^(2*I*f*x + 2*I*e))/
(a^2*f^4)
```

giac [B] time = 0.51, size = 433, normalized size = 1.60

$$32\,d^3f^4x^4 + 128\,cd^2f^4x^3 - 32i\,d^3f^3x^3e^{\left(4ifx+4ie\right)} + 128i\,d^3f^3x^3e^{\left(2ifx+2ie\right)} + 192\,c^2df^4x^2 - 96i\,cd^2f^3x^2e^{\left(4ifx+4ie\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/512*(32*d^3*f^4*x^4 + 128*c*d^2*f^4*x^3 - 32*I*d^3*f^3*x^3*e^(4*I*f*x + 4
*I*e) + 128*I*d^3*f^3*x^3*e^(2*I*f*x + 2*I*e) + 192*c^2*d*f^4*x^2 - 96*I*c*
d^2*f^3*x^2*e^(4*I*f*x + 4*I*e) + 384*I*c*d^2*f^3*x^2*e^(2*I*f*x + 2*I*e) +
 128*c^3*f^4*x - 96*I*c^2*d*f^3*x*e^(4*I*f*x + 4*I*e) + 24*d^3*f^2*x^2*e^(4
*I*f*x + 4*I*e) + 384*I*c^2*d*f^3*x*e^(2*I*f*x + 2*I*e) - 192*d^3*f^2*x^2*e
^(2*I*f*x + 2*I*e) - 32*I*c^3*f^3*e^(4*I*f*x + 4*I*e) + 48*c*d^2*f^2*x*e^(4
*I*f*x + 4*I*e) + 128*I*c^3*f^3*e^(2*I*f*x + 2*I*e) - 384*c*d^2*f^2*x*e^(2*
I*f*x + 2*I*e) + 24*c^2*d*f^2*e^(4*I*f*x + 4*I*e) + 12*I*d^3*f*x*e^(4*I*f*x
 + 4*I*e) - 192*c^2*d*f^2*e^(2*I*f*x + 2*I*e) - 192*I*d^3*f*x*e^(2*I*f*x +
2*I*e) + 12*I*c*d^2*f*e^(4*I*f*x + 4*I*e) - 192*I*c*d^2*f*e^(2*I*f*x + 2*I*
e) - 3*d^3*e^(4*I*f*x + 4*I*e) + 96*d^3*e^(2*I*f*x + 2*I*e))/(a^2*f^4)
```

maple [B] time = 2.16, size = 2261, normalized size = 8.37

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*cot(f*x+e))^2,x)
```

```
[Out] -1/a^2/f*(1/2*I*c^3*sin(f*x+e)^4+6*I/f^2*c*d^2*(1/4*(f*x+e)^2*sin(f*x+e)^4-
1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3
/32*(f*x+e)^2-1/32*sin(f*x+e)^4-3/32*sin(f*x+e)^2)+6*I/f*c^2*d*(1/4*(f*x+e)
*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e
)-12/f^2*c*d^2*e*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/16*(
f*x+e)^2+1/16*sin(f*x+e)^2-(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(
f*x+e)+3/8*f*x+3/8*e)-1/16*sin(f*x+e)^4)-6*I/f^3*d^3*e*(1/4*(f*x+e)^2*sin(f
*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+
3/8*e)+3/32*(f*x+e)^2-1/32*sin(f*x+e)^4-3/32*sin(f*x+e)^2)+6*I/f^3*d^3*e^2*
(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/3
2*f*x-3/32*e)-1/f^3*d^3*((f*x+e)^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-3/4*(f*x+e)^2*cos(f*x+e)^2+3/2*(f*x+e)*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*
x+1/2*e)-3/8*(f*x+e)^2-3/8*sin(f*x+e)^2-3/8*(f*x+e)^4)+2/f^3*d^3*((f*x+e)^3
*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3/16*(f*x+e)^2*cos(f*x+e)^2+3/8
*(f*x+e)*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-21/128*(f*x+e)^2-3/128*s
in(f*x+e)^2-3/32*(f*x+e)^4-(f*x+e)^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*co
s(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2*sin(f*x+e)^4+3/8*(f*x+e)*(-1/4*(sin(
f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3/128*sin(f*x+e)^4)+3/f*
c^2*d*e*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+6/f^2*c*d^2*e*((f*x+e)*(
-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+6
/f^2*c*d^2*e^2*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(f*x+e)*cos(f*x+e)+1/8*
f*x+1/8*e)-3/f^2*c*d^2*e^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-6/f*c
^2*d*e*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(f*x+e)*cos(f*x+e)+1/8*f*x+1/8*
e)-3/f*c^2*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e
)^2+1/4*sin(f*x+e)^2)+2*I/f^3*d^3*(1/4*(f*x+e)^3*sin(f*x+e)^4-3/4*(f*x+e)^2
*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/32*(f*x+e)
*sin(f*x+e)^4-3/128*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-27/256*f*x-27/
256*e+9/32*(f*x+e)*cos(f*x+e)^2-9/64*sin(f*x+e)*cos(f*x+e)+3/16*(f*x+e)^3)-
2/f^3*d^3*e^3*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(f*x+e)*cos(f*x+e)+1/8*f
*x+1/8*e)-3/f^3*d^3*e^2*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)
-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)+6/f^2*c*d^2*((f*x+e)^2*(-1/2*sin(f*x+e)*co
s(f*x+e)+1/2*f*x+1/2*e)-1/8*(f*x+e)*cos(f*x+e)^2+1/16*sin(f*x+e)*cos(f*x+e)
+7/64*f*x+7/64*e-1/12*(f*x+e)^3-(f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e
))*cos(f*x+e)+3/8*f*x+3/8*e)-1/8*(f*x+e)*sin(f*x+e)^4-1/32*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e))+3/f^3*d^3*e*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e
)+1/2*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x
+1/4*e-1/3*(f*x+e)^3)-3/f^2*c*d^2*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)-1/2*(f*x+e)*cos(f*x+e)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4
*e-1/3*(f*x+e)^3)-6/f^3*d^3*e*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*
```

```
x+1/2*e)-1/8*(f*x+e)*cos(f*x+e)^2+1/16*sin(f*x+e)*cos(f*x+e)+7/64*f*x+7/64*
e-1/12*(f*x+e)^3-(f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3
/8*f*x+3/8*e)-1/8*(f*x+e)*sin(f*x+e)^4-1/32*(sin(f*x+e)^3+3/2*sin(f*x+e))*c
os(f*x+e))+6/f^3*d^3*e^2*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e
)-1/16*(f*x+e)^2+1/16*sin(f*x+e)^2-(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+
e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/16*sin(f*x+e)^4)+6/f*c^2*d*((f*x+e)*(-1/2*s
in(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/16*(f*x+e)^2+1/16*sin(f*x+e)^2-(f*x+e
)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/16*sin(f*
x+e)^4)+1/f^3*d^3*e^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2*I/f^3*
d^3*e^3*sin(f*x+e)^4-3/2*I/f*c^2*d*e*sin(f*x+e)^4+3/2*I/f^2*c*d^2*e^2*sin(f
*x+e)^4-12*I/f^2*c*d^2*e*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)-c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*
f*x+1/2*e)+2*c^3*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(f*x+e)*cos(f*x+e)+1/
8*f*x+1/8*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 1.12, size = 294, normalized size = 1.09

$$e^{e\,2i+f\,x\,2i}\left(-\frac{\left(-4\,c^3\,f^3-c^2\,d\,f^2\,6i+6\,c\,d^2\,f+d^3\,3i\right)\,1i}{16\,a^2\,f^4}+\frac{d^3\,x^3\,1i}{4\,a^2\,f}+\frac{d\,x\,\left(2\,c^2\,f^2+c\,d\,f\,2i-d^2\right)\,3i}{8\,a^2\,f^3}+\frac{d^2\,x^2\,\left(2\,c\,f\right.}{8\,a^2\,}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*cot(e + f*x)*1i)^2,x)
```

```
[Out] exp(e*2i + f*x*2i)*((d^3*x^3*1i)/(4*a^2*f) - ((d^3*3i - 4*c^3*f^3 - c^2*d*f
^2*6i + 6*c*d^2*f)*1i)/(16*a^2*f^4) + (d*x*(2*c^2*f^2 - d^2 + c*d*f*2i)*3i)
/(8*a^2*f^3) + (d^2*x^2*(d*1i + 2*c*f)*3i)/(8*a^2*f^2)) - exp(e*4i + f*x*4i
)*((d^3*x^3*1i)/(16*a^2*f) - ((d^3*3i - 32*c^3*f^3 - c^2*d*f^2*24i + 12*c*d
^2*f)*1i)/(512*a^2*f^4) + (d*x*(8*c^2*f^2 - d^2 + c*d*f*4i)*3i)/(128*a^2*f^
3) + (d^2*x^2*(d*1i + 4*c*f)*3i)/(64*a^2*f^2)) + (c^3*x)/(4*a^2) + (d^3*x^4
)/(16*a^2) + (3*c^2*d*x^2)/(8*a^2) + (c*d^2*x^3)/(4*a^2)
```

sympy [A] time = 0.64, size = 653, normalized size = 2.42

$$\begin{cases}\frac{\left(2048ia^2c^3f^7e^{2ie}+6144ia^2c^2df^7xe^{2ie}-3072a^2c^2df^6e^{2ie}+6144ia^2cd^2f^7x^2e^{2ie}-6144a^2cd^2f^6xe^{2ie}-3072ia^2cd^2f^5e^{2ie}+2048ia^2d^3f^7x^3e^{2ie}-3072a^2d^3f^6x^2\right.}{} \\ \frac{x^4\left(d^3e^{4ie}-2d^3e^{2ie}\right)}{16a^2}+\frac{x^3\left(cd^2e^{4ie}-2cd^2e^{2ie}\right)}{4a^2}+\frac{x^2\left(3c^2de^{4ie}-6c^2de^{2ie}\right)}{8a^2}+\frac{x\left(c^3e^{4ie}-2c^3e^{2ie}\right)}{4a^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] Piecewise((((2048*I*a**2*c**3*f**7*exp(2*I*e) + 6144*I*a**2*c**2*d*f**7*x*e
xp(2*I*e) - 3072*a**2*c**2*d*f**6*exp(2*I*e) + 6144*I*a**2*c*d**2*f**7*x**2
*exp(2*I*e) - 6144*a**2*c*d**2*f**6*x*exp(2*I*e) - 3072*I*a**2*c*d**2*f**5*
exp(2*I*e) + 2048*I*a**2*d**3*f**7*x**3*exp(2*I*e) - 3072*a**2*d**3*f**6*x*
*2*exp(2*I*e) - 3072*I*a**2*d**3*f**5*x*exp(2*I*e) + 1536*a**2*d**3*f**4*ex
p(2*I*e))*exp(2*I*f*x) + (-512*I*a**2*c**3*f**7*exp(4*I*e) - 1536*I*a**2*c*
*2*d*f**7*x*exp(4*I*e) + 384*a**2*c**2*d*f**6*exp(4*I*e) - 1536*I*a**2*c*d*
```

```
*2*f**7*x**2*exp(4*I*e) + 768*a**2*c*d**2*f**6*x*exp(4*I*e) + 192*I*a**2*c*
d**2*f**5*exp(4*I*e) - 512*I*a**2*d**3*f**7*x**3*exp(4*I*e) + 384*a**2*d**3
*f**6*x**2*exp(4*I*e) + 192*I*a**2*d**3*f**5*x*exp(4*I*e) - 48*a**2*d**3*f*
*4*exp(4*I*e))*exp(4*I*f*x))/(8192*a**4*f**8), Ne(8192*a**4*f**8, 0)), (x**
4*(d**3*exp(4*I*e) - 2*d**3*exp(2*I*e))/(16*a**2) + x**3*(c*d**2*exp(4*I*e)
 - 2*c*d**2*exp(2*I*e))/(4*a**2) + x**2*(3*c**2*d*exp(4*I*e) - 6*c**2*d*exp
(2*I*e))/(8*a**2) + x*(c**3*exp(4*I*e) - 2*c**3*exp(2*I*e))/(4*a**2), True)
) + c**3*x/(4*a**2) + 3*c**2*d*x**2/(8*a**2) + c*d**2*x**3/(4*a**2) + d**3*
x**4/(16*a**2)
```

3.23 $\int \frac{(c+dx)^2}{(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=202

$$-\frac{d(c+dx)e^{2ie+2ifx}}{4a^2f^2}+\frac{d(c+dx)e^{4ie+4ifx}}{32a^2f^2}+\frac{i(c+dx)^2e^{2ie+2ifx}}{4a^2f}-\frac{i(c+dx)^2e^{4ie+4ifx}}{16a^2f}+\frac{(c+dx)^3}{12a^2d}-\frac{id^2e^{2ie+2ifx}}{8a^2f^3}+\frac{id^2e^{4ie+4if}}{128a^2f^3}$$

```
[Out] -1/8*I*d^2*exp(2*I*e+2*I*f*x)/a^2/f^3+1/128*I*d^2*exp(4*I*e+4*I*f*x)/a^2/f^
3-1/4*d*exp(2*I*e+2*I*f*x)*(d*x+c)/a^2/f^2+1/32*d*exp(4*I*e+4*I*f*x)*(d*x+c
)/a^2/f^2+1/4*I*exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^2/f-1/16*I*exp(4*I*e+4*I*f*x
)*(d*x+c)^2/a^2/f+1/12*(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3729, 2176, 2194}

$$-\frac{d(c+dx)e^{2ie+2ifx}}{4a^2f^2}+\frac{d(c+dx)e^{4ie+4ifx}}{32a^2f^2}+\frac{i(c+dx)^2e^{2ie+2ifx}}{4a^2f}-\frac{i(c+dx)^2e^{4ie+4ifx}}{16a^2f}+\frac{(c+dx)^3}{12a^2d}-\frac{id^2e^{2ie+2ifx}}{8a^2f^3}+\frac{id^2e^{4ie+4if}}{128a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] ((-I/8)*d^2*E^((2*I)*e + (2*I)*f*x))/(a^2*f^3) + ((I/128)*d^2*E^((4*I)*e +
(4*I)*f*x))/(a^2*f^3) - (d*E^((2*I)*e + (2*I)*f*x)*(c + d*x))/(4*a^2*f^2) +
 (d*E^((4*I)*e + (4*I)*f*x)*(c + d*x))/(32*a^2*f^2) + ((I/4)*E^((2*I)*e + (
2*I)*f*x)*(c + d*x)^2)/(a^2*f) - ((I/16)*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^
2)/(a^2*f) + (c + d*x)^3/(12*a^2*d)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+ia\cot(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^2}{4a^2} - \frac{e^{2ie+2ifx}(c+dx)^2}{2a^2} + \frac{e^{4ie+4ifx}(c+dx)^2}{4a^2}\right) dx$$
$$= \frac{(c+dx)^3}{12a^2d} + \frac{\int e^{4ie+4ifx}(c+dx)^2\,dx}{4a^2} - \frac{\int e^{2ie+2ifx}(c+dx)^2\,dx}{2a^2}$$
$$= \frac{ie^{2ie+2ifx}(c+dx)^2}{4a^2f} - \frac{ie^{4ie+4ifx}(c+dx)^2}{16a^2f} + \frac{(c+dx)^3}{12a^2d} + \frac{(id)\int e^{4ie+4ifx}(c+dx)\,dx}{8a^2f}$$
$$= -\frac{de^{2ie+2ifx}(c+dx)}{4a^2f^2} + \frac{de^{4ie+4ifx}(c+dx)}{32a^2f^2} + \frac{ie^{2ie+2ifx}(c+dx)^2}{4a^2f} - \frac{ie^{4ie+4ifx}(c+dx)^2}{16a^2f}$$
$$= -\frac{id^2e^{2ie+2ifx}}{8a^2f^3} + \frac{id^2e^{4ie+4ifx}}{128a^2f^3} - \frac{de^{2ie+2ifx}(c+dx)}{4a^2f^2} + \frac{de^{4ie+4ifx}(c+dx)}{32a^2f^2} + \frac{ie^{2ie+2ifx}(c$$

Mathematica [A] time = 0.74, size = 255, normalized size = 1.26

$$\frac{32f^3x\left(3c^2+3cdx+d^2x^2\right) + 48(\cos(2e)+i\sin(2e))\cos(2fx)((1+i)cf+d(-1+(1+i)fx))((1+i)cf+d((1+$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] (32*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 48*((1 + I)*c*f + d*(-1 + (1 + I)*f
*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e])
 - 3*((2 + 2*I)*c*f + d*(-1 + (2 + 2*I)*f*x))*((2 + 2*I)*c*f + d*(I + (2 +
2*I)*f*x))*Cos[4*f*x]*(Cos[4*e] + I*Sin[4*e]) + (48*I)*((1 + I)*c*f + d*(-1
 + (1 + I)*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*(Cos[2*e] + I*Sin[2*e]
)*Sin[2*f*x] - 3*(d - (2 + 2*I)*c*f - (2 + 2*I)*d*f*x)*(d + (2 - 2*I)*c*f +
 (2 - 2*I)*d*f*x)*(Cos[4*e] + I*Sin[4*e])*Sin[4*f*x])/(384*a^2*f^3)
```

fricas [A] time = 1.51, size = 151, normalized size = 0.75

$$\frac{32\,d^2f^3x^3 + 96\,cdf^3x^2 + 96\,c^2f^3x + \left(-24i\,d^2f^2x^2 - 24i\,c^2f^2 + 12\,cdf + 3i\,d^2 + \left(-48i\,cdf^2 + 12\,d^2f\right)x\right)e^{(4ifx+}}{384\,a^2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/384*(32*d^2*f^3*x^3 + 96*c*d*f^3*x^2 + 96*c^2*f^3*x + (-24*I*d^2*f^2*x^2
 - 24*I*c^2*f^2 + 12*c*d*f + 3*I*d^2 + (-48*I*c*d*f^2 + 12*d^2*f)*x)*e^(4*I*
f*x + 4*I*e) + (96*I*d^2*f^2*x^2 + 96*I*c^2*f^2 - 96*c*d*f - 48*I*d^2 + (19
2*I*c*d*f^2 - 96*d^2*f)*x)*e^(2*I*f*x + 2*I*e))/(a^2*f^3)
```

giac [A] time = 0.50, size = 247, normalized size = 1.22

$$\frac{32\,d^2f^3x^3 + 96\,cdf^3x^2 - 24i\,d^2f^2x^2e^{(4ifx+4ie)} + 96i\,d^2f^2x^2e^{(2ifx+2ie)} + 96\,c^2f^3x - 48i\,cdf^2xe^{(4ifx+4ie)} + 192i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/384*(32*d^2*f^3*x^3 + 96*c*d*f^3*x^2 - 24*I*d^2*f^2*x^2*e^(4*I*f*x + 4*I*
e) + 96*I*d^2*f^2*x^2*e^(2*I*f*x + 2*I*e) + 96*c^2*f^3*x - 48*I*c*d*f^2*x*e
^(4*I*f*x + 4*I*e) + 192*I*c*d*f^2*x*e^(2*I*f*x + 2*I*e) - 24*I*c^2*f^2*e^(
4*I*f*x + 4*I*e) + 12*d^2*f*x*e^(4*I*f*x + 4*I*e) + 96*I*c^2*f^2*e^(2*I*f*x
```

```
 + 2*I*e) - 96*d^2*f*x*e^(2*I*f*x + 2*I*e) + 12*c*d*f*e^(4*I*f*x + 4*I*e) -
 96*c*d*f*e^(2*I*f*x + 2*I*e) + 3*I*d^2*e^(4*I*f*x + 4*I*e) - 48*I*d^2*e^(2
*I*f*x + 2*I*e))/(a^2*f^3)
```

maple [B] time = 1.95, size = 1073, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x)
```

```
[Out] -1/a^2/f*(-I/f*c*d*e*sin(f*x+e)^4-4*I/f^2*d^2*e*(1/4*(f*x+e)*sin(f*x+e)^4+1
/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)+2*I/f^2*d^2*(
1/4*(f*x+e)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*
cos(f*x+e)+3/8*f*x+3/8*e)+3/32*(f*x+e)^2-1/32*sin(f*x+e)^4-3/32*sin(f*x+e)^
2)+1/2*I/f^2*d^2*e^2*sin(f*x+e)^4+4*I/f*c*d*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*
(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)+1/2*I*c^2*sin(f*x
+e)^4+2/f^2*d^2*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/8*(
f*x+e)*cos(f*x+e)^2+1/16*sin(f*x+e)*cos(f*x+e)+7/64*f*x+7/64*e-1/12*(f*x+e)
^3-(f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-
1/8*(f*x+e)*sin(f*x+e)^4-1/32*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e))+4/f
*c*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/16*(f*x+e)^2+1/1
6*sin(f*x+e)^2-(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f
*x+3/8*e)-1/16*sin(f*x+e)^4)-4/f^2*d^2*e*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-1/16*(f*x+e)^2+1/16*sin(f*x+e)^2-(f*x+e)*(-1/4*(sin(f*x+e
)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/16*sin(f*x+e)^4)+2*c^2*(-1/
4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(f*x+e)*cos(f*x+e)+1/8*f*x+1/8*e)-4/f*c*d*
e*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(f*x+e)*cos(f*x+e)+1/8*f*x+1/8*e)+2/
f^2*d^2*e^2*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(f*x+e)*cos(f*x+e)+1/8*f*x
+1/8*e)-1/f^2*d^2*((f*x+e)^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/2
*(f*x+e)*cos(f*x+e)^2+1/4*sin(f*x+e)*cos(f*x+e)+1/4*f*x+1/4*e-1/3*(f*x+e)^3
)-2/f*c*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2
+1/4*sin(f*x+e)^2)+2/f^2*d^2*e*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x
+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)+2/f*c*d*e*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/f^2*d^2*
e^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.58, size = 186, normalized size = 0.92

$$e^{e\,2i+f\,x\,2i}\left(\frac{\left(2\,c^2\,f^2+c\,d\,f\,2i-d^2\right)1i}{8\,a^2\,f^3}+\frac{d^2\,x^2\,1i}{4\,a^2\,f}+\frac{d\,x\,\left(2\,c\,f+d\,1i\right)1i}{4\,a^2\,f^2}\right)-e^{e\,4i+f\,x\,4i}\left(\frac{\left(8\,c^2\,f^2+c\,d\,f\,4i-d^2\right)1i}{128\,a^2\,f^3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*cot(e + f*x)*1i)^2,x)
```

```
[Out] exp(e*2i + f*x*2i)*(((2*c^2*f^2 - d^2 + c*d*f*2i)*1i)/(8*a^2*f^3) + (d^2*x^
2*1i)/(4*a^2*f) + (d*x*(d*1i + 2*c*f)*1i)/(4*a^2*f^2)) - exp(e*4i + f*x*4i)
*(((8*c^2*f^2 - d^2 + c*d*f*4i)*1i)/(128*a^2*f^3) + (d^2*x^2*1i)/(16*a^2*f)
```

```
 + (d*x*(d*1i + 4*c*f)*1i)/(32*a^2*f^2)) + (c^2*x)/(4*a^2) + (d^2*x^3)/(12*
a^2) + (c*d*x^2)/(4*a^2)
```

sympy [A] time = 0.48, size = 406, normalized size = 2.01

$$\begin{cases} \frac{\left(256ia^2c^2f^5e^{2ie}+512ia^2cdf^5xe^{2ie}-256a^2cdf^4e^{2ie}+256ia^2d^2f^5x^2e^{2ie}-256a^2d^2f^4xe^{2ie}-128ia^2d^2f^3e^{2ie}\right)e^{2ifx}+\left(-64ia^2c^2f^5e^{4ie}-128ia^2cdf^5xe^{4ie}+32\right.}{1024a^4f^6} \\ \frac{x^3\left(d^2e^{4ie}-2d^2e^{2ie}\right)}{12a^2}+\frac{x^2\left(cde^{4ie}-2cde^{2ie}\right)}{4a^2}+\frac{x\left(c^2e^{4ie}-2c^2e^{2ie}\right)}{4a^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] Piecewise((((256*I*a**2*c**2*f**5*exp(2*I*e) + 512*I*a**2*c*d*f**5*x*exp(2*
I*e) - 256*a**2*c*d*f**4*exp(2*I*e) + 256*I*a**2*d**2*f**5*x**2*exp(2*I*e)
- 256*a**2*d**2*f**4*x*exp(2*I*e) - 128*I*a**2*d**2*f**3*exp(2*I*e))*exp(2*
I*f*x) + (-64*I*a**2*c**2*f**5*exp(4*I*e) - 128*I*a**2*c*d*f**5*x*exp(4*I*e
) + 32*a**2*c*d*f**4*exp(4*I*e) - 64*I*a**2*d**2*f**5*x**2*exp(4*I*e) + 32*
a**2*d**2*f**4*x*exp(4*I*e) + 8*I*a**2*d**2*f**3*exp(4*I*e))*exp(4*I*f*x))/
(1024*a**4*f**6), Ne(1024*a**4*f**6, 0)), (x**3*(d**2*exp(4*I*e) - 2*d**2*e
xp(2*I*e))/(12*a**2) + x**2*(c*d*exp(4*I*e) - 2*c*d*exp(2*I*e))/(4*a**2) +
x*(c**2*exp(4*I*e) - 2*c**2*exp(2*I*e))/(4*a**2), True)) + c**2*x/(4*a**2)
+ c*d*x**2/(4*a**2) + d**2*x**3/(12*a**2)
```

3.24 $\int \frac{c+dx}{(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=151

$$-\frac{i(c+dx)}{4f\left(a^2+ia^2\cot(e+fx)\right)}+\frac{x(c+dx)}{4a^2}+\frac{3d}{16f^2\left(a^2+ia^2\cot(e+fx)\right)}+\frac{3idx}{16a^2f}-\frac{dx^2}{8a^2}-\frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2}+\frac{}{16f^2(}$$

```
[Out] 3/16*I*d*x/a^2/f-1/8*d*x^2/a^2+1/4*x*(d*x+c)/a^2+1/16*d/f^2/(a+I*a*cot(f*x+
e))^2-1/4*I*(d*x+c)/f/(a+I*a*cot(f*x+e))^2+3/16*d/f^2/(a^2+I*a^2*cot(f*x+e)
)-1/4*I*(d*x+c)/f/(a^2+I*a^2*cot(f*x+e))
```

Rubi [A] time = 0.14, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3479, 8, 3730}

$$-\frac{i(c+dx)}{4f\left(a^2+ia^2\cot(e+fx)\right)}+\frac{x(c+dx)}{4a^2}+\frac{3d}{16f^2\left(a^2+ia^2\cot(e+fx)\right)}+\frac{3idx}{16a^2f}-\frac{dx^2}{8a^2}-\frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2}+\frac{}{16f^2(}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] (((3*I)/16)*d*x)/(a^2*f) - (d*x^2)/(8*a^2) + (x*(c + d*x))/(4*a^2) + d/(16*
f^2*(a + I*a*Cot[e + f*x])^2) - ((I/4)*(c + d*x))/(f*(a + I*a*Cot[e + f*x])
^2) + (3*d)/(16*f^2*(a^2 + I*a^2*Cot[e + f*x])) - ((I/4)*(c + d*x))/(f*(a^2
 + I*a^2*Cot[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3730

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Dist[(c + d*x)
^m, u, x] - Dist[d*m, Int[Dist[(c + d*x)^(m - 1), u, x], x], x]] /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+ia\cot(e+fx))^2}\,dx = \frac{x(c+dx)}{4a^2} - \frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{4f\left(a^2+ia^2\cot(e+fx)\right)} - d\int\left(\frac{x}{4a^2} - \frac{}{4}\right.$$

$$= -\frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} - \frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{4f\left(a^2+ia^2\cot(e+fx)\right)} + \frac{(id)\int}{}$$

$$= -\frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} + \frac{d}{16f^2(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2} + \frac{}{8f^2\left(}$$

$$= \frac{idx}{8a^2f} - \frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} + \frac{d}{16f^2(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2} +$$

$$= \frac{3idx}{16a^2f} - \frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} + \frac{d}{16f^2(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2}$$

Mathematica [A] time = 0.61, size = 165, normalized size = 1.09

$$\frac{8i(2cf+d(2fx+i))\cos(2(e+fx)) + (-4icf-4idfx+d)\cos(4(e+fx)) - 16cf\sin(2(e+fx)) + 4cf\sin(4(e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] (-8*d*e^2 + 16*c*e*f + 16*c*f^2*x + 8*d*f^2*x^2 + (8*I)*(2*c*f + d*(I + 2*f
*x))*Cos[2*(e + f*x)] + (d - (4*I)*c*f - (4*I)*d*f*x)*Cos[4*(e + f*x)] - (8
*I)*d*Sin[2*(e + f*x)] - 16*c*f*Sin[2*(e + f*x)] - 16*d*f*x*Sin[2*(e + f*x)
] + I*d*Sin[4*(e + f*x)] + 4*c*f*Sin[4*(e + f*x)] + 4*d*f*x*Sin[4*(e + f*x)
])/(64*a^2*f^2)
```

fricas [A] time = 0.76, size = 69, normalized size = 0.46

$$\frac{8\,df^2x^2 + 16\,cf^2x + \left(-4i\,dfx - 4i\,cf + d\right)e^{\left(4ifx+4ie\right)} + \left(16i\,dfx + 16i\,cf - 8\,d\right)e^{\left(2ifx+2ie\right)}}{64\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/64*(8*d*f^2*x^2 + 16*c*f^2*x + (-4*I*d*f*x - 4*I*c*f + d)*e^(4*I*f*x + 4*
I*e) + (16*I*d*f*x + 16*I*c*f - 8*d)*e^(2*I*f*x + 2*I*e))/(a^2*f^2)
```

giac [A] time = 1.73, size = 108, normalized size = 0.72

$$\frac{8\,df^2x^2 + 16\,cf^2x - 4i\,dfxe^{\left(4ifx+4ie\right)} + 16i\,dfxe^{\left(2ifx+2ie\right)} - 4i\,cfe^{\left(4ifx+4ie\right)} + 16i\,cfe^{\left(2ifx+2ie\right)} + de^{\left(4ifx+4ie\right)} - 8}{64\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*d*f^2*x^2 + 16*c*f^2*x - 4*I*d*f*x*e^(4*I*f*x + 4*I*e) + 16*I*d*f*x
*e^(2*I*f*x + 2*I*e) - 4*I*c*f*e^(4*I*f*x + 4*I*e) + 16*I*c*f*e^(2*I*f*x +
2*I*e) + d*e^(4*I*f*x + 4*I*e) - 8*d*e^(2*I*f*x + 2*I*e))/(a^2*f^2)
```

maple [B] time = 1.66, size = 390, normalized size = 2.58

$$\frac{2id\left(\frac{(fx+e)\left(\sin^4(fx+e)\right)}{4}+\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{16}-\frac{3fx}{32}-\frac{3e}{32}\right)}{f}+\frac{ic\left(\sin^4(fx+e)\right)}{2}-\frac{ide\left(\sin^4(fx+e)\right)}{2f}+\frac{2d\left((fx+e)\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\right.\right.}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*cot(f*x+e))^2,x)
```

```
[Out] -1/a^2/f*(2*I/f*d*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+
e))*cos(f*x+e)-3/32*f*x-3/32*e)+1/2*I*c*sin(f*x+e)^4-1/2*I/f*d*e*sin(f*x+e)
^4+2/f*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/16*(f*x+e)^2
+1/16*sin(f*x+e)^2-(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3
/8*f*x+3/8*e)-1/16*sin(f*x+e)^4)+2*c*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(
f*x+e)*cos(f*x+e)+1/8*f*x+1/8*e)-2*e/f*d*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*
sin(f*x+e)*cos(f*x+e)+1/8*f*x+1/8*e)-1/f*d*((f*x+e)*(-1/2*sin(f*x+e)*cos(f*
x+e)+1/2*f*x+1/2*e)-1/4*(f*x+e)^2+1/4*sin(f*x+e)^2)-c*(-1/2*sin(f*x+e)*cos(
f*x+e)+1/2*f*x+1/2*e)+e/f*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.34, size = 102, normalized size = 0.68

$$\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\left(\frac{\left(2\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,a^2\,f^2}+\frac{d\,x\,1\mathrm{i}}{4\,a^2\,f}\right)-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\left(\frac{\left(4\,c\,f+d\,1\mathrm{i}\right)\,1\mathrm{i}}{64\,a^2\,f^2}+\frac{d\,x\,1\mathrm{i}}{16\,a^2\,f}\right)+\frac{d\,x^2}{8\,a^2}+\frac{c\,x}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*cot(e + f*x)*1i)^2,x)
```

```
[Out] exp(e*2i + f*x*2i)*(((d*1i + 2*c*f)*1i)/(8*a^2*f^2) + (d*x*1i)/(4*a^2*f)) -
 exp(e*4i + f*x*4i)*(((d*1i + 4*c*f)*1i)/(64*a^2*f^2) + (d*x*1i)/(16*a^2*f)
) + (d*x^2)/(8*a^2) + (c*x)/(4*a^2)
```

sympy [A] time = 0.36, size = 214, normalized size = 1.42

$$\begin{cases}\frac{\left(128ia^2cf^3e^{2ie}+128ia^2df^3xe^{2ie}-64a^2df^2e^{2ie}\right)e^{2ifx}+\left(-32ia^2cf^3e^{4ie}-32ia^2df^3xe^{4ie}+8a^2df^2e^{4ie}\right)e^{4ifx}}{512a^4f^4} & \text{for } 512a^4f^4\neq 0 \\ \frac{x^2\left(de^{4ie}-2de^{2ie}\right)}{8a^2}+\frac{x\left(ce^{4ie}-2ce^{2ie}\right)}{4a^2} & \text{otherwise}\end{cases}+\frac{cx}{4a^2}+\frac{dx^2}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] Piecewise((((128*I*a**2*c*f**3*exp(2*I*e) + 128*I*a**2*d*f**3*x*exp(2*I*e)
- 64*a**2*d*f**2*exp(2*I*e))*exp(2*I*f*x) + (-32*I*a**2*c*f**3*exp(4*I*e) -
 32*I*a**2*d*f**3*x*exp(4*I*e) + 8*a**2*d*f**2*exp(4*I*e))*exp(4*I*f*x))/(5
12*a**4*f**4), Ne(512*a**4*f**4, 0)), (x**2*(d*exp(4*I*e) - 2*d*exp(2*I*e))
/(8*a**2) + x*(c*exp(4*I*e) - 2*c*exp(2*I*e))/(4*a**2), True)) + c*x/(4*a**
2) + d*x**2/(8*a**2)
```

3.25 $\int \frac{1}{(c+dx)(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=305

$$-\frac{i\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)}{2a^2d}+\frac{i\mathrm{Ci}\left(4xf+\frac{4cf}{d}\right)\sin\left(4e-\frac{4cf}{d}\right)}{4a^2d}-\frac{\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\cos\left(2e-\frac{2cf}{d}\right)}{2a^2d}+\frac{\mathrm{Ci}\left(4xf+\frac{4c}{d}\right.}{4}$$

```
[Out] 1/4*Ci(4*c*f/d+4*f*x)*cos(-4*e+4*c*f/d)/a^2/d-1/2*Ci(2*c*f/d+2*f*x)*cos(-2*
e+2*c*f/d)/a^2/d+1/4*ln(d*x+c)/a^2/d-1/2*I*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f
*x)/a^2/d+1/4*I*cos(-4*e+4*c*f/d)*Si(4*c*f/d+4*f*x)/a^2/d-1/4*I*Ci(4*c*f/d+
4*f*x)*sin(-4*e+4*c*f/d)/a^2/d+1/4*Si(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^2/
d+1/2*I*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^2/d-1/2*Si(2*c*f/d+2*f*x)*sin
(-2*e+2*c*f/d)/a^2/d
```

Rubi [A] time = 0.75, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3728, 3303, 3299, 3302, 3312}

$$-\frac{i\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{2a^2d}+\frac{i\mathrm{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4cf}{d}\right)}{4a^2d}-\frac{\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right.}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + I*a*Cot[e + f*x])^2),x]
```

```
[Out] -(Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) + (Cos[4*e
 - (4*c*f)/d]*CosIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + Log[c + d*x]/(4*a
^2*d) + ((I/4)*CosIntegral[(4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d])/(a^2*d)
 - ((I/2)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a^2*d) - ((
I/2)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^2*d) + (Sin[2*
e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) + ((I/4)*Cos[4*e -
 (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(a^2*d) - (Sin[4*e - (4*c*f)/d]
*SinIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3728

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+ia\cot(e+fx))^2}\,dx = \int \left(\frac{1}{4a^2(c+dx)} - \frac{\cos(2e+2fx)}{2a^2(c+dx)} + \frac{\cos^2(2e+2fx)}{4a^2(c+dx)} - \frac{i\sin(2e+2fx)}{2a^2(c+dx)} - \frac{\sin}{4}\right.$$

$$= \frac{\log(c+dx)}{4a^2d} + \frac{i\int \frac{\sin(4e+4fx)}{c+dx}\,dx}{4a^2} - \frac{i\int \frac{\sin(2e+2fx)}{c+dx}\,dx}{2a^2} + \frac{\int \frac{\cos^2(2e+2fx)}{c+dx}\,dx}{4a^2} - \frac{\int \frac{s}{}}{}$$

$$= \frac{\log(c+dx)}{4a^2d} - \frac{\int \left(\frac{1}{2(c+dx)} - \frac{\cos(4e+4fx)}{2(c+dx)}\right)dx}{4a^2} + \frac{\int \left(\frac{1}{2(c+dx)} + \frac{\cos(4e+4fx)}{2(c+dx)}\right)dx}{4a^2} + \frac{(}{}$$

$$= -\frac{\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} + \frac{i\mathrm{Ci}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4}{}\right.}{4a^2d}$$

$$= -\frac{\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} + \frac{i\mathrm{Ci}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4}{}\right.}{4a^2d}$$

$$= -\frac{\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} + \frac{i\mathrm{Ci}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4}{}\right.}{4a^2d}$$

Mathematica [A] time = 0.61, size = 136, normalized size = 0.45

$$\frac{-2\left(\mathrm{Ci}\left(\frac{2f(c+dx)}{d}\right)+i\mathrm{Si}\left(\frac{2f(c+dx)}{d}\right)\right)\left(\cos\left(2e-\frac{2cf}{d}\right)+i\sin\left(2e-\frac{2cf}{d}\right)\right)+\left(\mathrm{Ci}\left(\frac{4f(c+dx)}{d}\right)+i\mathrm{Si}\left(\frac{4f(c+dx)}{d}\right)\right)\left(\cos\left(4e-\frac{4}{}\right.\right.}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Cot[e + f*x])^2),x]
```

```
[Out] (Log[c + d*x] - 2*(Cos[2*e - (2*c*f)/d] + I*Sin[2*e - (2*c*f)/d])*(CosInteg
ral[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]) + (Cos[4*e - (4*
c*f)/d] + I*Sin[4*e - (4*c*f)/d])*(CosIntegral[(4*f*(c + d*x))/d] + I*SinIn
tegral[(4*f*(c + d*x))/d]))/(4*a^2*d)
```

fricas [A] time = 0.88, size = 80, normalized size = 0.26

$$\frac{\mathrm{Ei}\left(\frac{4i\,dfx+4i\,cf}{d}\right)e^{\left(\frac{4i\,de-4i\,cf}{d}\right)} - 2\,\mathrm{Ei}\left(\frac{2i\,dfx+2i\,cf}{d}\right)e^{\left(\frac{2i\,de-2i\,cf}{d}\right)} + \log\left(\frac{dx+c}{d}\right)}{4\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(Ei((4*I*d*f*x + 4*I*c*f)/d)*e^((4*I*d*e - 4*I*c*f)/d) - 2*Ei((2*I*d*f*
x + 2*I*c*f)/d)*e^((2*I*d*e - 2*I*c*f)/d) + log((d*x + c)/d))/(a^2*d)
```

giac [B] time = 1.83, size = 987, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*(cos(4*c*f/d)*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d) - I*cos(e)^4*cos
_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d) + 4*I*cos(4*c*f/d)*cos(e)^3*cos_i
ntegral(4*(d*f*x + c*f)/d)*sin(e) + 4*cos(e)^3*cos_integral(4*(d*f*x + c*f)
/d)*sin(4*c*f/d)*sin(e) - 6*cos(4*c*f/d)*cos(e)^2*cos_integral(4*(d*f*x + c
*f)/d)*sin(e)^2 + 6*I*cos(e)^2*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d)
*sin(e)^2 - 4*I*cos(4*c*f/d)*cos(e)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^
3 - 4*cos(e)*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e)^3 + cos(4*
c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4 - I*cos_integral(4*(d*f*x +
 c*f)/d)*sin(4*c*f/d)*sin(e)^4 + I*cos(4*c*f/d)*cos(e)^4*sin_integral(4*(d*
f*x + c*f)/d) + cos(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 4*c
os(4*c*f/d)*cos(e)^3*sin(e)*sin_integral(4*(d*f*x + c*f)/d) + 4*I*cos(e)^3*
sin(4*c*f/d)*sin(e)*sin_integral(4*(d*f*x + c*f)/d) - 6*I*cos(4*c*f/d)*cos(
e)^2*sin(e)^2*sin_integral(4*(d*f*x + c*f)/d) - 6*cos(e)^2*sin(4*c*f/d)*sin
(e)^2*sin_integral(4*(d*f*x + c*f)/d) + 4*cos(4*c*f/d)*cos(e)*sin(e)^3*sin_
integral(4*(d*f*x + c*f)/d) - 4*I*cos(e)*sin(4*c*f/d)*sin(e)^3*sin_integral
(4*(d*f*x + c*f)/d) + I*cos(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f)/
d) + sin(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f)/d) - 2*cos(2*c*f/d)
*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d) + 2*I*cos(e)^2*cos_integral(2*(d*
f*x + c*f)/d)*sin(2*c*f/d) - 4*I*cos(2*c*f/d)*cos(e)*cos_integral(2*(d*f*x
+ c*f)/d)*sin(e) - 4*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*si
n(e) + 2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 - 2*I*cos_in
tegral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e)^2 - 2*I*cos(2*c*f/d)*cos(e)^2
*sin_integral(2*(d*f*x + c*f)/d) - 2*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(
d*f*x + c*f)/d) + 4*cos(2*c*f/d)*cos(e)*sin(e)*sin_integral(2*(d*f*x + c*f)
/d) - 4*I*cos(e)*sin(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) + 2*I*
cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) + 2*sin(2*c*f/d)*sin(
e)^2*sin_integral(2*(d*f*x + c*f)/d) + log(d*x + c))/(a^2*d)
```

maple [A] time = 1.42, size = 382, normalized size = 1.25

$$-\frac{i\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{2a^2d}+\frac{i\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{2a^2d}+\frac{i\,\mathrm{Si}\left(4fx+4e+\frac{4cf-4de}{d}\right)\cos}{4a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x)
```

```
[Out] -1/2*I/a^2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d+1/2*I/a^2*Ci(2*
f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+1/4*I/a^2*Si(4*f*x+4*e+4*(c*f-d
*e)/d)*cos(4*(c*f-d*e)/d)/d-1/4*I/a^2*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*
f-d*e)/d)/d+1/4/a^2*ln((f*x+e)*d+c*f-d*e)/d+1/4/a^2*Si(4*f*x+4*e+4*(c*f-d*e
)/d)*sin(4*(c*f-d*e)/d)/d+1/4/a^2*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*
e)/d)/d-1/2/a^2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d-1/2/a^2*Ci
(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d
```

maxima [A] time = 0.96, size = 194, normalized size = 0.64

$$-\frac{f\cos\left(-\frac{4\left(de-cf\right)}{d}\right)E_1\left(-\frac{4i\left(fx+e\right)d-4ide+4icf}{d}\right)-2f\cos\left(-\frac{2\left(de-cf\right)}{d}\right)E_1\left(-\frac{2i\left(fx+e\right)d-2ide+2icf}{d}\right)+2if\,E_1\left(-\frac{2i\left(fx+e\right)d}{4a^2df}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(f*cos(-4*(d*e - c*f)/d)*exp_integral_e(1, -(4*I*(f*x + e)*d - 4*I*d*e
 + 4*I*c*f)/d) - 2*f*cos(-2*(d*e - c*f)/d)*exp_integral_e(1, -(2*I*(f*x + e
```

```
)*d - 2*I*d*e + 2*I*c*f)/d) + 2*I*f*exp_integral_e(1, -(2*I*(f*x + e)*d - 2
*I*d*e + 2*I*c*f)/d)*sin(-2*(d*e - c*f)/d) - I*f*exp_integral_e(1, -(4*I*(f
*x + e)*d - 4*I*d*e + 4*I*c*f)/d)*sin(-4*(d*e - c*f)/d) - f*log((f*x + e)*d
 - d*e + c*f))/(a^2*d*f)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + a\cot\left(e + f\,x\right)\,1\mathrm{i}\right)^2\,(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{c\cot^2\left(e+fx\right)-2ic\cot\left(e+fx\right)-c+dx\cot^2\left(e+fx\right)-2idx\cot\left(e+fx\right)-dx}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] -Integral(1/(c*cot(e + f*x)**2 - 2*I*c*cot(e + f*x) - c + d*x*cot(e + f*x)*
*2 - 2*I*d*x*cot(e + f*x) - d*x), x)/a**2
```

3.26 $\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=434

$$-\frac{f\mathrm{Ci}\left(4xf+\frac{4cf}{d}\right)\sin\left(4e-\frac{4cf}{d}\right)}{a^2d^2}+\frac{f\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{if\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\cos\left(2e-\frac{2cf}{d}\right)}{a^2d^2}+\frac{if\mathrm{Ci}\left(4xf\right.}{}$$

```
[Out] -1/4/a^2/d/(d*x+c)+I*f*Ci(4*c*f/d+4*f*x)*cos(-4*e+4*c*f/d)/a^2/d^2-I*f*Ci(2
*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^2/d^2+1/2*cos(2*f*x+2*e)/a^2/d/(d*x+c)-1/
4*cos(2*f*x+2*e)^2/a^2/d/(d*x+c)+f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a^2/
d^2-f*cos(-4*e+4*c*f/d)*Si(4*c*f/d+4*f*x)/a^2/d^2+f*Ci(4*c*f/d+4*f*x)*sin(-
4*e+4*c*f/d)/a^2/d^2+I*f*Si(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^2/d^2-f*Ci(2
*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^2/d^2-I*f*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*
f/d)/a^2/d^2+1/2*I*sin(2*f*x+2*e)/a^2/d/(d*x+c)+1/4*sin(2*f*x+2*e)^2/a^2/d/
(d*x+c)-1/4*I*sin(4*f*x+4*e)/a^2/d/(d*x+c)
```

Rubi [A] time = 0.68, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3728, 3297, 3303, 3299, 3302, 3313, 12}

$$-\frac{f\mathrm{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4cf}{d}\right)}{a^2d^2}+\frac{f\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{if\mathrm{CosIntegral}\left(\frac{2cf}{d}+\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])^2),x]
```

```
[Out] -1/(4*a^2*d*(c + d*x)) + Cos[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - Cos[2*e + 2
*f*x]^2/(4*a^2*d*(c + d*x)) - (I*f*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)
/d + 2*f*x])/(a^2*d^2) + (I*f*Cos[4*e - (4*c*f)/d]*CosIntegral[(4*c*f)/d +
4*f*x])/(a^2*d^2) - (f*CosIntegral[(4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d])
/(a^2*d^2) + (f*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a^2*d
^2) + ((I/2)*Sin[2*e + 2*f*x])/(a^2*d*(c + d*x)) + Sin[2*e + 2*f*x]^2/(4*a^
2*d*(c + d*x)) - ((I/4)*Sin[4*e + 4*f*x])/(a^2*d*(c + d*x)) + (f*Cos[2*e -
(2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (I*f*Sin[2*e - (2*c*
f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*Cos[4*e - (4*c*f)/d]*S
inIntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2) - (I*f*Sin[4*e - (4*c*f)/d]*SinInt
egral[(4*c*f)/d + 4*f*x])/(a^2*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3728

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))^2}\,dx = \int \left(\frac{1}{4a^2(c+dx)^2} - \frac{\cos(2e+2fx)}{2a^2(c+dx)^2} + \frac{\cos^2(2e+2fx)}{4a^2(c+dx)^2} - \frac{i\sin(2e+2fx)}{2a^2(c+dx)^2} - \text{s}$$

$$= -\frac{1}{4a^2d(c+dx)} + \frac{i\int \frac{\sin(4e+4fx)}{(c+dx)^2}\,dx}{4a^2} - \frac{i\int \frac{\sin(2e+2fx)}{(c+dx)^2}\,dx}{2a^2} + \frac{\int \frac{\cos^2(2e+2fx)}{(c+dx)^2}\,dx}{4a^2} -$$

$$= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} + \frac{i\sin(2e+2fx)}{2a^2d(c+dx)} + \frac{\text{sin}}{4a}$$

$$= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} + \frac{i\sin(2e+2fx)}{2a^2d(c+dx)} + \frac{\text{sin}}{4a}$$

$$= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} - \frac{if\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\right.}{a^2d^2}$$

$$= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} - \frac{if\cos\left(2e-\frac{2cf}{d}\right)\text{Ci}\left(\right.}{a^2d^2}$$

Mathematica [A] time = 0.59, size = 203, normalized size = 0.47

$$\frac{4f(c+dx)\left(\text{Ci}\left(\frac{2f(c+dx)}{d}\right)+i\text{Si}\left(\frac{2f(c+dx)}{d}\right)\right)\left(\sin\left(2e-\frac{2cf}{d}\right)-i\cos\left(2e-\frac{2cf}{d}\right)\right)+(c+dx)\left(\text{Ci}\left(\frac{4f(c+dx)}{d}\right)+i\text{Si}\left(\frac{4f(c+d}{d}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])^2),x]
```

```
[Out] (-d + 2*d*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]) - d*(Cos[4*(e + f*x)] + I
*Sin[4*(e + f*x)]) + 4*f*(c + d*x)*((-I)*Cos[2*e - (2*c*f)/d] + Sin[2*e - (
2*c*f)/d])*(CosIntegral[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/
d]) + (c + d*x)*((4*I)*f*Cos[4*e - (4*c*f)/d] - 4*f*Sin[4*e - (4*c*f)/d])*(
CosIntegral[(4*f*(c + d*x))/d] + I*SinIntegral[(4*f*(c + d*x))/d]))/(4*a^2*
d^2*(c + d*x))
```

fricas [A] time = 0.60, size = 129, normalized size = 0.30

$$\frac{\left(4idfx+4icf\right)\mathrm{Ei}\left(\frac{4idfx+4icf}{d}\right)e^{\left(\frac{4ide-4icf}{d}\right)}+\left(-4idfx-4icf\right)\mathrm{Ei}\left(\frac{2idfx+2icf}{d}\right)e^{\left(\frac{2ide-2icf}{d}\right)}-de^{(4ifx+4ie)}+2de^{(2ifx}}{4\left(a^2d^3x+a^2cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((4*I*d*f*x + 4*I*c*f)*Ei((4*I*d*f*x + 4*I*c*f)/d)*e^((4*I*d*e - 4*I*c*
f)/d) + (-4*I*d*f*x - 4*I*c*f)*Ei((2*I*d*f*x + 2*I*c*f)/d)*e^((2*I*d*e - 2*
I*c*f)/d) - d*e^(4*I*f*x + 4*I*e) + 2*d*e^(2*I*f*x + 2*I*e) - d)/(a^2*d^3*x
 + a^2*c*d^2)
```

giac [B] time = 64.09, size = 2369, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*(4*I*d*f*x*cos(4*c*f/d)*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d) + 4*d*
f*x*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d) - 16*d*f*x*cos(4*
c*f/d)*cos(e)^3*cos_integral(4*(d*f*x + c*f)/d)*sin(e) + 16*I*d*f*x*cos(e)^
3*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e) - 24*I*d*f*x*cos(4*c*
f/d)*cos(e)^2*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2 - 24*d*f*x*cos(e)^2*
cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e)^2 + 16*d*f*x*cos(4*c*f/
d)*cos(e)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3 - 16*I*d*f*x*cos(e)*cos_
integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e)^3 + 4*I*d*f*x*cos(4*c*f/d)*
cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4 + 4*d*f*x*cos_integral(4*(d*f*x +
c*f)/d)*sin(4*c*f/d)*sin(e)^4 - 4*d*f*x*cos(4*c*f/d)*cos(e)^4*sin_integral(
4*(d*f*x + c*f)/d) + 4*I*d*f*x*cos(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x
+ c*f)/d) - 16*I*d*f*x*cos(4*c*f/d)*cos(e)^3*sin(e)*sin_integral(4*(d*f*x +
 c*f)/d) - 16*d*f*x*cos(e)^3*sin(4*c*f/d)*sin(e)*sin_integral(4*(d*f*x + c*
f)/d) + 24*d*f*x*cos(4*c*f/d)*cos(e)^2*sin(e)^2*sin_integral(4*(d*f*x + c*f
)/d) - 24*I*d*f*x*cos(e)^2*sin(4*c*f/d)*sin(e)^2*sin_integral(4*(d*f*x + c*
f)/d) + 16*I*d*f*x*cos(4*c*f/d)*cos(e)*sin(e)^3*sin_integral(4*(d*f*x + c*f
)/d) + 16*d*f*x*cos(e)*sin(4*c*f/d)*sin(e)^3*sin_integral(4*(d*f*x + c*f)/d
) - 4*d*f*x*cos(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f)/d) + 4*I*d*f
*x*sin(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f)/d) + 4*I*c*f*cos(4*c*
f/d)*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d) + 4*c*f*cos(e)^4*cos_integral
(4*(d*f*x + c*f)/d)*sin(4*c*f/d) - 16*c*f*cos(4*c*f/d)*cos(e)^3*cos_integra
l(4*(d*f*x + c*f)/d)*sin(e) + 16*I*c*f*cos(e)^3*cos_integral(4*(d*f*x + c*f
)/d)*sin(4*c*f/d)*sin(e) - 24*I*c*f*cos(4*c*f/d)*cos(e)^2*cos_integral(4*(d
*f*x + c*f)/d)*sin(e)^2 - 24*c*f*cos(e)^2*cos_integral(4*(d*f*x + c*f)/d)*s
in(4*c*f/d)*sin(e)^2 + 16*c*f*cos(4*c*f/d)*cos(e)*cos_integral(4*(d*f*x + c
*f)/d)*sin(e)^3 - 16*I*c*f*cos(e)*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f
/d)*sin(e)^3 + 4*I*c*f*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^
4 + 4*c*f*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e)^4 - 4*c*f*cos
(4*c*f/d)*cos(e)^4*sin_integral(4*(d*f*x + c*f)/d) + 4*I*c*f*cos(e)^4*sin(4
*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 16*I*c*f*cos(4*c*f/d)*cos(e)^3*si
```

```
n(e)*sin_integral(4*(d*f*x + c*f)/d) - 16*c*f*cos(e)^3*sin(4*c*f/d)*sin(e)*
sin_integral(4*(d*f*x + c*f)/d) + 24*c*f*cos(4*c*f/d)*cos(e)^2*sin(e)^2*sin
_integral(4*(d*f*x + c*f)/d) - 24*I*c*f*cos(e)^2*sin(4*c*f/d)*sin(e)^2*sin_
integral(4*(d*f*x + c*f)/d) + 16*I*c*f*cos(4*c*f/d)*cos(e)*sin(e)^3*sin_int
egral(4*(d*f*x + c*f)/d) + 16*c*f*cos(e)*sin(4*c*f/d)*sin(e)^3*sin_integral
(4*(d*f*x + c*f)/d) - 4*c*f*cos(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c
*f)/d) + 4*I*c*f*sin(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f)/d) - 4*
I*d*f*x*cos(2*c*f/d)*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d) - 4*d*f*x*cos
(e)^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 8*d*f*x*cos(2*c*f/d)*c
os(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) - 8*I*d*f*x*cos(e)*cos_integra
l(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e) + 4*I*d*f*x*cos(2*c*f/d)*cos_integ
ral(2*(d*f*x + c*f)/d)*sin(e)^2 + 4*d*f*x*cos_integral(2*(d*f*x + c*f)/d)*s
in(2*c*f/d)*sin(e)^2 + 4*d*f*x*cos(2*c*f/d)*cos(e)^2*sin_integral(2*(d*f*x
+ c*f)/d) - 4*I*d*f*x*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d)
 + 8*I*d*f*x*cos(2*c*f/d)*cos(e)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) + 8
*d*f*x*cos(e)*sin(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) - 4*d*f*x
*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) + 4*I*d*f*x*sin(2*c*
f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) - d*cos(4*f*x)*cos(e)^4 - 4*I
*c*f*cos(2*c*f/d)*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d) - I*d*cos(e)^4*s
in(4*f*x) - 4*c*f*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) - 4
*I*d*cos(4*f*x)*cos(e)^3*sin(e) + 8*c*f*cos(2*c*f/d)*cos(e)*cos_integral(2*
(d*f*x + c*f)/d)*sin(e) + 4*d*cos(e)^3*sin(4*f*x)*sin(e) - 8*I*c*f*cos(e)*c
os_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e) + 6*d*cos(4*f*x)*cos(e)^
2*sin(e)^2 + 4*I*c*f*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2
+ 6*I*d*cos(e)^2*sin(4*f*x)*sin(e)^2 + 4*c*f*cos_integral(2*(d*f*x + c*f)/d
)*sin(2*c*f/d)*sin(e)^2 + 4*I*d*cos(4*f*x)*cos(e)*sin(e)^3 - 4*d*cos(e)*sin
(4*f*x)*sin(e)^3 - d*cos(4*f*x)*sin(e)^4 - I*d*sin(4*f*x)*sin(e)^4 + 4*c*f*
cos(2*c*f/d)*cos(e)^2*sin_integral(2*(d*f*x + c*f)/d) - 4*I*c*f*cos(e)^2*si
n(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*I*c*f*cos(2*c*f/d)*cos(e)*si
n(e)*sin_integral(2*(d*f*x + c*f)/d) + 8*c*f*cos(e)*sin(2*c*f/d)*sin(e)*sin
_integral(2*(d*f*x + c*f)/d) - 4*c*f*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(
d*f*x + c*f)/d) + 4*I*c*f*sin(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f
)/d) + 2*d*cos(2*f*x)*cos(e)^2 + 2*I*d*cos(e)^2*sin(2*f*x) + 4*I*d*cos(2*f*
x)*cos(e)*sin(e) - 4*d*cos(e)*sin(2*f*x)*sin(e) - 2*d*cos(2*f*x)*sin(e)^2 -
 2*I*d*sin(2*f*x)*sin(e)^2 - d)/(a^2*d^3*x + a^2*c*d^2)
```

maple [A] time = 1.43, size = 537, normalized size = 1.24

$$\frac{if^2\left(-\frac{2\sin\left(2fx+2e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}+\frac{\frac{4\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}}{d}\right)}{4}-\frac{if^2\left(-\frac{4\sin\left(4fx+4e\right)}{\left(\left(fx+e\right)d+cf-de\right)d}+\frac{\frac{16\,\mathrm{Si}\left(4fx+4e+\frac{4cf-4de}{d}\right)\sin\left(4\right.\right.}{d}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x)
```

```
[Out] -1/a^2/f*(1/4*I*f^2*(-2*sin(2*f*x+2*e)/((f*x+e)*d+c*f-d*e)/d+2*(2*Si(2*f*x+
2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2
*(c*f-d*e)/d)/d)/d)-1/16*I*f^2*(-4*sin(4*f*x+4*e)/((f*x+e)*d+c*f-d*e)/d+4*(
4*Si(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d+4*Ci(4*f*x+4*e+4*(c*f-d*
e)/d)*cos(4*(c*f-d*e)/d)/d)/d)+1/4*f^2/((f*x+e)*d+c*f-d*e)/d-1/16*f^2*(-4*c
os(4*f*x+4*e)/((f*x+e)*d+c*f-d*e)/d-4*(4*Si(4*f*x+4*e+4*(c*f-d*e)/d)*cos(4*
(c*f-d*e)/d)/d-4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d)/d)+1/4*f
^2*(-2*cos(2*f*x+2*e)/((f*x+e)*d+c*f-d*e)/d-2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d
)*cos(2*(c*f-d*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/
d))
```

maxima [A] time = 0.81, size = 214, normalized size = 0.49

$$-\frac{64 f^2 \cos\left(-\frac{4(de-cf)}{d}\right) E_2\left(-\frac{4i(fx+e)d-4ide+4icf}{d}\right) - 128 f^2 \cos\left(-\frac{2(de-cf)}{d}\right) E_2\left(-\frac{2i(fx+e)d-2ide+2icf}{d}\right) + 128i f^2 E_2}{256\left((fx+e)a^2d^2 - a^2d^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/256*(64*f^2*cos(-4*(d*e - c*f)/d)*exp_integral_e(2, -(4*I*(f*x + e)*d -
4*I*d*e + 4*I*c*f)/d) - 128*f^2*cos(-2*(d*e - c*f)/d)*exp_integral_e(2, -(2
*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d) + 128*I*f^2*exp_integral_e(2, -(2*I*
(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d)*sin(-2*(d*e - c*f)/d) - 64*I*f^2*exp_in
tegral_e(2, -(4*I*(f*x + e)*d - 4*I*d*e + 4*I*c*f)/d)*sin(-4*(d*e - c*f)/d)
 + 64*f^2)/(((f*x + e)*a^2*d^2 - a^2*d^2*e + a^2*c*d*f)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + a\cot\left(e + f\,x\right)\,1\mathrm{i}\right)^2\,(c + d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{c^2\cot^2(e+fx) - 2ic^2\cot(e+fx) - c^2 + 2cdx\cot^2(e+fx) - 4icdx\cot(e+fx) - 2cdx + d^2x^2\cot^2(e+fx) - 2id^2x^2\cot(e+fx) - d^2x^2}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] -Integral(1/(c**2*cot(e + f*x)**2 - 2*I*c**2*cot(e + f*x) - c**2 + 2*c*d*x*
cot(e + f*x)**2 - 4*I*c*d*x*cot(e + f*x) - 2*c*d*x + d**2*x**2*cot(e + f*x)
**2 - 2*I*d**2*x**2*cot(e + f*x) - d**2*x**2), x)/a**2
```

3.27 $\int \frac{(c+dx)^3}{(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=396

$$-\frac{9id^2(c+dx)e^{2ie+2ifx}}{32a^3f^3}+\frac{9id^2(c+dx)e^{4ie+4ifx}}{256a^3f^3}-\frac{id^2(c+dx)e^{6ie+6ifx}}{288a^3f^3}-\frac{9d(c+dx)^2e^{2ie+2ifx}}{32a^3f^2}+\frac{9d(c+dx)^2e^{4ie+4ifx}}{128a^3f^2}-\frac{d(c}{}$$

```
[Out] 9/64*d^3*exp(2*I*e+2*I*f*x)/a^3/f^4-9/1024*d^3*exp(4*I*e+4*I*f*x)/a^3/f^4+1
/1728*d^3*exp(6*I*e+6*I*f*x)/a^3/f^4-9/32*I*d^2*exp(2*I*e+2*I*f*x)*(d*x+c)/
a^3/f^3+9/256*I*d^2*exp(4*I*e+4*I*f*x)*(d*x+c)/a^3/f^3-1/288*I*d^2*exp(6*I*
e+6*I*f*x)*(d*x+c)/a^3/f^3-9/32*d*exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^3/f^2+9/12
8*d*exp(4*I*e+4*I*f*x)*(d*x+c)^2/a^3/f^2-1/96*d*exp(6*I*e+6*I*f*x)*(d*x+c)^
2/a^3/f^2+3/16*I*exp(2*I*e+2*I*f*x)*(d*x+c)^3/a^3/f-3/32*I*exp(4*I*e+4*I*f*
x)*(d*x+c)^3/a^3/f+1/48*I*exp(6*I*e+6*I*f*x)*(d*x+c)^3/a^3/f+1/32*(d*x+c)^4
/a^3/d
```

Rubi [A] time = 0.38, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3729, 2176, 2194}

$$-\frac{9id^2(c+dx)e^{2ie+2ifx}}{32a^3f^3}+\frac{9id^2(c+dx)e^{4ie+4ifx}}{256a^3f^3}-\frac{id^2(c+dx)e^{6ie+6ifx}}{288a^3f^3}-\frac{9d(c+dx)^2e^{2ie+2ifx}}{32a^3f^2}+\frac{9d(c+dx)^2e^{4ie+4ifx}}{128a^3f^2}-\frac{d(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (9*d^3*E^((2*I)*e + (2*I)*f*x))/(64*a^3*f^4) - (9*d^3*E^((4*I)*e + (4*I)*f*
x))/(1024*a^3*f^4) + (d^3*E^((6*I)*e + (6*I)*f*x))/(1728*a^3*f^4) - (((9*I)
/32)*d^2*E^((2*I)*e + (2*I)*f*x)*(c + d*x))/(a^3*f^3) + (((9*I)/256)*d^2*E^
((4*I)*e + (4*I)*f*x)*(c + d*x))/(a^3*f^3) - ((I/288)*d^2*E^((6*I)*e + (6*I
)*f*x)*(c + d*x))/(a^3*f^3) - (9*d*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^2)/(32
*a^3*f^2) + (9*d*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^2)/(128*a^3*f^2) - (d*E^
((6*I)*e + (6*I)*f*x)*(c + d*x)^2)/(96*a^3*f^2) + (((3*I)/16)*E^((2*I)*e +
(2*I)*f*x)*(c + d*x)^3)/(a^3*f) - (((3*I)/32)*E^((4*I)*e + (4*I)*f*x)*(c +
d*x)^3)/(a^3*f) + ((I/48)*E^((6*I)*e + (6*I)*f*x)*(c + d*x)^3)/(a^3*f) + (c
 + d*x)^4/(32*a^3*d)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+ia\cot(e+fx))^3}\,dx = \int \left(\frac{(c+dx)^3}{8a^3} - \frac{3e^{2ie+2ifx}(c+dx)^3}{8a^3} + \frac{3e^{4ie+4ifx}(c+dx)^3}{8a^3} - \frac{e^{6ie+6ifx}(c+dx)^3}{8a^3}\right) dx$$

$$= \frac{(c+dx)^4}{32a^3d} - \frac{\int e^{6ie+6ifx}(c+dx)^3\,dx}{8a^3} - \frac{3\int e^{2ie+2ifx}(c+dx)^3\,dx}{8a^3} + \frac{3\int e^{4ie+4ifx}(c+$$

$$= \frac{3ie^{2ie+2ifx}(c+dx)^3}{16a^3f} - \frac{3ie^{4ie+4ifx}(c+dx)^3}{32a^3f} + \frac{ie^{6ie+6ifx}(c+dx)^3}{48a^3f} + \frac{(c+dx)^4}{32a^3d} - \frac{(id)}{}$$

$$= -\frac{9de^{2ie+2ifx}(c+dx)^2}{32a^3f^2} + \frac{9de^{4ie+4ifx}(c+dx)^2}{128a^3f^2} - \frac{de^{6ie+6ifx}(c+dx)^2}{96a^3f^2} + \frac{3ie^{2ie+2ifx}(c}{16a^3f}$$

$$= -\frac{9id^2e^{2ie+2ifx}(c+dx)}{32a^3f^3} + \frac{9id^2e^{4ie+4ifx}(c+dx)}{256a^3f^3} - \frac{id^2e^{6ie+6ifx}(c+dx)}{288a^3f^3} - \frac{9de^{2ie+2ifx}}{32a^3}$$

$$= \frac{9d^3e^{2ie+2ifx}}{64a^3f^4} - \frac{9d^3e^{4ie+4ifx}}{1024a^3f^4} + \frac{d^3e^{6ie+6ifx}}{1728a^3f^4} - \frac{9id^2e^{2ie+2ifx}(c+dx)}{32a^3f^3} + \frac{9id^2e^{4ie+4ifx}(c}{256a^3f^3}$$

Mathematica [A] time = 2.85, size = 664, normalized size = 1.68

$$\frac{(\cos(3(e+fx)) + i\sin(3(e+fx)))\left(-3456ic^3f^4x\sin(3(e+fx)) + 7776c^3f^3\sin(e+fx) - 576c^3f^3\sin(3(e+f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] ((Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*((81*I)*(32*c^3*f^3 + 24*c^2*d*f^2
*(3*I + 4*f*x) + 12*c*d^2*f*(-7 + (12*I)*f*x + 8*f^2*x^2) + d^3*(-45*I - 84
*f*x + (72*I)*f^2*x^2 + 32*f^3*x^3))*Cos[e + f*x] + 16*(36*c^3*f^3*(I + 6*f
*x) + 18*c^2*d*f^2*(-1 + (6*I)*f*x + 18*f^2*x^2) + 6*c*d^2*f*(-I - 6*f*x +
(18*I)*f^2*x^2 + 36*f^3*x^3) + d^3*(1 - (6*I)*f*x - 18*f^2*x^2 + (36*I)*f^3
*x^3 + 54*f^4*x^4))*Cos[3*(e + f*x)] - (4131*I)*d^3*Sin[e + f*x] - 8748*c*d
^2*f*Sin[e + f*x] + (9720*I)*c^2*d*f^2*Sin[e + f*x] + 7776*c^3*f^3*Sin[e +
f*x] - 8748*d^3*f*x*Sin[e + f*x] + (19440*I)*c*d^2*f^2*x*Sin[e + f*x] + 233
28*c^2*d*f^3*x*Sin[e + f*x] + (9720*I)*d^3*f^2*x^2*Sin[e + f*x] + 23328*c*d
^2*f^3*x^2*Sin[e + f*x] + 7776*d^3*f^3*x^3*Sin[e + f*x] + (16*I)*d^3*Sin[3*
(e + f*x)] + 96*c*d^2*f*Sin[3*(e + f*x)] - (288*I)*c^2*d*f^2*Sin[3*(e + f*x
)] - 576*c^3*f^3*Sin[3*(e + f*x)] + 96*d^3*f*x*Sin[3*(e + f*x)] - (576*I)*c
*d^2*f^2*x*Sin[3*(e + f*x)] - 1728*c^2*d*f^3*x*Sin[3*(e + f*x)] - (3456*I)*
c^3*f^4*x*Sin[3*(e + f*x)] - (288*I)*d^3*f^2*x^2*Sin[3*(e + f*x)] - 1728*c*
d^2*f^3*x^2*Sin[3*(e + f*x)] - (5184*I)*c^2*d*f^4*x^2*Sin[3*(e + f*x)] - 57
6*d^3*f^3*x^3*Sin[3*(e + f*x)] - (3456*I)*c*d^2*f^4*x^3*Sin[3*(e + f*x)] -
(864*I)*d^3*f^4*x^4*Sin[3*(e + f*x)]))/(27648*a^3*f^4)
```

fricas [A] time = 0.95, size = 353, normalized size = 0.89

$$\frac{864\,d^3f^4x^4 + 3456\,cd^2f^4x^3 + 5184\,c^2df^4x^2 + 3456\,c^3f^4x + \left(576i\,d^3f^3x^3 + 576i\,c^3f^3 - 288\,c^2df^2 - 96i\,cd^2f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/27648*(864*d^3*f^4*x^4 + 3456*c*d^2*f^4*x^3 + 5184*c^2*d*f^4*x^2 + 3456*c
^3*f^4*x + (576*I*d^3*f^3*x^3 + 576*I*c^3*f^3 - 288*c^2*d*f^2 - 96*I*c*d^2*
f + 16*d^3 + (1728*I*c*d^2*f^3 - 288*d^3*f^2)*x^2 + (1728*I*c^2*d*f^3 - 576
*c*d^2*f^2 - 96*I*d^3*f)*x)*e^(6*I*f*x + 6*I*e) + (-2592*I*d^3*f^3*x^3 - 25
92*I*c^3*f^3 + 1944*c^2*d*f^2 + 972*I*c*d^2*f - 243*d^3 + (-7776*I*c*d^2*f^
```

```
3 + 1944*d^3*f^2)*x^2 + (-7776*I*c^2*d*f^3 + 3888*c*d^2*f^2 + 972*I*d^3*f)*
x)*e^(4*I*f*x + 4*I*e) + (5184*I*d^3*f^3*x^3 + 5184*I*c^3*f^3 - 7776*c^2*d*
f^2 - 7776*I*c*d^2*f + 3888*d^3 + (15552*I*c*d^2*f^3 - 7776*d^3*f^2)*x^2 +
(15552*I*c^2*d*f^3 - 15552*c*d^2*f^2 - 7776*I*d^3*f)*x)*e^(2*I*f*x + 2*I*e)
)/(a^3*f^4)
```

giac [B] time = 0.61, size = 623, normalized size = 1.57

$$\frac{864\,d^3f^4x^4 + 3456\,cd^2f^4x^3 + 576i\,d^3f^3x^3e^{(6i\,fx+6i\,e)} - 2592i\,d^3f^3x^3e^{(4i\,fx+4i\,e)} + 5184i\,d^3f^3x^3e^{(2i\,fx+2i\,e)} + 5184\,c^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/27648*(864*d^3*f^4*x^4 + 3456*c*d^2*f^4*x^3 + 576*I*d^3*f^3*x^3*e^(6*I*f*
x + 6*I*e) - 2592*I*d^3*f^3*x^3*e^(4*I*f*x + 4*I*e) + 5184*I*d^3*f^3*x^3*e^
(2*I*f*x + 2*I*e) + 5184*c^2*d*f^4*x^2 + 1728*I*c*d^2*f^3*x^2*e^(6*I*f*x +
6*I*e) - 7776*I*c*d^2*f^3*x^2*e^(4*I*f*x + 4*I*e) + 15552*I*c*d^2*f^3*x^2*e
^(2*I*f*x + 2*I*e) + 3456*c^3*f^4*x + 1728*I*c^2*d*f^3*x*e^(6*I*f*x + 6*I*e
) - 288*d^3*f^2*x^2*e^(6*I*f*x + 6*I*e) - 7776*I*c^2*d*f^3*x*e^(4*I*f*x + 4
*I*e) + 1944*d^3*f^2*x^2*e^(4*I*f*x + 4*I*e) + 15552*I*c^2*d*f^3*x*e^(2*I*f
*x + 2*I*e) - 7776*d^3*f^2*x^2*e^(2*I*f*x + 2*I*e) + 576*I*c^3*f^3*e^(6*I*f
*x + 6*I*e) - 576*c*d^2*f^2*x*e^(6*I*f*x + 6*I*e) - 2592*I*c^3*f^3*e^(4*I*f
*x + 4*I*e) + 3888*c*d^2*f^2*x*e^(4*I*f*x + 4*I*e) + 5184*I*c^3*f^3*e^(2*I*
f*x + 2*I*e) - 15552*c*d^2*f^2*x*e^(2*I*f*x + 2*I*e) - 288*c^2*d*f^2*e^(6*I
*f*x + 6*I*e) - 96*I*d^3*f*x*e^(6*I*f*x + 6*I*e) + 1944*c^2*d*f^2*e^(4*I*f*
x + 4*I*e) + 972*I*d^3*f*x*e^(4*I*f*x + 4*I*e) - 7776*c^2*d*f^2*e^(2*I*f*x
+ 2*I*e) - 7776*I*d^3*f*x*e^(2*I*f*x + 2*I*e) - 96*I*c*d^2*f*e^(6*I*f*x + 6
*I*e) + 972*I*c*d^2*f*e^(4*I*f*x + 4*I*e) - 7776*I*c*d^2*f*e^(2*I*f*x + 2*I
*e) + 16*d^3*e^(6*I*f*x + 6*I*e) - 243*d^3*e^(4*I*f*x + 4*I*e) + 3888*d^3*e
^(2*I*f*x + 2*I*e))/(a^3*f^4)
```

maple [B] time = 1.72, size = 3997, normalized size = 10.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*cot(f*x+e))^3,x)
```

```
[Out] 1/f^4/a^3*(-3*d^3*e*((f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)+3/8*f*x+3/8*e)+1/8*(f*x+e)*sin(f*x+e)^4+1/32*(sin(f*x+e)^3+3/2*sin(f*x+e
))*cos(f*x+e)+9/64*f*x+9/64*e-3/8*(f*x+e)*cos(f*x+e)^2+3/16*sin(f*x+e)*cos(
f*x+e)-1/4*(f*x+e)^3)+3*d^3*e^2*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e)
)*cos(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/16*sin(f*x+e)^4+3/16*sin(f*x+e
)^2)+3*c*d^2*e^2*f*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3
/8*e)+12*c^2*d*e*f^2*(-1/6*sin(f*x+e)^3*cos(f*x+e)^3-1/8*sin(f*x+e)*cos(f*x
+e)^3+1/16*sin(f*x+e)*cos(f*x+e)+1/16*f*x+1/16*e)+12*I*c*d^2*f*(1/4*(f*x+e)
^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+
3/8*f*x+3/8*e)+1/24*(f*x+e)^2-1/72*sin(f*x+e)^4-1/24*sin(f*x+e)^2-1/6*(f*x+
e)^2*sin(f*x+e)^6+1/3*(f*x+e)*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin
(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/108*sin(f*x+e)^6)-6*c*d^2*e*f*((f*x+
e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+
e)^2+1/16*sin(f*x+e)^4+3/16*sin(f*x+e)^2)-3*c^2*d*e*f^2*(-1/4*(sin(f*x+e)^3
+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-12*c*d^2*e^2*f*(-1/6*sin(f*x+e)^
3*cos(f*x+e)^3-1/8*sin(f*x+e)*cos(f*x+e)^3+1/16*sin(f*x+e)*cos(f*x+e)+1/16*
f*x+1/16*e)+12*I*c*d^2*e^2*f*(-1/6*sin(f*x+e)^2*cos(f*x+e)^4-1/12*cos(f*x+e
)^4)+18*I*c*d^2*e*f*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*
x+e))*cos(f*x+e)-3/32*f*x-3/32*e)-12*I*d^3*e*(1/4*(f*x+e)^2*sin(f*x+e)^4-1/
2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/2
```

```
4*(f*x+e)^2-1/72*sin(f*x+e)^4-1/24*sin(f*x+e)^2-1/6*(f*x+e)^2*sin(f*x+e)^6+
1/3*(f*x+e)*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e
)+5/16*f*x+5/16*e)+1/108*sin(f*x+e)^6)+3*c^2*d*f^2*((f*x+e)*(-1/4*(sin(f*x+
e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/16*sin(f*x+
e)^4+3/16*sin(f*x+e)^2)-3/4*I*c^3*f^3*sin(f*x+e)^4+4*I*c^3*f^3*(-1/6*sin(f*
x+e)^2*cos(f*x+e)^4-1/12*cos(f*x+e)^4)-9*I*d^3*e^2*(1/4*(f*x+e)*sin(f*x+e)^
4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)-12*d^3*e^2
*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3
2*(f*x+e)^2+1/96*sin(f*x+e)^4+1/32*sin(f*x+e)^2-(f*x+e)*(-1/6*(sin(f*x+e)^5
+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/36*sin(f*x
+e)^6)+c^3*f^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e
)-d^3*e^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*I
*d^3*(1/4*(f*x+e)^3*sin(f*x+e)^4-3/4*(f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(
f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/32*(f*x+e)*sin(f*x+e)^4-3/128*(sin(f*x+
e)^3+3/2*sin(f*x+e))*cos(f*x+e)-27/256*f*x-27/256*e+9/32*(f*x+e)*cos(f*x+e)
^2-9/64*sin(f*x+e)*cos(f*x+e)+3/16*(f*x+e)^3)+12*d^3*e*((f*x+e)^2*(-1/4*(si
n(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/48*(f*x+e)*sin(f*x+e
)^4+1/192*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+47/1152*f*x+47/1152*e-1/
16*(f*x+e)*cos(f*x+e)^2+1/32*sin(f*x+e)*cos(f*x+e)-1/24*(f*x+e)^3-(f*x+e)^2
*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+
5/16*e)-1/18*(f*x+e)*sin(f*x+e)^6-1/108*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8
*sin(f*x+e))*cos(f*x+e))+4*I*d^3*(1/4*(f*x+e)^3*sin(f*x+e)^4-3/4*(f*x+e)^2*
(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/24*(f*x+e)*
sin(f*x+e)^4-1/96*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-1/18*f*x-1/18*e+
1/8*(f*x+e)*cos(f*x+e)^2-1/16*sin(f*x+e)*cos(f*x+e)+1/12*(f*x+e)^3-1/6*(f*x
+e)^3*sin(f*x+e)^6+1/2*(f*x+e)^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*
sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/36*(f*x+e)*sin(f*x+e)^6+1/216*(si
n(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e))+4*d^3*e^3*(-1/6*si
n(f*x+e)^3*cos(f*x+e)^3-1/8*sin(f*x+e)*cos(f*x+e)^3+1/16*sin(f*x+e)*cos(f*x
+e)+1/16*f*x+1/16*e)-4*c^3*f^3*(-1/6*sin(f*x+e)^3*cos(f*x+e)^3-1/8*sin(f*x+
e)*cos(f*x+e)^3+1/16*sin(f*x+e)*cos(f*x+e)+1/16*f*x+1/16*e)+12*I*c^2*d*f^2*
(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-1/2
4*f*x-1/24*e-1/6*(f*x+e)*sin(f*x+e)^6-1/36*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+1
5/8*sin(f*x+e))*cos(f*x+e))-9/4*I*c*d^2*e^2*f*sin(f*x+e)^4-12*I*c^2*d*e*f^2
*(-1/6*sin(f*x+e)^2*cos(f*x+e)^4-1/12*cos(f*x+e)^4)-9*I*c*d^2*f*(1/4*(f*x+e
)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)
+3/8*f*x+3/8*e)+3/32*(f*x+e)^2-1/32*sin(f*x+e)^4-3/32*sin(f*x+e)^2)-9*I*c^2
*d*f^2*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x
+e)-3/32*f*x-3/32*e)+24*c*d^2*e*f*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+
e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/32*(f*x+e)^2+1/96*sin(f*x+e)^4+1/32*sin(f*x
+e)^2-(f*x+e)*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x
+e)+5/16*f*x+5/16*e)-1/36*sin(f*x+e)^6)+9*I*d^3*e*(1/4*(f*x+e)^2*sin(f*x+e)
^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e
)+3/32*(f*x+e)^2-1/32*sin(f*x+e)^4-3/32*sin(f*x+e)^2)-12*c^2*d*f^2*((f*x+e)
*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/32*(f*x+e)
^2+1/96*sin(f*x+e)^4+1/32*sin(f*x+e)^2-(f*x+e)*(-1/6*(sin(f*x+e)^5+5/4*sin(
f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/36*sin(f*x+e)^6)-4*
I*d^3*e^3*(-1/6*sin(f*x+e)^2*cos(f*x+e)^4-1/12*cos(f*x+e)^4)-12*c*d^2*f*((f
*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/48*
(f*x+e)*sin(f*x+e)^4+1/192*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+47/1152
*f*x+47/1152*e-1/16*(f*x+e)*cos(f*x+e)^2+1/32*sin(f*x+e)*cos(f*x+e)-1/24*(f
*x+e)^3-(f*x+e)^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos
(f*x+e)+5/16*f*x+5/16*e)-1/18*(f*x+e)*sin(f*x+e)^6-1/108*(sin(f*x+e)^5+5/4*
sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e))+12*I*d^3*e^2*(1/4*(f*x+e)*sin(f*x
+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-1/24*f*x-1/24*e-1/6*(f*
x+e)*sin(f*x+e)^6-1/36*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(
f*x+e))-24*I*c*d^2*e*f*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin
(f*x+e))*cos(f*x+e)-1/24*f*x-1/24*e-1/6*(f*x+e)*sin(f*x+e)^6-1/36*(sin(f*x+
e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e))+9/4*I*c^2*d*e*f^2*sin(f*
```

```
x+e)^4+3/4*I*d^3*e^3*sin(f*x+e)^4+3*c*d^2*f*((f*x+e)^2*(-1/4*(sin(f*x+e)^3+
3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/8*(f*x+e)*sin(f*x+e)^4+1/32*(si
n(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+9/64*f*x+9/64*e-3/8*(f*x+e)*cos(f*x+e
)^2+3/16*sin(f*x+e)*cos(f*x+e)-1/4*(f*x+e)^3)-4*d^3*((f*x+e)^3*(-1/4*(sin(f
*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/32*(f*x+e)^2*sin(f*x+e)
^4-1/16*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*
e)-47/768*(f*x+e)^2+11/2304*sin(f*x+e)^4-25/768*sin(f*x+e)^2-3/32*(f*x+e)^2
*cos(f*x+e)^2+3/16*(f*x+e)*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3/64*(
f*x+e)^4-(f*x+e)^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*co
s(f*x+e)+5/16*f*x+5/16*e)-1/12*(f*x+e)^2*sin(f*x+e)^6+1/6*(f*x+e)*(-1/6*(si
n(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/
216*sin(f*x+e)^6)+d^3*((f*x+e)^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*
x+e)+3/8*f*x+3/8*e)+3/16*(f*x+e)^2*sin(f*x+e)^4-3/8*(f*x+e)*(-1/4*(sin(f*x+
e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-27/128*(f*x+e)^2-3/128*sin(f
*x+e)^4-45/128*sin(f*x+e)^2-9/16*(f*x+e)^2*cos(f*x+e)^2+9/8*(f*x+e)*(1/2*si
n(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-9/32*(f*x+e)^4))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 1.62, size = 418, normalized size = 1.06

$$e^{e\,2i+f\,x\,2i}\left(-\frac{\left(-12\,c^3\,f^3-c^2\,d\,f^2\,18i+18\,c\,d^2\,f+d^3\,9i\right)1i}{64\,a^3\,f^4}+\frac{d^3\,x^3\,3i}{16\,a^3\,f}+\frac{d\,x\,\left(2\,c^2\,f^2+c\,d\,f\,2i-d^2\right)9i}{32\,a^3\,f^3}+\frac{d^2\,x^2\,\left(2\right.}{32}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*cot(e + f*x)*1i)^3,x)
```

```
[Out] exp(e*2i + f*x*2i)*((d^3*x^3*3i)/(16*a^3*f) - ((d^3*9i - 12*c^3*f^3 - c^2*d
*f^2*18i + 18*c*d^2*f)*1i)/(64*a^3*f^4) + (d*x*(2*c^2*f^2 - d^2 + c*d*f*2i)
*9i)/(32*a^3*f^3) + (d^2*x^2*(d*1i + 2*c*f)*9i)/(32*a^3*f^2)) - exp(e*4i +
f*x*4i)*((d^3*x^3*3i)/(32*a^3*f) - ((d^3*9i - 96*c^3*f^3 - c^2*d*f^2*72i +
36*c*d^2*f)*1i)/(1024*a^3*f^4) + (d*x*(8*c^2*f^2 - d^2 + c*d*f*4i)*9i)/(256
*a^3*f^3) + (d^2*x^2*(d*1i + 4*c*f)*9i)/(128*a^3*f^2)) + exp(e*6i + f*x*6i)
*((d^3*x^3*1i)/(48*a^3*f) - ((d^3*1i - 36*c^3*f^3 - c^2*d*f^2*18i + 6*c*d^2
*f)*1i)/(1728*a^3*f^4) + (d*x*(18*c^2*f^2 - d^2 + c*d*f*6i)*1i)/(288*a^3*f^
3) + (d^2*x^2*(d*1i + 6*c*f)*1i)/(96*a^3*f^2)) + (c^3*x)/(8*a^3) + (d^3*x^4
)/(32*a^3) + (3*c^2*d*x^2)/(16*a^3) + (c*d^2*x^3)/(8*a^3)
```

sympy [A] time = 0.84, size = 935, normalized size = 2.36

$$\begin{cases}-\frac{\left(-21233664ia^6c^3f^{11}e^{2ie}-63700992ia^6c^2df^{11}xe^{2ie}+31850496a^6c^2df^{10}e^{2ie}-63700992ia^6cd^2f^{11}x^2e^{2ie}+63700992a^6cd^2f^{10}xe^{2ie}+31850496ia^6cd^2f^9e^{2}\right.}{} \\ \frac{x^4\left(-d^3e^{6ie}+3d^3e^{4ie}-3d^3e^{2ie}\right)}{32a^3}+\frac{x^3\left(-cd^2e^{6ie}+3cd^2e^{4ie}-3cd^2e^{2ie}\right)}{8a^3}+\frac{x^2\left(-3c^2de^{6ie}+9c^2de^{4ie}-9c^2de^{2ie}\right)}{16a^3}+\frac{x\left(-c^3e^{6ie}+3c^3e^{4ie}-3c^3e^{2ie}\right)}{8a^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] Piecewise((-(-21233664*I*a**6*c**3*f**11*exp(2*I*e) - 63700992*I*a**6*c**2
*d*f**11*x*exp(2*I*e) + 31850496*a**6*c**2*d*f**10*exp(2*I*e) - 63700992*I*
```

```
a**6*c*d**2*f**11*x**2*exp(2*I*e) + 63700992*a**6*c*d**2*f**10*x*exp(2*I*e)
 + 31850496*I*a**6*c*d**2*f**9*exp(2*I*e) - 21233664*I*a**6*d**3*f**11*x**3
*exp(2*I*e) + 31850496*a**6*d**3*f**10*x**2*exp(2*I*e) + 31850496*I*a**6*d*
*3*f**9*x*exp(2*I*e) - 15925248*a**6*d**3*f**8*exp(2*I*e))*exp(2*I*f*x) + (
10616832*I*a**6*c**3*f**11*exp(4*I*e) + 31850496*I*a**6*c**2*d*f**11*x*exp(
4*I*e) - 7962624*a**6*c**2*d*f**10*exp(4*I*e) + 31850496*I*a**6*c*d**2*f**1
1*x**2*exp(4*I*e) - 15925248*a**6*c*d**2*f**10*x*exp(4*I*e) - 3981312*I*a**
6*c*d**2*f**9*exp(4*I*e) + 10616832*I*a**6*d**3*f**11*x**3*exp(4*I*e) - 796
2624*a**6*d**3*f**10*x**2*exp(4*I*e) - 3981312*I*a**6*d**3*f**9*x*exp(4*I*e
) + 995328*a**6*d**3*f**8*exp(4*I*e))*exp(4*I*f*x) + (-2359296*I*a**6*c**3*
f**11*exp(6*I*e) - 7077888*I*a**6*c**2*d*f**11*x*exp(6*I*e) + 1179648*a**6*
c**2*d*f**10*exp(6*I*e) - 7077888*I*a**6*c*d**2*f**11*x**2*exp(6*I*e) + 235
9296*a**6*c*d**2*f**10*x*exp(6*I*e) + 393216*I*a**6*c*d**2*f**9*exp(6*I*e)
- 2359296*I*a**6*d**3*f**11*x**3*exp(6*I*e) + 1179648*a**6*d**3*f**10*x**2*
exp(6*I*e) + 393216*I*a**6*d**3*f**9*x*exp(6*I*e) - 65536*a**6*d**3*f**8*ex
p(6*I*e))*exp(6*I*f*x))/(113246208*a**9*f**12), Ne(113246208*a**9*f**12, 0)
), (x**4*(-d**3*exp(6*I*e) + 3*d**3*exp(4*I*e) - 3*d**3*exp(2*I*e))/(32*a**
3) + x**3*(-c*d**2*exp(6*I*e) + 3*c*d**2*exp(4*I*e) - 3*c*d**2*exp(2*I*e))/
(8*a**3) + x**2*(-3*c**2*d*exp(6*I*e) + 9*c**2*d*exp(4*I*e) - 9*c**2*d*exp(
2*I*e))/(16*a**3) + x*(-c**3*exp(6*I*e) + 3*c**3*exp(4*I*e) - 3*c**3*exp(2*
I*e))/(8*a**3), True)) + c**3*x/(8*a**3) + 3*c**2*d*x**2/(16*a**3) + c*d**2
*x**3/(8*a**3) + d**3*x**4/(32*a**3)
```

3.28 $\int \frac{(c+dx)^2}{(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=294

$$-\frac{3d(c+dx)e^{2ie+2ifx}}{16a^3f^2}+\frac{3d(c+dx)e^{4ie+4ifx}}{64a^3f^2}-\frac{d(c+dx)e^{6ie+6ifx}}{144a^3f^2}+\frac{3i(c+dx)^2e^{2ie+2ifx}}{16a^3f}-\frac{3i(c+dx)^2e^{4ie+4ifx}}{32a^3f}+\frac{i(c+dx)^2}{48a^3}$$

```
[Out] -3/32*I*d^2*exp(2*I*e+2*I*f*x)/a^3/f^3+3/256*I*d^2*exp(4*I*e+4*I*f*x)/a^3/f
^3-1/864*I*d^2*exp(6*I*e+6*I*f*x)/a^3/f^3-3/16*d*exp(2*I*e+2*I*f*x)*(d*x+c)
/a^3/f^2+3/64*d*exp(4*I*e+4*I*f*x)*(d*x+c)/a^3/f^2-1/144*d*exp(6*I*e+6*I*f*
x)*(d*x+c)/a^3/f^2+3/16*I*exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^3/f-3/32*I*exp(4*I
*e+4*I*f*x)*(d*x+c)^2/a^3/f+1/48*I*exp(6*I*e+6*I*f*x)*(d*x+c)^2/a^3/f+1/24*
(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.27, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3729, 2176, 2194}

$$-\frac{3d(c+dx)e^{2ie+2ifx}}{16a^3f^2}+\frac{3d(c+dx)e^{4ie+4ifx}}{64a^3f^2}-\frac{d(c+dx)e^{6ie+6ifx}}{144a^3f^2}+\frac{3i(c+dx)^2e^{2ie+2ifx}}{16a^3f}-\frac{3i(c+dx)^2e^{4ie+4ifx}}{32a^3f}+\frac{i(c+dx)^2}{48a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (((-3*I)/32)*d^2*E^((2*I)*e + (2*I)*f*x))/(a^3*f^3) + (((3*I)/256)*d^2*E^((
4*I)*e + (4*I)*f*x))/(a^3*f^3) - ((I/864)*d^2*E^((6*I)*e + (6*I)*f*x))/(a^3
*f^3) - (3*d*E^((2*I)*e + (2*I)*f*x)*(c + d*x))/(16*a^3*f^2) + (3*d*E^((4*I
)*e + (4*I)*f*x)*(c + d*x))/(64*a^3*f^2) - (d*E^((6*I)*e + (6*I)*f*x)*(c +
d*x))/(144*a^3*f^2) + (((3*I)/16)*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^2)/(a^3
*f) - (((3*I)/32)*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^2)/(a^3*f) + ((I/48)*E^
((6*I)*e + (6*I)*f*x)*(c + d*x)^2)/(a^3*f) + (c + d*x)^3/(24*a^3*d)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+ia\cot(e+fx))^3}\,dx = \int \left(\frac{(c+dx)^2}{8a^3} - \frac{3e^{2ie+2ifx}(c+dx)^2}{8a^3} + \frac{3e^{4ie+4ifx}(c+dx)^2}{8a^3} - \frac{e^{6ie+6ifx}(c+dx)^2}{8a^3}\right) dx$$
$$= \frac{(c+dx)^3}{24a^3d} - \frac{\int e^{6ie+6ifx}(c+dx)^2\,dx}{8a^3} - \frac{3\int e^{2ie+2ifx}(c+dx)^2\,dx}{8a^3} + \frac{3\int e^{4ie+4ifx}(c+$$
$$= \frac{3ie^{2ie+2ifx}(c+dx)^2}{16a^3f} - \frac{3ie^{4ie+4ifx}(c+dx)^2}{32a^3f} + \frac{ie^{6ie+6ifx}(c+dx)^2}{48a^3f} + \frac{(c+dx)^3}{24a^3d} - \frac{(id)}{}$$
$$= -\frac{3de^{2ie+2ifx}(c+dx)}{16a^3f^2} + \frac{3de^{4ie+4ifx}(c+dx)}{64a^3f^2} - \frac{de^{6ie+6ifx}(c+dx)}{144a^3f^2} + \frac{3ie^{2ie+2ifx}(c+d}{16a^3f}$$
$$= -\frac{3id^2e^{2ie+2ifx}}{32a^3f^3} + \frac{3id^2e^{4ie+4ifx}}{256a^3f^3} - \frac{id^2e^{6ie+6ifx}}{864a^3f^3} - \frac{3de^{2ie+2ifx}(c+dx)}{16a^3f^2} + \frac{3de^{4ie+4ifx}(c}{64a^3f^2}$$

Mathematica [A] time = 0.81, size = 369, normalized size = 1.26

$$\frac{288f^3x\left(3c^2+3cdx+d^2x^2\right) + 648(\cos(2e)+i\sin(2e))\cos(2fx)((1+i)cf+d(-1+(1+i)fx))((1+i)cf+d((}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (288*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 648*((1 + I)*c*f + d*(-1 + (1 + I)
*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e
]) - 81*((2 + 2*I)*c*f + d*(-1 + (2 + 2*I)*f*x))*((2 + 2*I)*c*f + d*(I + (2
 + 2*I)*f*x))*Cos[4*f*x]*(Cos[4*e] + I*Sin[4*e]) + 8*((3 + 3*I)*c*f + d*(-1
 + (3 + 3*I)*f*x))*((3 + 3*I)*c*f + d*(I + (3 + 3*I)*f*x))*Cos[6*f*x]*(Cos[
6*e] + I*Sin[6*e]) + (648*I)*((1 + I)*c*f + d*(-1 + (1 + I)*f*x))*((1 + I)*
c*f + d*(I + (1 + I)*f*x))*(Cos[2*e] + I*Sin[2*e])*Sin[2*f*x] - 81*(d - (2
 + 2*I)*c*f - (2 + 2*I)*d*f*x)*(d + (2 - 2*I)*c*f + (2 - 2*I)*d*f*x)*(Cos[4*
e] + I*Sin[4*e])*Sin[4*f*x] + (8*I)*((3 + 3*I)*c*f + d*(-1 + (3 + 3*I)*f*x)
)*((3 + 3*I)*c*f + d*(I + (3 + 3*I)*f*x))*(Cos[6*e] + I*Sin[6*e])*Sin[6*f*x
])/(6912*a^3*f^3)
```

fricas [A] time = 0.81, size = 207, normalized size = 0.70

$$\frac{288\,d^2f^3x^3 + 864\,cdf^3x^2 + 864\,c^2f^3x + \left(144i\,d^2f^2x^2 + 144i\,c^2f^2 - 48\,cdf - 8i\,d^2 + \left(288i\,cdf^2 - 48\,d^2f\right)x\right)e^{(6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6912*(288*d^2*f^3*x^3 + 864*c*d*f^3*x^2 + 864*c^2*f^3*x + (144*I*d^2*f^2*
x^2 + 144*I*c^2*f^2 - 48*c*d*f - 8*I*d^2 + (288*I*c*d*f^2 - 48*d^2*f)*x)*e^
(6*I*f*x + 6*I*e) + (-648*I*d^2*f^2*x^2 - 648*I*c^2*f^2 + 324*c*d*f + 81*I*
d^2 + (-1296*I*c*d*f^2 + 324*d^2*f)*x)*e^(4*I*f*x + 4*I*e) + (1296*I*d^2*f^
2*x^2 + 1296*I*c^2*f^2 - 1296*c*d*f - 648*I*d^2 + (2592*I*c*d*f^2 - 1296*d^
2*f)*x)*e^(2*I*f*x + 2*I*e))/(a^3*f^3)
```

giac [A] time = 0.55, size = 351, normalized size = 1.19

$$\frac{288\,d^2f^3x^3 + 864\,cdf^3x^2 + 144i\,d^2f^2x^2e^{\left(6i\,fx+6i\,e\right)} - 648i\,d^2f^2x^2e^{\left(4i\,fx+4i\,e\right)} + 1296i\,d^2f^2x^2e^{\left(2i\,fx+2i\,e\right)} + 864\,c^2f}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/6912*(288*d^2*f^3*x^3 + 864*c*d*f^3*x^2 + 144*I*d^2*f^2*x^2*e^(6*I*f*x +
6*I*e) - 648*I*d^2*f^2*x^2*e^(4*I*f*x + 4*I*e) + 1296*I*d^2*f^2*x^2*e^(2*I*
f*x + 2*I*e) + 864*c^2*f^3*x + 288*I*c*d*f^2*x*e^(6*I*f*x + 6*I*e) - 1296*I
*c*d*f^2*x*e^(4*I*f*x + 4*I*e) + 2592*I*c*d*f^2*x*e^(2*I*f*x + 2*I*e) + 144
*I*c^2*f^2*e^(6*I*f*x + 6*I*e) - 48*d^2*f*x*e^(6*I*f*x + 6*I*e) - 648*I*c^2
*f^2*e^(4*I*f*x + 4*I*e) + 324*d^2*f*x*e^(4*I*f*x + 4*I*e) + 1296*I*c^2*f^2
*e^(2*I*f*x + 2*I*e) - 1296*d^2*f*x*e^(2*I*f*x + 2*I*e) - 48*c*d*f*e^(6*I*f
*x + 6*I*e) + 324*c*d*f*e^(4*I*f*x + 4*I*e) - 1296*c*d*f*e^(2*I*f*x + 2*I*e
) - 8*I*d^2*e^(6*I*f*x + 6*I*e) + 81*I*d^2*e^(4*I*f*x + 4*I*e) - 648*I*d^2*
e^(2*I*f*x + 2*I*e))/(a^3*f^3)
```

maple [B] time = 1.48, size = 1843, normalized size = 6.27

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x)
```

```
[Out] 1/f^3/a^3*(-6*I*c*d*f*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(
f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)+4*I*c^2*f^2*(-1/6*sin(f*x+e)^2*cos(f*x+
e)^4-1/12*cos(f*x+e)^4)-8*I*d^2*e*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e
)^3+3/2*sin(f*x+e))*cos(f*x+e)-1/24*f*x-1/24*e-1/6*(f*x+e)*sin(f*x+e)^6-1/3
6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e))+8*I*c*d*f*(1/
4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-1/24*f
*x-1/24*e-1/6*(f*x+e)*sin(f*x+e)^6-1/36*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8
*sin(f*x+e))*cos(f*x+e))-3*I*d^2*(1/4*(f*x+e)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-
1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3/32*(f*x+e)^2-
1/32*sin(f*x+e)^4-3/32*sin(f*x+e)^2)+4*I*d^2*e^2*(-1/6*sin(f*x+e)^2*cos(f*x
+e)^4-1/12*cos(f*x+e)^4)-4*d^2*((f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e
))*cos(f*x+e)+3/8*f*x+3/8*e)+1/48*(f*x+e)*sin(f*x+e)^4+1/192*(sin(f*x+e)^3+
3/2*sin(f*x+e))*cos(f*x+e)+47/1152*f*x+47/1152*e-1/16*(f*x+e)*cos(f*x+e)^2+
1/32*sin(f*x+e)*cos(f*x+e)-1/24*(f*x+e)^3-(f*x+e)^2*(-1/6*(sin(f*x+e)^5+5/4
*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/18*(f*x+e)*sin
(f*x+e)^6-1/108*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e))
-8*c*d*f*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/
8*e)-1/32*(f*x+e)^2+1/96*sin(f*x+e)^4+1/32*sin(f*x+e)^2-(f*x+e)*(-1/6*(sin(
f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/36
*sin(f*x+e)^6)+8*d^2*e*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x
+e)+3/8*f*x+3/8*e)-1/32*(f*x+e)^2+1/96*sin(f*x+e)^4+1/32*sin(f*x+e)^2-(f*x+
e)*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*
x+5/16*e)-1/36*sin(f*x+e)^6)-4*c^2*f^2*(-1/6*sin(f*x+e)^3*cos(f*x+e)^3-1/8*
sin(f*x+e)*cos(f*x+e)^3+1/16*sin(f*x+e)*cos(f*x+e)+1/16*f*x+1/16*e)+8*c*d*e
*f*(-1/6*sin(f*x+e)^3*cos(f*x+e)^3-1/8*sin(f*x+e)*cos(f*x+e)^3+1/16*sin(f*x
+e)*cos(f*x+e)+1/16*f*x+1/16*e)-4*d^2*e^2*(-1/6*sin(f*x+e)^3*cos(f*x+e)^3-1
/8*sin(f*x+e)*cos(f*x+e)^3+1/16*sin(f*x+e)*cos(f*x+e)+1/16*f*x+1/16*e)+3/2*
I*c*d*e*f*sin(f*x+e)^4-3/4*I*c^2*f^2*sin(f*x+e)^4+6*I*d^2*e*(1/4*(f*x+e)*si
n(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)+4
*I*d^2*(1/4*(f*x+e)^2*sin(f*x+e)^4-1/2*(f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(
f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/24*(f*x+e)^2-1/72*sin(f*x+e)^4-1/24*sin
(f*x+e)^2-1/6*(f*x+e)^2*sin(f*x+e)^6+1/3*(f*x+e)*(-1/6*(sin(f*x+e)^5+5/4*si
n(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/108*sin(f*x+e)^6)
-3/4*I*d^2*e^2*sin(f*x+e)^4-8*I*c*d*e*f*(-1/6*sin(f*x+e)^2*cos(f*x+e)^4-1/1
2*cos(f*x+e)^4)+d^2*((f*x+e)^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)+3/8*f*x+3/8*e)+1/8*(f*x+e)*sin(f*x+e)^4+1/32*(sin(f*x+e)^3+3/2*sin(f*x+e
))*cos(f*x+e)+9/64*f*x+9/64*e-3/8*(f*x+e)*cos(f*x+e)^2+3/16*sin(f*x+e)*cos(
f*x+e)-1/4*(f*x+e)^3)+2*c*d*f*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*
cos(f*x+e)+3/8*f*x+3/8*e)-3/16*(f*x+e)^2+1/16*sin(f*x+e)^4+3/16*sin(f*x+e)^
2)-2*d^2*e*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+
```

```
3/8*e)-3/16*(f*x+e)^2+1/16*sin(f*x+e)^4+3/16*sin(f*x+e)^2)+c^2*f^2*(-1/4*(s
in(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*c*d*e*f*(-1/4*(sin(
f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+d^2*e^2*(-1/4*(sin(f*x+e
)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**B**] time = 1.00, size = 263, normalized size = 0.89

$$e^{e\,2i+f\,x\,2i}\left(\frac{\left(6\,c^2\,f^2+c\,d\,f\,6i-3\,d^2\right)1i}{32\,a^3\,f^3}+\frac{d^2\,x^2\,3i}{16\,a^3\,f}+\frac{d\,x\,\left(2\,c\,f+d\,1i\right)3i}{16\,a^3\,f^2}\right)-e^{e\,4i+f\,x\,4i}\left(\frac{\left(24\,c^2\,f^2+c\,d\,f\,12i-3\,d\right.}{256\,a^3\,f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*cot(e + f*x)*1i)^3,x)
```

```
[Out] exp(e*2i + f*x*2i)*(((6*c^2*f^2 - 3*d^2 + c*d*f*6i)*1i)/(32*a^3*f^3) + (d^2
*x^2*3i)/(16*a^3*f) + (d*x*(d*1i + 2*c*f)*3i)/(16*a^3*f^2)) - exp(e*4i + f*
x*4i)*(((24*c^2*f^2 - 3*d^2 + c*d*f*12i)*1i)/(256*a^3*f^3) + (d^2*x^2*3i)/(
32*a^3*f) + (d*x*(d*1i + 4*c*f)*3i)/(64*a^3*f^2)) + exp(e*6i + f*x*6i)*(((1
8*c^2*f^2 - d^2 + c*d*f*6i)*1i)/(864*a^3*f^3) + (d^2*x^2*1i)/(48*a^3*f) + (
d*x*(d*1i + 6*c*f)*1i)/(144*a^3*f^2)) + (c^2*x)/(8*a^3) + (d^2*x^3)/(24*a^3
) + (c*d*x^2)/(8*a^3)
```

sympy [**A**] time = 0.64, size = 578, normalized size = 1.97

$$\begin{cases} -\frac{\left(-1327104ia^6c^2f^8e^{2ie}-2654208ia^6cdf^8xe^{2ie}+1327104a^6cdf^7e^{2ie}-1327104ia^6d^2f^8x^2e^{2ie}+1327104a^6d^2f^7xe^{2ie}+663552ia^6d^2f^6e^{2ie}\right)e^{2ifx}+\left(663\right.}{} \\ \frac{x^3\left(-d^2e^{6ie}+3d^2e^{4ie}-3d^2e^{2ie}\right)}{24a^3}+\frac{x^2\left(-cde^{6ie}+3cde^{4ie}-3cde^{2ie}\right)}{8a^3}+\frac{x\left(-c^2e^{6ie}+3c^2e^{4ie}-3c^2e^{2ie}\right)}{8a^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] Piecewise((-((-1327104*I*a**6*c**2*f**8*exp(2*I*e) - 2654208*I*a**6*c*d*f**
8*x*exp(2*I*e) + 1327104*a**6*c*d*f**7*exp(2*I*e) - 1327104*I*a**6*d**2*f**
8*x**2*exp(2*I*e) + 1327104*a**6*d**2*f**7*x*exp(2*I*e) + 663552*I*a**6*d**
2*f**6*exp(2*I*e))*exp(2*I*f*x) + (663552*I*a**6*c**2*f**8*exp(4*I*e) + 132
7104*I*a**6*c*d*f**8*x*exp(4*I*e) - 331776*a**6*c*d*f**7*exp(4*I*e) + 66355
2*I*a**6*d**2*f**8*x**2*exp(4*I*e) - 331776*a**6*d**2*f**7*x*exp(4*I*e) - 8
2944*I*a**6*d**2*f**6*exp(4*I*e))*exp(4*I*f*x) + (-147456*I*a**6*c**2*f**8*
exp(6*I*e) - 294912*I*a**6*c*d*f**8*x*exp(6*I*e) + 49152*a**6*c*d*f**7*exp(
6*I*e) - 147456*I*a**6*d**2*f**8*x**2*exp(6*I*e) + 49152*a**6*d**2*f**7*x*e
xp(6*I*e) + 8192*I*a**6*d**2*f**6*exp(6*I*e))*exp(6*I*f*x))/(7077888*a**9*f
**9), Ne(7077888*a**9*f**9, 0)), (x**3*(-d**2*exp(6*I*e) + 3*d**2*exp(4*I*e
) - 3*d**2*exp(2*I*e))/(24*a**3) + x**2*(-c*d*exp(6*I*e) + 3*c*d*exp(4*I*e)
 - 3*c*d*exp(2*I*e))/(8*a**3) + x*(-c**2*exp(6*I*e) + 3*c**2*exp(4*I*e) - 3
*c**2*exp(2*I*e))/(8*a**3), True)) + c**2*x/(8*a**3) + c*d*x**2/(8*a**3) +
d**2*x**3/(24*a**3)
```

3.29 $\int \frac{c+dx}{(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=209

$$-\frac{i(c+dx)}{8f\left(a^3+ia^3\cot(e+fx)\right)}+\frac{x(c+dx)}{8a^3}+\frac{11d}{96f^2\left(a^3+ia^3\cot(e+fx)\right)}+\frac{11idx}{96a^3f}-\frac{dx^2}{16a^3}-\frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2}-\frac{}{6f}$$

```
[Out] 11/96*I*d*x/a^3/f-1/16*d*x^2/a^3+1/8*x*(d*x+c)/a^3+1/36*d/f^2/(a+I*a*cot(f*
x+e))^3-1/6*I*(d*x+c)/f/(a+I*a*cot(f*x+e))^3+5/96*d/a/f^2/(a+I*a*cot(f*x+e)
)^2-1/8*I*(d*x+c)/a/f/(a+I*a*cot(f*x+e))^2+11/96*d/f^2/(a^3+I*a^3*cot(f*x+e
))-1/8*I*(d*x+c)/f/(a^3+I*a^3*cot(f*x+e))
```

Rubi [A] time = 0.22, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3479, 8, 3730}

$$-\frac{i(c+dx)}{8f\left(a^3+ia^3\cot(e+fx)\right)}+\frac{x(c+dx)}{8a^3}+\frac{11d}{96f^2\left(a^3+ia^3\cot(e+fx)\right)}+\frac{11idx}{96a^3f}-\frac{dx^2}{16a^3}-\frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2}-\frac{}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (((11*I)/96)*d*x)/(a^3*f) - (d*x^2)/(16*a^3) + (x*(c + d*x))/(8*a^3) + d/(3
6*f^2*(a + I*a*Cot[e + f*x])^3) - ((I/6)*(c + d*x))/(f*(a + I*a*Cot[e + f*x
])^3) + (5*d)/(96*a*f^2*(a + I*a*Cot[e + f*x])^2) - ((I/8)*(c + d*x))/(a*f*
(a + I*a*Cot[e + f*x])^2) + (11*d)/(96*f^2*(a^3 + I*a^3*Cot[e + f*x])) - ((
I/8)*(c + d*x))/(f*(a^3 + I*a^3*Cot[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3730

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Dist[(c + d*x)
^m, u, x] - Dist[d*m, Int[Dist[(c + d*x)^(m - 1), u, x], x], x]] /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+ia\cot(e+fx))^3}\,dx = \frac{x(c+dx)}{8a^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2} - \frac{i(c+d}{8f\left(a^3+ia^3\cot\right.}$$

$$= -\frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2} - \frac{}{8f\left(a\right.}$$

$$= -\frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3} + \frac{}{32a}$$

$$= \frac{idx}{16a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3}$$

$$= \frac{3idx}{32a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3}$$

$$= \frac{11idx}{96a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3}$$

Mathematica [A] time = 0.66, size = 244, normalized size = 1.17

$$\frac{108i(2cf+d(2fx+i))\cos(2(e+fx)) + 27(-4icf-4idfx+d)\cos(4(e+fx)) - 216cf\sin(2(e+fx)) + 108cf}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (-72*d*e^2 + 144*c*e*f + 144*c*f^2*x + 72*d*f^2*x^2 + (108*I)*(2*c*f + d*(I
 + 2*f*x))*Cos[2*(e + f*x)] + 27*(d - (4*I)*c*f - (4*I)*d*f*x)*Cos[4*(e + f
*x)] - 4*d*Cos[6*(e + f*x)] + (24*I)*c*f*Cos[6*(e + f*x)] + (24*I)*d*f*x*Co
s[6*(e + f*x)] - (108*I)*d*Sin[2*(e + f*x)] - 216*c*f*Sin[2*(e + f*x)] - 21
6*d*f*x*Sin[2*(e + f*x)] + (27*I)*d*Sin[4*(e + f*x)] + 108*c*f*Sin[4*(e + f
*x)] + 108*d*f*x*Sin[4*(e + f*x)] - (4*I)*d*Sin[6*(e + f*x)] - 24*c*f*Sin[6
*(e + f*x)] - 24*d*f*x*Sin[6*(e + f*x)])/(1152*a^3*f^2)
```

fricas [A] time = 0.66, size = 94, normalized size = 0.45

$$\frac{72\,df^2x^2 + 144\,cf^2x + \left(24i\,dfx + 24i\,cf - 4\,d\right)e^{\left(6ifx+6ie\right)} + \left(-108i\,dfx - 108i\,cf + 27\,d\right)e^{\left(4ifx+4ie\right)} + \left(216i\,df\right.}{1152\,a^3f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/1152*(72*d*f^2*x^2 + 144*c*f^2*x + (24*I*d*f*x + 24*I*c*f - 4*d)*e^(6*I*f
*x + 6*I*e) + (-108*I*d*f*x - 108*I*c*f + 27*d)*e^(4*I*f*x + 4*I*e) + (216*
I*d*f*x + 216*I*c*f - 108*d)*e^(2*I*f*x + 2*I*e))/(a^3*f^2)
```

giac [A] time = 2.08, size = 151, normalized size = 0.72

$$\frac{72\,df^2x^2 + 144\,cf^2x + 24i\,dfxe^{\left(6ifx+6ie\right)} - 108i\,dfxe^{\left(4ifx+4ie\right)} + 216i\,dfxe^{\left(2ifx+2ie\right)} + 24i\,cfe^{\left(6ifx+6ie\right)} - 108i\,c}{1152\,a^3f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/1152*(72*d*f^2*x^2 + 144*c*f^2*x + 24*I*d*f*x*e^(6*I*f*x + 6*I*e) - 108*I
*d*f*x*e^(4*I*f*x + 4*I*e) + 216*I*d*f*x*e^(2*I*f*x + 2*I*e) + 24*I*c*f*e^(
6*I*f*x + 6*I*e) - 108*I*c*f*e^(4*I*f*x + 4*I*e) + 216*I*c*f*e^(2*I*f*x + 2
*I*e) - 4*d*e^(6*I*f*x + 6*I*e) + 27*d*e^(4*I*f*x + 4*I*e) - 108*d*e^(2*I*f
*x + 2*I*e))/(a^3*f^2)
```

maple [B] time = 1.06, size = 653, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*cot(f*x+e))^3,x)
```

```
[Out] 1/f^2/a^3*(4*I*d*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*sin(f*x+e
))*cos(f*x+e)-1/24*f*x-1/24*e-1/6*(f*x+e)*sin(f*x+e)^6-1/36*(sin(f*x+e)^5+5
/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e))+4*I*c*f*(-1/6*sin(f*x+e)^2*cos
(f*x+e)^4-1/12*cos(f*x+e)^4)-4*I*d*e*(-1/6*sin(f*x+e)^2*cos(f*x+e)^4-1/12*c
os(f*x+e)^4)-4*d*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/
8*f*x+3/8*e)-1/32*(f*x+e)^2+1/96*sin(f*x+e)^4+1/32*sin(f*x+e)^2-(f*x+e)*(-1
/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16
*e)-1/36*sin(f*x+e)^6)-4*c*f*(-1/6*sin(f*x+e)^3*cos(f*x+e)^3-1/8*sin(f*x+e)
*cos(f*x+e)^3+1/16*sin(f*x+e)*cos(f*x+e)+1/16*f*x+1/16*e)+4*d*e*(-1/6*sin(f
*x+e)^3*cos(f*x+e)^3-1/8*sin(f*x+e)*cos(f*x+e)^3+1/16*sin(f*x+e)*cos(f*x+e)
+1/16*f*x+1/16*e)-3*I*d*(1/4*(f*x+e)*sin(f*x+e)^4+1/16*(sin(f*x+e)^3+3/2*si
n(f*x+e))*cos(f*x+e)-3/32*f*x-3/32*e)-3/4*I*c*f*sin(f*x+e)^4+3/4*I*d*e*sin(
f*x+e)^4+d*((f*x+e)*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+
3/8*e)-3/16*(f*x+e)^2+1/16*sin(f*x+e)^4+3/16*sin(f*x+e)^2)+c*f*(-1/4*(sin(f
*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-d*e*(-1/4*(sin(f*x+e)^3+3
/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.62, size = 144, normalized size = 0.69

$$e^{e\,2i+f\,x\,2i}\left(\frac{\left(6\,c\,f+d\,3i\right)\,1i}{32\,a^3\,f^2}+\frac{d\,x\,3i}{16\,a^3\,f}\right)-e^{e\,4i+f\,x\,4i}\left(\frac{\left(12\,c\,f+d\,3i\right)\,1i}{128\,a^3\,f^2}+\frac{d\,x\,3i}{32\,a^3\,f}\right)+e^{e\,6i+f\,x\,6i}\left(\frac{\left(6\,c\,f+d\,1i\right)\,1i}{288\,a^3\,f^2}+\frac{d}{48}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*cot(e + f*x)*1i)^3,x)
```

```
[Out] exp(e*2i + f*x*2i)*(((d*3i + 6*c*f)*1i)/(32*a^3*f^2) + (d*x*3i)/(16*a^3*f))
 - exp(e*4i + f*x*4i)*(((d*3i + 12*c*f)*1i)/(128*a^3*f^2) + (d*x*3i)/(32*a^
3*f)) + exp(e*6i + f*x*6i)*(((d*1i + 6*c*f)*1i)/(288*a^3*f^2) + (d*x*1i)/(4
8*a^3*f)) + (d*x^2)/(16*a^3) + (c*x)/(8*a^3)
```

sympy [A] time = 0.46, size = 301, normalized size = 1.44

$$\begin{cases} -\frac{\left(-221184ia^6cf^5e^{2ie}-221184ia^6df^5xe^{2ie}+110592a^6df^4e^{2ie}\right)e^{2ifx}+\left(110592ia^6cf^5e^{4ie}+110592ia^6df^5xe^{4ie}-27648a^6df^4e^{4ie}\right)e^{4ifx}+\left(-24576ia^6cf^5e^{6ie}-\right.}{1179648a^9f^6} \\ \frac{x^2\left(-de^{6ie}+3de^{4ie}-3de^{2ie}\right)}{16a^3}+\frac{x\left(-ce^{6ie}+3ce^{4ie}-3ce^{2ie}\right)}{8a^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] Piecewise((-((-221184*I*a**6*c*f**5*exp(2*I*e) - 221184*I*a**6*d*f**5*x*exp
(2*I*e) + 110592*a**6*d*f**4*exp(2*I*e))*exp(2*I*f*x) + (110592*I*a**6*c*f*
*5*exp(4*I*e) + 110592*I*a**6*d*f**5*x*exp(4*I*e) - 27648*a**6*d*f**4*exp(4
*I*e))*exp(4*I*f*x) + (-24576*I*a**6*c*f**5*exp(6*I*e) - 24576*I*a**6*d*f**
5*x*exp(6*I*e) + 4096*a**6*d*f**4*exp(6*I*e))*exp(6*I*f*x))/(1179648*a**9*f
**6), Ne(1179648*a**9*f**6, 0)), (x**2*(-d*exp(6*I*e) + 3*d*exp(4*I*e) - 3*
d*exp(2*I*e))/(16*a**3) + x*(-c*exp(6*I*e) + 3*c*exp(4*I*e) - 3*c*exp(2*I*e
))/(8*a**3), True)) + c*x/(8*a**3) + d*x**2/(16*a**3)
```

3.30 $\int \frac{1}{(c+dx)(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=449

$$-\frac{3i\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)}{8a^3d}-\frac{i\mathrm{Ci}\left(6xf+\frac{6cf}{d}\right)\sin\left(6e-\frac{6cf}{d}\right)}{8a^3d}+\frac{3i\mathrm{Ci}\left(4xf+\frac{4cf}{d}\right)\sin\left(4e-\frac{4cf}{d}\right)}{8a^3d}-\frac{3\mathrm{Ci}\left(2xf+\frac{2}{}\right.}{}$$

```
[Out] -1/8*Ci(6*c*f/d+6*f*x)*cos(-6*e+6*c*f/d)/a^3/d+3/8*Ci(4*c*f/d+4*f*x)*cos(-4
*e+4*c*f/d)/a^3/d-3/8*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^3/d+1/8*ln(d*x+
c)/a^3/d-3/8*I*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a^3/d+3/8*I*cos(-4*e+4*c
*f/d)*Si(4*c*f/d+4*f*x)/a^3/d-1/8*I*cos(-6*e+6*c*f/d)*Si(6*c*f/d+6*f*x)/a^3
/d+1/8*I*Ci(6*c*f/d+6*f*x)*sin(-6*e+6*c*f/d)/a^3/d-1/8*Si(6*c*f/d+6*f*x)*si
n(-6*e+6*c*f/d)/a^3/d-3/8*I*Ci(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^3/d+3/8*S
i(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^3/d+3/8*I*Ci(2*c*f/d+2*f*x)*sin(-2*e+2
*c*f/d)/a^3/d-3/8*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^3/d
```

Rubi [A] time = 1.73, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 53, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3728, 3303, 3299, 3302, 3312, 4406, 4428}

$$-\frac{3i\mathrm{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{8a^3d}-\frac{i\mathrm{CosIntegral}\left(\frac{6cf}{d}+6fx\right)\sin\left(6e-\frac{6cf}{d}\right)}{8a^3d}+\frac{3i\mathrm{CosIntegral}\left(\frac{4cf}{d}+4fx\right.}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + I*a*Cot[e + f*x])^3),x]
```

```
[Out] (-3*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*Cos
[4*e - (4*c*f)/d]*CosIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) - (Cos[6*e - (6
*c*f)/d]*CosIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + Log[c + d*x]/(8*a^3*d)
 - ((I/8)*CosIntegral[(6*c*f)/d + 6*f*x]*Sin[6*e - (6*c*f)/d])/(a^3*d) + ((
(3*I)/8)*CosIntegral[(4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d])/(a^3*d) - (((
3*I)/8)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a^3*d) - (((3
*I)/8)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^3*d) + (3*Si
n[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (((3*I)/8)*C
os[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(a^3*d) - (3*Sin[4*e -
(4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) - ((I/8)*Cos[6*e - (6*
c*f)/d]*SinIntegral[(6*c*f)/d + 6*f*x])/(a^3*d) + (Sin[6*e - (6*c*f)/d]*Sin
Integral[(6*c*f)/d + 6*f*x])/(8*a^3*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3728

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4428

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d
_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x
]^p*Sin[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] &&
 IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+ia\cot(e+fx))^3}\,dx = \int\left(\frac{1}{8a^3(c+dx)} - \frac{3\cos(2e+2fx)}{8a^3(c+dx)} + \frac{3\cos^2(2e+2fx)}{8a^3(c+dx)} - \frac{\cos^3(2e+2fx)}{8a^3(c+dx)}\right.$$

$$= \frac{\log(c+dx)}{8a^3d} + \frac{i\int\frac{\sin^3(2e+2fx)}{c+dx}\,dx}{8a^3} - \frac{(3i)\int\frac{\sin(2e+2fx)}{c+dx}\,dx}{8a^3} - \frac{(3i)\int\frac{\cos^2(2e+2f}{c}}{8a}$$

$$= \frac{\log(c+dx)}{8a^3d} + \frac{i\int\left(\frac{3\sin(2e+2fx)}{4(c+dx)} - \frac{\sin(6e+6fx)}{4(c+dx)}\right)dx}{8a^3} - \frac{(3i)\int\left(\frac{\sin(2e+2fx)}{4(c+dx)} + \frac{\sin}{4}\right.}{8a^3}$$

$$= -\frac{3\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d} + \frac{3i\mathrm{Ci}\left(\frac{4cf}{d}+4fx\right)\sin\left(\right.}{8a^3d}$$

$$= -\frac{3\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d} + \frac{3i\mathrm{Ci}\left(\frac{4cf}{d}+4fx\right)\sin\left(\right.}{8a^3d}$$

$$= -\frac{3\cos\left(2e-\frac{2cf}{d}\right)\mathrm{Ci}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} - \frac{\cos\left(6e-\frac{6cf}{d}\right)\mathrm{Ci}\left(\frac{6cf}{d}+6fx\right)}{8a^3d} + \frac{\log}{8}$$

Mathematica [A] time = 0.58, size = 197, normalized size = 0.44

$$\frac{-3\left(\mathrm{Ci}\left(\frac{2f(c+dx)}{d}\right)+i\mathrm{Si}\left(\frac{2f(c+dx)}{d}\right)\right)\left(\cos\left(2e-\frac{2cf}{d}\right)+i\sin\left(2e-\frac{2cf}{d}\right)\right)+3\left(\mathrm{Ci}\left(\frac{4f(c+dx)}{d}\right)+i\mathrm{Si}\left(\frac{4f(c+dx)}{d}\right)\right)\left(\cos\left(4\right.\right.}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Cot[e + f*x])^3),x]
```

```
[Out] (Log[c + d*x] - 3*(Cos[2*e - (2*c*f)/d] + I*Sin[2*e - (2*c*f)/d])*(CosInteg
ral[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]) + 3*(Cos[4*e - (
4*c*f)/d] + I*Sin[4*e - (4*c*f)/d])*(CosIntegral[(4*f*(c + d*x))/d] + I*Sin
Integral[(4*f*(c + d*x))/d]) - (Cos[6*e - (6*c*f)/d] + I*Sin[6*e - (6*c*f)/
d])*(CosIntegral[(6*f*(c + d*x))/d] + I*SinIntegral[(6*f*(c + d*x))/d]))/(8
*a^3*d)
```

fricas [A] time = 0.83, size = 113, normalized size = 0.25

$$-\frac{\operatorname{Ei}\left(\frac{6i\,d\,f\,x+6i\,c\,f}{d}\right)e^{\left(\frac{6i\,d\,e-6i\,c\,f}{d}\right)}-3\operatorname{Ei}\left(\frac{4i\,d\,f\,x+4i\,c\,f}{d}\right)e^{\left(\frac{4i\,d\,e-4i\,c\,f}{d}\right)}+3\operatorname{Ei}\left(\frac{2i\,d\,f\,x+2i\,c\,f}{d}\right)e^{\left(\frac{2i\,d\,e-2i\,c\,f}{d}\right)}-\log\left(\frac{d\,x+c}{d}\right)}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(Ei((6*I*d*f*x + 6*I*c*f)/d)*e^((6*I*d*e - 6*I*c*f)/d) - 3*Ei((4*I*d*f
*x + 4*I*c*f)/d)*e^((4*I*d*e - 4*I*c*f)/d) + 3*Ei((2*I*d*f*x + 2*I*c*f)/d)*
e^((2*I*d*e - 2*I*c*f)/d) - log((d*x + c)/d))/(a^3*d)
```

giac [B] time = 1.54, size = 1887, normalized size = 4.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/8*(cos(6*c*f/d)*cos(e)^6*cos_integral(6*(d*f*x + c*f)/d) - I*cos(e)^6*co
s_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d) + 6*I*cos(6*c*f/d)*cos(e)^5*cos_
integral(6*(d*f*x + c*f)/d)*sin(e) + 6*cos(e)^5*cos_integral(6*(d*f*x + c*f
)/d)*sin(6*c*f/d)*sin(e) - 15*cos(6*c*f/d)*cos(e)^4*cos_integral(6*(d*f*x +
 c*f)/d)*sin(e)^2 + 15*I*cos(e)^4*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f
/d)*sin(e)^2 - 20*I*cos(6*c*f/d)*cos(e)^3*cos_integral(6*(d*f*x + c*f)/d)*s
in(e)^3 - 20*cos(e)^3*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d)*sin(e)^3
 + 15*cos(6*c*f/d)*cos(e)^2*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^4 - 15*I
*cos(e)^2*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d)*sin(e)^4 + 6*I*cos(6
*c*f/d)*cos(e)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^5 + 6*cos(e)*cos_inte
gral(6*(d*f*x + c*f)/d)*sin(6*c*f/d)*sin(e)^5 - cos(6*c*f/d)*cos_integral(6
*(d*f*x + c*f)/d)*sin(e)^6 + I*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d)
*sin(e)^6 + I*cos(6*c*f/d)*cos(e)^6*sin_integral(6*(d*f*x + c*f)/d) + cos(e
)^6*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) - 6*cos(6*c*f/d)*cos(e)^5*
sin(e)*sin_integral(6*(d*f*x + c*f)/d) + 6*I*cos(e)^5*sin(6*c*f/d)*sin(e)*s
in_integral(6*(d*f*x + c*f)/d) - 15*I*cos(6*c*f/d)*cos(e)^4*sin(e)^2*sin_in
tegral(6*(d*f*x + c*f)/d) - 15*cos(e)^4*sin(6*c*f/d)*sin(e)^2*sin_integral(
6*(d*f*x + c*f)/d) + 20*cos(6*c*f/d)*cos(e)^3*sin(e)^3*sin_integral(6*(d*f*
x + c*f)/d) - 20*I*cos(e)^3*sin(6*c*f/d)*sin(e)^3*sin_integral(6*(d*f*x + c
*f)/d) + 15*I*cos(6*c*f/d)*cos(e)^2*sin(e)^4*sin_integral(6*(d*f*x + c*f)/d
) + 15*cos(e)^2*sin(6*c*f/d)*sin(e)^4*sin_integral(6*(d*f*x + c*f)/d) - 6*c
os(6*c*f/d)*cos(e)*sin(e)^5*sin_integral(6*(d*f*x + c*f)/d) + 6*I*cos(e)*si
n(6*c*f/d)*sin(e)^5*sin_integral(6*(d*f*x + c*f)/d) - I*cos(6*c*f/d)*sin(e)
^6*sin_integral(6*(d*f*x + c*f)/d) - sin(6*c*f/d)*sin(e)^6*sin_integral(6*(
d*f*x + c*f)/d) - 3*cos(4*c*f/d)*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d) +
 3*I*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d) - 12*I*cos(4*c*f
/d)*cos(e)^3*cos_integral(4*(d*f*x + c*f)/d)*sin(e) - 12*cos(e)^3*cos_integ
ral(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e) + 18*cos(4*c*f/d)*cos(e)^2*cos_i
ntegral(4*(d*f*x + c*f)/d)*sin(e)^2 - 18*I*cos(e)^2*cos_integral(4*(d*f*x +
 c*f)/d)*sin(4*c*f/d)*sin(e)^2 + 12*I*cos(4*c*f/d)*cos(e)*cos_integral(4*(d
*f*x + c*f)/d)*sin(e)^3 + 12*cos(e)*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c
*f/d)*sin(e)^3 - 3*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4 +
3*I*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e)^4 - 3*I*cos(4*c*f/d
```

```
)*cos(e)^4*sin_integral(4*(d*f*x + c*f)/d) - 3*cos(e)^4*sin(4*c*f/d)*sin_in
tegral(4*(d*f*x + c*f)/d) + 12*cos(4*c*f/d)*cos(e)^3*sin(e)*sin_integral(4*
(d*f*x + c*f)/d) - 12*I*cos(e)^3*sin(4*c*f/d)*sin(e)*sin_integral(4*(d*f*x
+ c*f)/d) + 18*I*cos(4*c*f/d)*cos(e)^2*sin(e)^2*sin_integral(4*(d*f*x + c*f
)/d) + 18*cos(e)^2*sin(4*c*f/d)*sin(e)^2*sin_integral(4*(d*f*x + c*f)/d) -
12*cos(4*c*f/d)*cos(e)*sin(e)^3*sin_integral(4*(d*f*x + c*f)/d) + 12*I*cos(
e)*sin(4*c*f/d)*sin(e)^3*sin_integral(4*(d*f*x + c*f)/d) - 3*I*cos(4*c*f/d)
*sin(e)^4*sin_integral(4*(d*f*x + c*f)/d) - 3*sin(4*c*f/d)*sin(e)^4*sin_int
egral(4*(d*f*x + c*f)/d) + 3*cos(2*c*f/d)*cos(e)^2*cos_integral(2*(d*f*x +
c*f)/d) - 3*I*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 6*I*c
os(2*c*f/d)*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) + 6*cos(e)*cos_in
tegral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e) - 3*cos(2*c*f/d)*cos_integral
(2*(d*f*x + c*f)/d)*sin(e)^2 + 3*I*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*
f/d)*sin(e)^2 + 3*I*cos(2*c*f/d)*cos(e)^2*sin_integral(2*(d*f*x + c*f)/d) +
 3*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - 6*cos(2*c*f/d)*c
os(e)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) + 6*I*cos(e)*sin(2*c*f/d)*sin(
e)*sin_integral(2*(d*f*x + c*f)/d) - 3*I*cos(2*c*f/d)*sin(e)^2*sin_integral
(2*(d*f*x + c*f)/d) - 3*sin(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/
d) - log(d*x + c))/(a^3*d)
```

maple [A] time = 0.64, size = 560, normalized size = 1.25

$$\frac{3i\,\mathrm{Si}\left(4fx+4e+\frac{4cf-4de}{d}\right)\cos\left(\frac{4cf-4de}{d}\right)}{8a^3d}-\frac{3i\,\mathrm{Ci}\left(4fx+4e+\frac{4cf-4de}{d}\right)\sin\left(\frac{4cf-4de}{d}\right)}{8a^3d}-\frac{3i\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos}{8a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x)
```

```
[Out] 3/8*I/a^3*Si(4*f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d-3/8*I/a^3*Ci(4*f
*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d-3/8*I/a^3*Si(2*f*x+2*e+2*(c*f-d*
e)/d)*cos(2*(c*f-d*e)/d)/d+3/8*I/a^3*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f
-d*e)/d)/d-3/8/a^3*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d-3/8/a^3
*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-1/8/a^3*Si(6*f*x+6*e+6*(c
*f-d*e)/d)*sin(6*(c*f-d*e)/d)/d-1/8/a^3*Ci(6*f*x+6*e+6*(c*f-d*e)/d)*cos(6*(
c*f-d*e)/d)/d+3/8/a^3*Si(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)/d+3/8/
a^3*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d+1/8/a^3*ln((f*x+e)*d+c
*f-d*e)/d-1/8*I/a^3*Si(6*f*x+6*e+6*(c*f-d*e)/d)*cos(6*(c*f-d*e)/d)/d+1/8*I/
a^3*Ci(6*f*x+6*e+6*(c*f-d*e)/d)*sin(6*(c*f-d*e)/d)/d
```

maxima [A] time = 0.90, size = 275, normalized size = 0.61

$$f\cos\left(-\frac{6\left(de-cf\right)}{d}\right)E_1\left(-\frac{6i\left(fx+e\right)d-6i\,de+6i\,cf}{d}\right)-3\,f\cos\left(-\frac{4\left(de-cf\right)}{d}\right)E_1\left(-\frac{4i\left(fx+e\right)d-4i\,de+4i\,cf}{d}\right)+3\,f\cos\left(-\frac{2\left(de-cf\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(f*cos(-6*(d*e - c*f)/d)*exp_integral_e(1, -(6*I*(f*x + e)*d - 6*I*d*e
+ 6*I*c*f)/d) - 3*f*cos(-4*(d*e - c*f)/d)*exp_integral_e(1, -(4*I*(f*x + e)
*d - 4*I*d*e + 4*I*c*f)/d) + 3*f*cos(-2*(d*e - c*f)/d)*exp_integral_e(1, -(
2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d) - 3*I*f*exp_integral_e(1, -(2*I*(f*
x + e)*d - 2*I*d*e + 2*I*c*f)/d)*sin(-2*(d*e - c*f)/d) + 3*I*f*exp_integral
_e(1, -(4*I*(f*x + e)*d - 4*I*d*e + 4*I*c*f)/d)*sin(-4*(d*e - c*f)/d) - I*f
*exp_integral_e(1, -(6*I*(f*x + e)*d - 6*I*d*e + 6*I*c*f)/d)*sin(-6*(d*e -
c*f)/d) + f*log((f*x + e)*d - d*e + c*f))/(a^3*d*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + a\cot\left(e + f\,x\right)\,1\mathrm{i}\right)^3\,\left(c + d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{1}{c\cot^3\left(e+fx\right)-3ic\cot^2\left(e+fx\right)-3c\cot\left(e+fx\right)+ic+dx\cot^3\left(e+fx\right)-3idx\cot^2\left(e+fx\right)-3dx\cot\left(e+fx\right)+idx}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] I*Integral(1/(c*cot(e + f*x)**3 - 3*I*c*cot(e + f*x)**2 - 3*c*cot(e + f*x)
+ I*c + d*x*cot(e + f*x)**3 - 3*I*d*x*cot(e + f*x)**2 - 3*d*x*cot(e + f*x)
+ I*d*x), x)/a**3
```

3.31 $\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=712

$$\frac{3f\mathrm{Ci}\left(6xf+\frac{6cf}{d}\right)\sin\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}-\frac{3f\mathrm{Ci}\left(4xf+\frac{4cf}{d}\right)\sin\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}+\frac{3f\mathrm{Ci}\left(2xf+\frac{2cf}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3if\mathrm{Ci}\left(2$$

```
[Out] -1/8/a^3/d/(d*x+c)-1/8*I*sin(2*f*x+2*e)^3/a^3/d/(d*x+c)+3/2*I*f*Ci(4*c*f/d+
4*f*x)*cos(-4*e+4*c*f/d)/a^3/d^2+3/32*I*sin(6*f*x+6*e)/a^3/d/(d*x+c)+9/32*c
os(2*f*x+2*e)/a^3/d/(d*x+c)-3/8*cos(2*f*x+2*e)^2/a^3/d/(d*x+c)+1/8*cos(2*f*
x+2*e)^3/a^3/d/(d*x+c)+3/32*cos(6*f*x+6*e)/a^3/d/(d*x+c)+3/4*f*cos(-2*e+2*c
*f/d)*Si(2*c*f/d+2*f*x)/a^3/d^2-3/2*f*cos(-4*e+4*c*f/d)*Si(4*c*f/d+4*f*x)/a
^3/d^2+3/4*f*cos(-6*e+6*c*f/d)*Si(6*c*f/d+6*f*x)/a^3/d^2-3/4*f*Ci(6*c*f/d+6
*f*x)*sin(-6*e+6*c*f/d)/a^3/d^2-3/4*I*f*Si(6*c*f/d+6*f*x)*sin(-6*e+6*c*f/d)
/a^3/d^2+3/2*f*Ci(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^3/d^2-3/4*I*f*Si(2*c*f
/d+2*f*x)*sin(-2*e+2*c*f/d)/a^3/d^2-3/4*f*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/
d)/a^3/d^2-3/4*I*f*Ci(6*c*f/d+6*f*x)*cos(-6*e+6*c*f/d)/a^3/d^2+15/32*I*sin(
2*f*x+2*e)/a^3/d/(d*x+c)+3/8*sin(2*f*x+2*e)^2/a^3/d/(d*x+c)-3/8*I*sin(4*f*x
+4*e)/a^3/d/(d*x+c)-3/4*I*f*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^3/d^2+3/2
*I*f*Si(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^3/d^2
```

Rubi [A] time = 1.64, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3728, 3297, 3303, 3299, 3302, 3313, 12, 4406, 4428}

$$\frac{3f\mathrm{CosIntegral}\left(\frac{6cf}{d}+6fx\right)\sin\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}-\frac{3f\mathrm{CosIntegral}\left(\frac{4cf}{d}+4fx\right)\sin\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}+\frac{3f\mathrm{CosIntegral}\left(\frac{2cf}{d}-\right.}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])^3),x]
```

```
[Out] -1/(8*a^3*d*(c + d*x)) + (9*Cos[2*e + 2*f*x])/(32*a^3*d*(c + d*x)) - (3*Cos
[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) + Cos[2*e + 2*f*x]^3/(8*a^3*d*(c + d*x
)) + (3*Cos[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) - (((3*I)/4)*f*Cos[2*e - (2*
c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a^3*d^2) + (((3*I)/2)*f*Cos[4*e -
(4*c*f)/d]*CosIntegral[(4*c*f)/d + 4*f*x])/(a^3*d^2) - (((3*I)/4)*f*Cos[6*e
 - (6*c*f)/d]*CosIntegral[(6*c*f)/d + 6*f*x])/(a^3*d^2) + (3*f*CosIntegral[
(6*c*f)/d + 6*f*x]*Sin[6*e - (6*c*f)/d])/(4*a^3*d^2) - (3*f*CosIntegral[(4*
c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d])/(2*a^3*d^2) + (3*f*CosIntegral[(2*c*f
)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(4*a^3*d^2) + (((15*I)/32)*Sin[2*e + 2*f
*x])/(a^3*d*(c + d*x)) + (3*Sin[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) - ((I/8
)*Sin[2*e + 2*f*x]^3)/(a^3*d*(c + d*x)) - (((3*I)/8)*Sin[4*e + 4*f*x])/(a^3
*d*(c + d*x)) + (((3*I)/32)*Sin[6*e + 6*f*x])/(a^3*d*(c + d*x)) + (3*f*Cos[
2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) + (((3*I)/4)*f
*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^3*d^2) - (3*f*Cos[
4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) - (((3*I)/2)*f
*Sin[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(a^3*d^2) + (3*f*Cos[
6*e - (6*c*f)/d]*SinIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2) + (((3*I)/4)*f
*Sin[6*e - (6*c*f)/d]*SinIntegral[(6*c*f)/d + 6*f*x])/(a^3*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3728

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4428

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d
_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x
]^p*Sin[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] &&
 IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))^3}\,dx = \int \left(\frac{1}{8a^3(c+dx)^2} - \frac{3\cos(2e+2fx)}{8a^3(c+dx)^2} + \frac{3\cos^2(2e+2fx)}{8a^3(c+dx)^2} - \frac{\cos^3(2e+2f}{8a^3(c+dx)}\right.$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{i\int \frac{\sin^3(2e+2fx)}{(c+dx)^2}\,dx}{8a^3} - \frac{(3i)\int \frac{\sin(2e+2fx)}{(c+dx)^2}\,dx}{8a^3} - \frac{(3i)\int \frac{\cos^2(2e}{}}{}$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{3\cos(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)}$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{3\cos(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)}$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)}$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)}$$

$$= -\frac{1}{8a^3d(c+dx)} + \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)}$$

Mathematica [A] time = 0.64, size = 292, normalized size = 0.41

$$\frac{6f(c+dx)\left(\mathrm{Ci}\left(\frac{2f(c+dx)}{d}\right) + i\mathrm{Si}\left(\frac{2f(c+dx)}{d}\right)\right)\left(\sin\left(2e-\frac{2cf}{d}\right) - i\cos\left(2e-\frac{2cf}{d}\right)\right) + 12if(c+dx)\left(\mathrm{Ci}\left(\frac{4f(c+dx)}{d}\right) + i\mathrm{Si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])^3),x]
```

```
[Out] (-d + 3*d*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]) - 3*d*(Cos[4*(e + f*x)] +
 I*Sin[4*(e + f*x)]) + d*(Cos[6*(e + f*x)] + I*Sin[6*(e + f*x)]) + 6*f*(c +
 d*x)*((-I)*Cos[2*e - (2*c*f)/d] + Sin[2*e - (2*c*f)/d])*(CosIntegral[(2*f*
(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]) + (12*I)*f*(c + d*x)*(Cos
[4*e - (4*c*f)/d] + I*Sin[4*e - (4*c*f)/d])*(CosIntegral[(4*f*(c + d*x))/d]
 + I*SinIntegral[(4*f*(c + d*x))/d]) + 6*f*(c + d*x)*((-I)*Cos[6*e - (6*c*f
)/d] + Sin[6*e - (6*c*f)/d])*(CosIntegral[(6*f*(c + d*x))/d] + I*SinIntegra
l[(6*f*(c + d*x))/d]))/(8*a^3*d^2*(c + d*x))
```

fricas [A] time = 0.76, size = 180, normalized size = 0.25

$$\frac{\left(-6idfx-6icf\right)\mathrm{Ei}\left(\frac{6idfx+6icf}{d}\right)e^{\left(\frac{6ide-6icf}{d}\right)} + \left(12idfx+12icf\right)\mathrm{Ei}\left(\frac{4idfx+4icf}{d}\right)e^{\left(\frac{4ide-4icf}{d}\right)} + \left(-6idfx-6icf\right)\mathrm{Ei}}{8\left(a^3d^3x+a^3cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/8*((-6*I*d*f*x - 6*I*c*f)*Ei((6*I*d*f*x + 6*I*c*f)/d)*e^((6*I*d*e - 6*I*c
*f)/d) + (12*I*d*f*x + 12*I*c*f)*Ei((4*I*d*f*x + 4*I*c*f)/d)*e^((4*I*d*e -
4*I*c*f)/d) + (-6*I*d*f*x - 6*I*c*f)*Ei((2*I*d*f*x + 2*I*c*f)/d)*e^((2*I*d*
```

```
e - 2*I*c*f)/d) + d*e^(6*I*f*x + 6*I*e) - 3*d*e^(4*I*f*x + 4*I*e) + 3*d*e^(
2*I*f*x + 2*I*e) - d)/(a^3*d^3*x + a^3*c*d^2)
```

giac [B] time = 87.91, size = 4524, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/8*(-6*I*d*f*x*cos(6*c*f/d)*cos(e)^6*cos_integral(6*(d*f*x + c*f)/d) - 6*d
*f*x*cos(e)^6*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d) + 36*d*f*x*cos(6
*c*f/d)*cos(e)^5*cos_integral(6*(d*f*x + c*f)/d)*sin(e) - 36*I*d*f*x*cos(e)
^5*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d)*sin(e) + 90*I*d*f*x*cos(6*c
*f/d)*cos(e)^4*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^2 + 90*d*f*x*cos(e)^4
*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d)*sin(e)^2 - 120*d*f*x*cos(6*c*
f/d)*cos(e)^3*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^3 + 120*I*d*f*x*cos(e)
^3*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d)*sin(e)^3 - 90*I*d*f*x*cos(6
*c*f/d)*cos(e)^2*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^4 - 90*d*f*x*cos(e)
^2*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d)*sin(e)^4 + 36*d*f*x*cos(6*c
*f/d)*cos(e)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^5 - 36*I*d*f*x*cos(e)*c
os_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d)*sin(e)^5 + 6*I*d*f*x*cos(6*c*f/
d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^6 + 6*d*f*x*cos_integral(6*(d*f*x
 + c*f)/d)*sin(6*c*f/d)*sin(e)^6 + 6*d*f*x*cos(6*c*f/d)*cos(e)^6*sin_integr
al(6*(d*f*x + c*f)/d) - 6*I*d*f*x*cos(e)^6*sin(6*c*f/d)*sin_integral(6*(d*f
*x + c*f)/d) + 36*I*d*f*x*cos(6*c*f/d)*cos(e)^5*sin(e)*sin_integral(6*(d*f*
x + c*f)/d) + 36*d*f*x*cos(e)^5*sin(6*c*f/d)*sin(e)*sin_integral(6*(d*f*x +
 c*f)/d) - 90*d*f*x*cos(6*c*f/d)*cos(e)^4*sin(e)^2*sin_integral(6*(d*f*x +
c*f)/d) + 90*I*d*f*x*cos(e)^4*sin(6*c*f/d)*sin(e)^2*sin_integral(6*(d*f*x +
 c*f)/d) - 120*I*d*f*x*cos(6*c*f/d)*cos(e)^3*sin(e)^3*sin_integral(6*(d*f*x
 + c*f)/d) - 120*d*f*x*cos(e)^3*sin(6*c*f/d)*sin(e)^3*sin_integral(6*(d*f*x
 + c*f)/d) + 90*d*f*x*cos(6*c*f/d)*cos(e)^2*sin(e)^4*sin_integral(6*(d*f*x
+ c*f)/d) - 90*I*d*f*x*cos(e)^2*sin(6*c*f/d)*sin(e)^4*sin_integral(6*(d*f*x
 + c*f)/d) + 36*I*d*f*x*cos(6*c*f/d)*cos(e)*sin(e)^5*sin_integral(6*(d*f*x
+ c*f)/d) + 36*d*f*x*cos(e)*sin(6*c*f/d)*sin(e)^5*sin_integral(6*(d*f*x + c
*f)/d) - 6*d*f*x*cos(6*c*f/d)*sin(e)^6*sin_integral(6*(d*f*x + c*f)/d) + 6*
I*d*f*x*sin(6*c*f/d)*sin(e)^6*sin_integral(6*(d*f*x + c*f)/d) - 6*I*c*f*cos
(6*c*f/d)*cos(e)^6*cos_integral(6*(d*f*x + c*f)/d) - 6*c*f*cos(e)^6*cos_int
egral(6*(d*f*x + c*f)/d)*sin(6*c*f/d) + 36*c*f*cos(6*c*f/d)*cos(e)^5*cos_in
tegral(6*(d*f*x + c*f)/d)*sin(e) - 36*I*c*f*cos(e)^5*cos_integral(6*(d*f*x
+ c*f)/d)*sin(6*c*f/d)*sin(e) + 90*I*c*f*cos(6*c*f/d)*cos(e)^4*cos_integral
(6*(d*f*x + c*f)/d)*sin(e)^2 + 90*c*f*cos(e)^4*cos_integral(6*(d*f*x + c*f)
/d)*sin(6*c*f/d)*sin(e)^2 - 120*c*f*cos(6*c*f/d)*cos(e)^3*cos_integral(6*(d
*f*x + c*f)/d)*sin(e)^3 + 120*I*c*f*cos(e)^3*cos_integral(6*(d*f*x + c*f)/d
)*sin(6*c*f/d)*sin(e)^3 - 90*I*c*f*cos(6*c*f/d)*cos(e)^2*cos_integral(6*(d*
f*x + c*f)/d)*sin(e)^4 - 90*c*f*cos(e)^2*cos_integral(6*(d*f*x + c*f)/d)*si
n(6*c*f/d)*sin(e)^4 + 36*c*f*cos(6*c*f/d)*cos(e)*cos_integral(6*(d*f*x + c*
f)/d)*sin(e)^5 - 36*I*c*f*cos(e)*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/
d)*sin(e)^5 + 6*I*c*f*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^6
 + 6*c*f*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d)*sin(e)^6 + 6*c*f*cos(
6*c*f/d)*cos(e)^6*sin_integral(6*(d*f*x + c*f)/d) - 6*I*c*f*cos(e)^6*sin(6*
c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 36*I*c*f*cos(6*c*f/d)*cos(e)^5*sin
(e)*sin_integral(6*(d*f*x + c*f)/d) + 36*c*f*cos(e)^5*sin(6*c*f/d)*sin(e)*s
in_integral(6*(d*f*x + c*f)/d) - 90*c*f*cos(6*c*f/d)*cos(e)^4*sin(e)^2*sin_
integral(6*(d*f*x + c*f)/d) + 90*I*c*f*cos(e)^4*sin(6*c*f/d)*sin(e)^2*sin_i
ntegral(6*(d*f*x + c*f)/d) - 120*I*c*f*cos(6*c*f/d)*cos(e)^3*sin(e)^3*sin_i
ntegral(6*(d*f*x + c*f)/d) - 120*c*f*cos(e)^3*sin(6*c*f/d)*sin(e)^3*sin_int
egral(6*(d*f*x + c*f)/d) + 90*c*f*cos(6*c*f/d)*cos(e)^2*sin(e)^4*sin_integr
al(6*(d*f*x + c*f)/d) - 90*I*c*f*cos(e)^2*sin(6*c*f/d)*sin(e)^4*sin_integra
l(6*(d*f*x + c*f)/d) + 36*I*c*f*cos(6*c*f/d)*cos(e)*sin(e)^5*sin_integral(6
```

```
*(d*f*x + c*f)/d) + 36*c*f*cos(e)*sin(6*c*f/d)*sin(e)^5*sin_integral(6*(d*f
*x + c*f)/d) - 6*c*f*cos(6*c*f/d)*sin(e)^6*sin_integral(6*(d*f*x + c*f)/d)
+ 6*I*c*f*sin(6*c*f/d)*sin(e)^6*sin_integral(6*(d*f*x + c*f)/d) + 12*I*d*f*
x*cos(4*c*f/d)*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d) + 12*d*f*x*cos(e)^4
*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d) - 48*d*f*x*cos(4*c*f/d)*cos(e
)^3*cos_integral(4*(d*f*x + c*f)/d)*sin(e) + 48*I*d*f*x*cos(e)^3*cos_integr
al(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e) - 72*I*d*f*x*cos(4*c*f/d)*cos(e)^
2*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2 - 72*d*f*x*cos(e)^2*cos_integral
(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e)^2 + 48*d*f*x*cos(4*c*f/d)*cos(e)*co
s_integral(4*(d*f*x + c*f)/d)*sin(e)^3 - 48*I*d*f*x*cos(e)*cos_integral(4*(
d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e)^3 + 12*I*d*f*x*cos(4*c*f/d)*cos_integra
l(4*(d*f*x + c*f)/d)*sin(e)^4 + 12*d*f*x*cos_integral(4*(d*f*x + c*f)/d)*si
n(4*c*f/d)*sin(e)^4 - 12*d*f*x*cos(4*c*f/d)*cos(e)^4*sin_integral(4*(d*f*x
+ c*f)/d) + 12*I*d*f*x*cos(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d
) - 48*I*d*f*x*cos(4*c*f/d)*cos(e)^3*sin(e)*sin_integral(4*(d*f*x + c*f)/d)
 - 48*d*f*x*cos(e)^3*sin(4*c*f/d)*sin(e)*sin_integral(4*(d*f*x + c*f)/d) +
72*d*f*x*cos(4*c*f/d)*cos(e)^2*sin(e)^2*sin_integral(4*(d*f*x + c*f)/d) - 7
2*I*d*f*x*cos(e)^2*sin(4*c*f/d)*sin(e)^2*sin_integral(4*(d*f*x + c*f)/d) +
48*I*d*f*x*cos(4*c*f/d)*cos(e)*sin(e)^3*sin_integral(4*(d*f*x + c*f)/d) + 4
8*d*f*x*cos(e)*sin(4*c*f/d)*sin(e)^3*sin_integral(4*(d*f*x + c*f)/d) - 12*d
*f*x*cos(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f)/d) + 12*I*d*f*x*sin
(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f)/d) + d*cos(6*f*x)*cos(e)^6
+ 12*I*c*f*cos(4*c*f/d)*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d) + I*d*cos(
e)^6*sin(6*f*x) + 12*c*f*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f
/d) + 6*I*d*cos(6*f*x)*cos(e)^5*sin(e) - 48*c*f*cos(4*c*f/d)*cos(e)^3*cos_i
ntegral(4*(d*f*x + c*f)/d)*sin(e) - 6*d*cos(e)^5*sin(6*f*x)*sin(e) + 48*I*c
*f*cos(e)^3*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e) - 15*d*cos(
6*f*x)*cos(e)^4*sin(e)^2 - 72*I*c*f*cos(4*c*f/d)*cos(e)^2*cos_integral(4*(d
*f*x + c*f)/d)*sin(e)^2 - 15*I*d*cos(e)^4*sin(6*f*x)*sin(e)^2 - 72*c*f*cos(
e)^2*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e)^2 - 20*I*d*cos(6*f
*x)*cos(e)^3*sin(e)^3 + 48*c*f*cos(4*c*f/d)*cos(e)*cos_integral(4*(d*f*x +
c*f)/d)*sin(e)^3 + 20*d*cos(e)^3*sin(6*f*x)*sin(e)^3 - 48*I*c*f*cos(e)*cos_
integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d)*sin(e)^3 + 15*d*cos(6*f*x)*cos(e)^
2*sin(e)^4 + 12*I*c*f*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4
 + 15*I*d*cos(e)^2*sin(6*f*x)*sin(e)^4 + 12*c*f*cos_integral(4*(d*f*x + c*f
)/d)*sin(4*c*f/d)*sin(e)^4 + 6*I*d*cos(6*f*x)*cos(e)*sin(e)^5 - 6*d*cos(e)*
sin(6*f*x)*sin(e)^5 - d*cos(6*f*x)*sin(e)^6 - I*d*sin(6*f*x)*sin(e)^6 - 12*
c*f*cos(4*c*f/d)*cos(e)^4*sin_integral(4*(d*f*x + c*f)/d) + 12*I*c*f*cos(e)
^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 48*I*c*f*cos(4*c*f/d)*cos
(e)^3*sin(e)*sin_integral(4*(d*f*x + c*f)/d) - 48*c*f*cos(e)^3*sin(4*c*f/d)
*sin(e)*sin_integral(4*(d*f*x + c*f)/d) + 72*c*f*cos(4*c*f/d)*cos(e)^2*sin(
e)^2*sin_integral(4*(d*f*x + c*f)/d) - 72*I*c*f*cos(e)^2*sin(4*c*f/d)*sin(e
)^2*sin_integral(4*(d*f*x + c*f)/d) + 48*I*c*f*cos(4*c*f/d)*cos(e)*sin(e)^3
*sin_integral(4*(d*f*x + c*f)/d) + 48*c*f*cos(e)*sin(4*c*f/d)*sin(e)^3*sin_
integral(4*(d*f*x + c*f)/d) - 12*c*f*cos(4*c*f/d)*sin(e)^4*sin_integral(4*(
d*f*x + c*f)/d) + 12*I*c*f*sin(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*
f)/d) - 6*I*d*f*x*cos(2*c*f/d)*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d) - 6
*d*f*x*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 12*d*f*x*cos
(2*c*f/d)*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) - 12*I*d*f*x*cos(e)
*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e) + 6*I*d*f*x*cos(2*c*f/
d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 + 6*d*f*x*cos_integral(2*(d*f*x
 + c*f)/d)*sin(2*c*f/d)*sin(e)^2 + 6*d*f*x*cos(2*c*f/d)*cos(e)^2*sin_integr
al(2*(d*f*x + c*f)/d) - 6*I*d*f*x*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f
*x + c*f)/d) + 12*I*d*f*x*cos(2*c*f/d)*cos(e)*sin(e)*sin_integral(2*(d*f*x
+ c*f)/d) + 12*d*f*x*cos(e)*sin(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f
)/d) - 6*d*f*x*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) + 6*I*
d*f*x*sin(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) - 3*d*cos(4*f*x
)*cos(e)^4 - 6*I*c*f*cos(2*c*f/d)*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)
- 3*I*d*cos(e)^4*sin(4*f*x) - 6*c*f*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d
```

```
)*sin(2*c*f/d) - 12*I*d*cos(4*f*x)*cos(e)^3*sin(e) + 12*c*f*cos(2*c*f/d)*co
s(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) + 12*d*cos(e)^3*sin(4*f*x)*sin(
e) - 12*I*c*f*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e) +
18*d*cos(4*f*x)*cos(e)^2*sin(e)^2 + 6*I*c*f*cos(2*c*f/d)*cos_integral(2*(d*
f*x + c*f)/d)*sin(e)^2 + 18*I*d*cos(e)^2*sin(4*f*x)*sin(e)^2 + 6*c*f*cos_in
tegral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)*sin(e)^2 + 12*I*d*cos(4*f*x)*cos(e)*
sin(e)^3 - 12*d*cos(e)*sin(4*f*x)*sin(e)^3 - 3*d*cos(4*f*x)*sin(e)^4 - 3*I*
d*sin(4*f*x)*sin(e)^4 + 6*c*f*cos(2*c*f/d)*cos(e)^2*sin_integral(2*(d*f*x +
 c*f)/d) - 6*I*c*f*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) +
12*I*c*f*cos(2*c*f/d)*cos(e)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) + 12*c*
f*cos(e)*sin(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) - 6*c*f*cos(2*
c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) + 6*I*c*f*sin(2*c*f/d)*sin(
e)^2*sin_integral(2*(d*f*x + c*f)/d) + 3*d*cos(2*f*x)*cos(e)^2 + 3*I*d*cos(
e)^2*sin(2*f*x) + 6*I*d*cos(2*f*x)*cos(e)*sin(e) - 6*d*cos(e)*sin(2*f*x)*si
n(e) - 3*d*cos(2*f*x)*sin(e)^2 - 3*I*d*sin(2*f*x)*sin(e)^2 - d)/(a^3*d^3*x
+ a^3*c*d^2)
```

maple [A] time = 0.64, size = 756, normalized size = 1.06

$$-\frac{f\left(\frac{3i\left(-\frac{2\sin(2fx+2e)}{((fx+e)d+cf-de)d}+\frac{\frac{4\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}}{d}\right)}{16}+\frac{i\left(-\frac{6\sin(6fx+6e)}{((fx+e)d+cf-de)d}+\frac{\frac{36\,\mathrm{Si}\left(6fx+6e+\frac{6cf-6de}{d}\right)\sin\left(\frac{6}{}\right.}{d}}{48}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x)
```

```
[Out] -f/a^3*(3/16*I*(-2*sin(2*f*x+2*e)/((f*x+e)*d+c*f-d*e)/d+2*(2*Si(2*f*x+2*e+2
*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f
-d*e)/d)/d)/d)+1/48*I*(-6*sin(6*f*x+6*e)/((f*x+e)*d+c*f-d*e)/d+6*(6*Si(6*f*
x+6*e+6*(c*f-d*e)/d)*sin(6*(c*f-d*e)/d)/d+6*Ci(6*f*x+6*e+6*(c*f-d*e)/d)*cos
(6*(c*f-d*e)/d)/d)/d)+3/8*cos(4*f*x+4*e)/((f*x+e)*d+c*f-d*e)/d+3/8*(4*Si(4*
f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d-4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*s
in(4*(c*f-d*e)/d)/d)/d+1/8/((f*x+e)*d+c*f-d*e)/d-3/8*cos(2*f*x+2*e)/((f*x+e
)*d+c*f-d*e)/d-3/8*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci
(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d)/d-1/8*cos(6*f*x+6*e)/((f*x+
e)*d+c*f-d*e)/d-1/8*(6*Si(6*f*x+6*e+6*(c*f-d*e)/d)*cos(6*(c*f-d*e)/d)/d-6*C
i(6*f*x+6*e+6*(c*f-d*e)/d)*sin(6*(c*f-d*e)/d)/d)/d-3/32*I*(-4*sin(4*f*x+4*e
)/((f*x+e)*d+c*f-d*e)/d+4*(4*Si(4*f*x+4*e+4*(c*f-d*e)/d)*sin(4*(c*f-d*e)/d)
/d+4*Ci(4*f*x+4*e+4*(c*f-d*e)/d)*cos(4*(c*f-d*e)/d)/d))
```

maxima [A] time = 1.65, size = 300, normalized size = 0.42

$$\frac{8192\,f^2\cos\left(-\frac{6\left(de-cf\right)}{d}\right)E_2\left(-\frac{6i\left(fx+e\right)d-6i\,de+6i\,cf}{d}\right)-24576\,f^2\cos\left(-\frac{4\left(de-cf\right)}{d}\right)E_2\left(-\frac{4i\left(fx+e\right)d-4i\,de+4i\,cf}{d}\right)+24576\,f^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/65536*(8192*f^2*cos(-6*(d*e - c*f)/d)*exp_integral_e(2, -(6*I*(f*x + e)*d
 - 6*I*d*e + 6*I*c*f)/d) - 24576*f^2*cos(-4*(d*e - c*f)/d)*exp_integral_e(2
, -(4*I*(f*x + e)*d - 4*I*d*e + 4*I*c*f)/d) + 24576*f^2*cos(-2*(d*e - c*f)/
d)*exp_integral_e(2, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d) - 24576*I*f^
2*exp_integral_e(2, -(2*I*(f*x + e)*d - 2*I*d*e + 2*I*c*f)/d)*sin(-2*(d*e -
```

```
 c*f)/d) + 24576*I*f^2*exp_integral_e(2, -(4*I*(f*x + e)*d - 4*I*d*e + 4*I*
c*f)/d)*sin(-4*(d*e - c*f)/d) - 8192*I*f^2*exp_integral_e(2, -(6*I*(f*x + e
)*d - 6*I*d*e + 6*I*c*f)/d)*sin(-6*(d*e - c*f)/d) - 8192*f^2)/(((f*x + e)*a
^3*d^2 - a^3*d^2*e + a^3*c*d*f)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + a\cot\left(e + f\,x\right)\,1\mathrm{i}\right)^3\,(c + d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\int \frac{1}{c^2\cot^3\left(e+fx\right)-3ic^2\cot^2\left(e+fx\right)-3c^2\cot\left(e+fx\right)+ic^2+2cdx\cot^3\left(e+fx\right)-6icdx\cot^2\left(e+fx\right)-6cdx\cot\left(e+fx\right)+2icdx+d^2x^2\cot^3\left(e+fx\right)-3id^2x^2\,\mathrm{c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] I*Integral(1/(c**2*cot(e + f*x)**3 - 3*I*c**2*cot(e + f*x)**2 - 3*c**2*cot(
e + f*x) + I*c**2 + 2*c*d*x*cot(e + f*x)**3 - 6*I*c*d*x*cot(e + f*x)**2 - 6
*c*d*x*cot(e + f*x) + 2*I*c*d*x + d**2*x**2*cot(e + f*x)**3 - 3*I*d**2*x**2
*cot(e + f*x)**2 - 3*d**2*x**2*cot(e + f*x) + I*d**2*x**2), x)/a**3
```

3.32 $\int (c+dx)^m (a+ia\cot(e+fx))^2\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((c+dx)^m (a+ia\cot(e+fx))^2, x\right)$$

```
[Out] Unintegrable((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m (a+ia\cot(e+fx))^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + I*a*Cot[e + f*x])^2, x]
```

Rubi steps

$$\int (c+dx)^m (a+ia\cot(e+fx))^2\,dx = \int (c+dx)^m (a+ia\cot(e+fx))^2\,dx$$

Mathematica [A] time = 10.38, size = 0, normalized size = 0.00

$$\int (c+dx)^m (a+ia\cot(e+fx))^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x])^2, x]
```

fricas [A] time = 0.71, size = 0, normalized size = 0.00

$$\frac{2i\,(dx+c)^m a^2 + \left(fe^{(2ifx+2ie)} - f\right)\text{integral}\left(\frac{\left(4a^2dfx+4a^2cf+2ia^2dm\right)(dx+c)^m}{dfx+cf-\left(dfx+cf\right)e^{(2ifx+2ie)}}, x\right)}{fe^{(2ifx+2ie)} - f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (2*I*(d*x + c)^m*a^2 + (f*e^(2*I*f*x + 2*I*e) - f)*integral((4*a^2*d*f*x +
4*a^2*c*f + 2*I*a^2*d*m)*(d*x + c)^m/(d*f*x + c*f - (d*f*x + c*f)*e^(2*I*f*
x + 2*I*e)), x))/(f*e^(2*I*f*x + 2*I*e) - f)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(i\,a\cot\left(fx+e\right)+a\right)^2 (dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*cot(f*x + e) + a)^2*(d*x + c)^m, x)
```

maple [A] time = 0.71, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(a+ia\cot\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx+c)^{m+1}a^2}{d(m+1)}-\int -\frac{(dx+c)^m a^2\cos\left(4fx+4e\right)^2+4\,(dx+c)^m a^2\cos\left(2fx+2e\right)^2+(dx+c)^m a^2\sin\left(4fx+4\right.}{2\left(2\cos\left(2fx+2e\right)-1\right)\cos\left(4fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(m + 1)*a^2/(d*(m + 1)) - integrate(-((d*x + c)^m*a^2*cos(4*f*x +
 4*e)^2 + 4*(d*x + c)^m*a^2*cos(2*f*x + 2*e)^2 + (d*x + c)^m*a^2*sin(4*f*x
+ 4*e)^2 - 4*(d*x + c)^m*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d*x + c
)^m*a^2*sin(2*f*x + 2*e)^2 + 4*(d*x + c)^m*a^2*cos(2*f*x + 2*e) - 3*(d*x +
c)^m*a^2 - 2*(2*(d*x + c)^m*a^2*cos(2*f*x + 2*e) + (d*x + c)^m*a^2)*cos(4*f
*x + 4*e))/(2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^
2 - 4*cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1), x) - I*integrate
(-4*((d*x + c)^m*a^2*sin(4*f*x + 4*e) - 2*(d*x + c)^m*a^2*sin(2*f*x + 2*e))
/(2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*cos(
2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) -
 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+a\cot\left(e+f\,x\right)\,1\mathrm{i}\right)^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(e + f*x)*1i)^2*(c + d*x)^m,x)
```

```
[Out] int((a + a*cot(e + f*x)*1i)^2*(c + d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int (c+dx)^m\cot^2\left(e+fx\right)dx+\int\left(-2i\,(c+dx)^m\cot\left(e+fx\right)\right)dx+\int\left(-(c+dx)^m\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral((c + d*x)**m*cot(e + f*x)**2, x) + Integral(-2*I*(c + d*x)*
*m*cot(e + f*x), x) + Integral(-(c + d*x)**m, x))
```

3.33 $\int (c+dx)^m (a+ia\cot(e+fx))\,dx$

Optimal. Leaf size=24

$$\text{Int}\left((c+dx)^m (a+ia\cot(e+fx)), x\right)$$

```
[Out] Unintegrable((d*x+c)^m*(a+I*a*cot(f*x+e)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m (a+ia\cot(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + I*a*Cot[e + f*x]),x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + I*a*Cot[e + f*x]), x]
```

Rubi steps

$$\int (c+dx)^m (a+ia\cot(e+fx))\,dx = \int (c+dx)^m (a+ia\cot(e+fx))\,dx$$

Mathematica [A] time = 5.74, size = 0, normalized size = 0.00

$$\int (c+dx)^m (a+ia\cot(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x]),x]
```

```
[Out] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x]), x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{2\,(dx+c)^m a}{e^{(2ifx+2ie)}-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(-2*(d*x + c)^m*a/(e^(2*I*f*x + 2*I*e) - 1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(i\,a\cot\left(fx+e\right)+a\right)(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((I*a*cot(f*x + e) + a)*(d*x + c)^m, x)
```

maple [A] time = 1.01, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(a+ia\cot\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*cot(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+I*a*cot(f*x+e)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$2i\,a\int \frac{(dx+c)^m \sin\left(2fx+2e\right)}{\cos\left(2fx+2e\right)^2+\sin\left(2fx+2e\right)^2-2\cos\left(2fx+2e\right)+1}\,dx+\frac{(dx+c)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*I*a*integrate((d*x + c)^m*sin(2*f*x + 2*e)/(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 - 2*cos(2*f*x + 2*e) + 1), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+a\cot\left(e+f\,x\right)\,1\mathrm{i}\right)\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cot(e + f*x)*1i)*(c + d*x)^m,x)
```

```
[Out] int((a + a*cot(e + f*x)*1i)*(c + d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int \left(-i\,(c+dx)^m\right)\,dx+\int (c+dx)^m\cot\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*cot(f*x+e)),x)
```

```
[Out] I*a*(Integral(-I*(c + d*x)**m, x) + Integral((c + d*x)**m*cot(e + f*x), x))
```

3.34 $\int \frac{(c+dx)^m}{a+ia\cot(e+fx)}\,dx$

Optimal. Leaf size=98

$$\frac{(c+dx)^{m+1}}{2ad(m+1)}+\frac{i2^{-m-2}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2if(c+dx)}{d}\right)}{af}$$

```
[Out] 1/2*(d*x+c)^(1+m)/a/d/(1+m)+I*2^(-2-m)*exp(2*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1
+m,-2*I*f*(d*x+c)/d)/a/f/((-I*f*(d*x+c)/d)^m)
```

Rubi [A] time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3727, 2181}

$$\frac{(c+dx)^{m+1}}{2ad(m+1)}+\frac{i2^{-m-2}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if(c+dx)}{d}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] (c + d*x)^(1 + m)/(2*a*d*(1 + m)) + (I*2^(-2 - m)*E^((2*I)*(e - (c*f)/d))*(
c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(a*f*(((-I)*f*(c + d*x))/d
)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3727

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + Dist[1/(2*a), Int[(c +
 d*x)^m*E^((2*a*(e + f*x))/b), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[a^2 + b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^m}{a+ia\cot(e+fx)}\,dx &= \frac{(c+dx)^{1+m}}{2ad(1+m)}+\frac{\int e^{2i\left(e+\frac{\pi}{2}+fx\right)}(c+dx)^m\,dx}{2a}\\ &= \frac{(c+dx)^{1+m}}{2ad(1+m)}+\frac{i2^{-2-m}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{af}\end{aligned}$$

Mathematica [A] time = 1.29, size = 190, normalized size = 1.94

$$\frac{2^{-m-2}(c+dx)^m\left(\frac{if(c+dx)}{d}\right)^m\left(\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(\cos\left(e-\frac{cf}{d}\right)+i\sin\left(e-\frac{cf}{d}\right)\right)\left(f2^{m+1}(c+dx)\left(-\frac{if(c+dx)}{d}\right)^m\left(\cos\left(e-\frac{cf}{d}\right)-\right.\right.}{adf(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] (2^(-2 - m)*(c + d*x)^m*((I*f*(c + d*x))/d)^m*(2^(1 + m)*f*(c + d*x)*(((-I)
*f*(c + d*x))/d)^m*(Cos[e - (c*f)/d] - I*Sin[e - (c*f)/d]) + I*d*(1 + m)*Ga
mma[1 + m, ((-2*I)*f*(c + d*x))/d]*(Cos[e - (c*f)/d] + I*Sin[e - (c*f)/d]))
*(Cos[e - (c*f)/d] + I*Sin[e - (c*f)/d]))/(a*d*f*(1 + m)*((f^2*(c + d*x)^2)
/d^2)^m)
```

fricas [A] time = 0.54, size = 84, normalized size = 0.86

$$\frac{(i\,dm + i\,d)e^{\left(-\frac{dm\log\left(-\frac{2if}{d}\right)-2ide+2icf}{d}\right)}\Gamma\left(m+1,\frac{-2idfx-2icf}{d}\right)+2\left(dfx+cf\right)(dx+c)^m}{4\left(adfm+adf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*((I*d*m + I*d)*e^(-(d*m*log(-2*I*f/d) - 2*I*d*e + 2*I*c*f)/d)*gamma(m +
 1, (-2*I*d*f*x - 2*I*c*f)/d) + 2*(d*f*x + c*f)*(d*x + c)^m)/(a*d*f*m + a*d
*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{i\,a\cot\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*cot(f*x + e) + a), x)
```

maple [F] time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a+ia\cot\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*cot(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+I*a*cot(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(dm+d)\int(dx+c)^m\cos\left(2fx+2e\right)\,dx+(i\,dm+i\,d)\int(dx+c)^m\sin\left(2fx+2e\right)\,dx-e^{\left(m\log(dx+c)+\log(dx+c)\right.}}{2\,(adm+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*((d*m + d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) + (I*d*m + I*d)*
integrate((d*x + c)^m*sin(2*f*x + 2*e), x) - e^(m*log(d*x + c) + log(d*x +
c)))/(a*d*m + a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^m}{a+a\cot\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*cot(e + f*x)*1i),x)
```

```
[Out] int((c + d*x)^m/(a + a*cot(e + f*x)*1i), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{(c+dx)^m}{\cot\left(e+fx\right)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*cot(f*x+e)),x)
```

```
[Out] -I*Integral((c + d*x)**m/(cot(e + f*x) - I), x)/a
```

3.35 $\int \frac{(c+dx)^m}{(a+ia\cot(e+fx))^2}\,dx$

Optimal. Leaf size=171

$$\frac{i2^{-m-2}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2if(c+dx)}{d}\right)}{a^2f}-\frac{i4^{-m-2}e^{4i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\right.}{a^2f}$$

```
[Out] 1/4*(d*x+c)^(1+m)/a^2/d/(1+m)+I*2^(-2-m)*exp(2*I*(e-c*f/d))*(d*x+c)^m*GAMMA
(1+m,-2*I*f*(d*x+c)/d)/a^2/f/((-I*f*(d*x+c)/d)^m)-I*4^(-2-m)*exp(4*I*(e-c*f
/d))*(d*x+c)^m*GAMMA(1+m,-4*I*f*(d*x+c)/d)/a^2/f/((-I*f*(d*x+c)/d)^m)
```

Rubi [A] time = 0.18, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3729, 2181}

$$\frac{i2^{-m-2}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if(c+dx)}{d}\right)}{a^2f}-\frac{i4^{-m-2}e^{4i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gar}}{a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(4*a^2*d*(1 + m)) + (I*2^(-2 - m)*E^((2*I)*(e - (c*f)/d))
*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(a^2*f*(((-I)*f*(c + d*x
))/d)^m) - (I*4^(-2 - m)*E^((4*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, (
(-4*I)*f*(c + d*x))/d])/(a^2*f*(((-I)*f*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^m}{(a+ia\cot(e+fx))^2}\,dx &= \int\left(\frac{(c+dx)^m}{4a^2}-\frac{e^{2ie+2ifx}(c+dx)^m}{2a^2}+\frac{e^{4ie+4ifx}(c+dx)^m}{4a^2}\right)dx\\ &= \frac{(c+dx)^{1+m}}{4a^2d(1+m)}+\frac{\int e^{4ie+4ifx}(c+dx)^m\,dx}{4a^2}-\frac{\int e^{2ie+2ifx}(c+dx)^m\,dx}{2a^2}\\ &= \frac{(c+dx)^{1+m}}{4a^2d(1+m)}+\frac{i2^{-2-m}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{a^2f}-\frac{i4^{-}}{}\end{aligned}$$

Mathematica [A] time = 2.83, size = 152, normalized size = 0.89

$$\frac{(c+dx)^m\left(i2^{2-m}e^{2i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2if(c+dx)}{d}\right)-i4^{-m}e^{4i\left(e-\frac{cf}{d}\right)}\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{4if(c+dx)}{d}\right)+\frac{4f}{d}\right.}{16a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Cot[e + f*x])^2,x]
```

```
[Out] ((c + d*x)^m*((4*f*(c + d*x))/(d*(1 + m)) + (I*2^(2 - m)*E^((2*I)*(e - (c*f
)/d))*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(((-I)*f*(c + d*x))/d)^m - (I*E
^((4*I)*(e - (c*f)/d))*Gamma[1 + m, ((-4*I)*f*(c + d*x))/d])/(4^m*(((-I)*f*
(c + d*x))/d)^m)))/(16*a^2*f)
```

fricas [A] time = 0.68, size = 140, normalized size = 0.82

$$\frac{(4i\,dm+4i\,d)e^{\left(-\frac{dm\log\left(-\frac{2if}{d}\right)-2ide+2icf}{d}\right)}\Gamma\left(m+1,\frac{-2idfx-2icf}{d}\right)+(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(-\frac{4if}{d}\right)-4ide+4icf}{d}\right)}\Gamma\left(m+1,\frac{-4idfx-4icf}{d}\right.}{16\left(a^2dfm+a^2df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*((4*I*d*m + 4*I*d)*e^(-(d*m*log(-2*I*f/d) - 2*I*d*e + 2*I*c*f)/d)*gamm
a(m + 1, (-2*I*d*f*x - 2*I*c*f)/d) + (-I*d*m - I*d)*e^(-(d*m*log(-4*I*f/d)
- 4*I*d*e + 4*I*c*f)/d)*gamma(m + 1, (-4*I*d*f*x - 4*I*c*f)/d) + 4*(d*f*x +
 c*f)*(d*x + c)^m)/(a^2*d*f*m + a^2*d*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(i\,a\cot\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*cot(f*x + e) + a)^2, x)
```

maple [F] time = 2.10, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(a+ia\cot\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dm+d)\int(dx+c)^m\cos\left(4fx+4e\right)\,dx-2\,(dm+d)\int(dx+c)^m\cos\left(2fx+2e\right)\,dx+(i\,dm+i\,d)\int(dx+c)^m\,s}{4\left(a^2dm+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((d*m + d)*integrate((d*x + c)^m*cos(4*f*x + 4*e), x) - 2*(d*m + d)*int
egrate((d*x + c)^m*cos(2*f*x + 2*e), x) + (I*d*m + I*d)*integrate((d*x + c)
^m*sin(4*f*x + 4*e), x) + (-2*I*d*m - 2*I*d)*integrate((d*x + c)^m*sin(2*f*
x + 2*e), x) + e^(m*log(d*x + c) + log(d*x + c)))/(a^2*d*m + a^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^m}{\left(a+a\cot\left(e+f\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*cot(e + f*x)*1i)^2,x)
```

```
[Out] int((c + d*x)^m/(a + a*cot(e + f*x)*1i)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(c+dx)^m}{\cot^2\left(e+fx\right)-2i\cot\left(e+fx\right)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] -Integral((c + d*x)**m/(cot(e + f*x)**2 - 2*I*cot(e + f*x) - 1), x)/a**2
```

3.36 $\int \frac{(c+dx)^m}{(a+ia\cot(e+fx))^3}\,dx$

Optimal. Leaf size=251

$$\frac{3i2^{-m-4}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2if(c+dx)}{d}\right)}{a^3f}-\frac{3i2^{-2m-5}e^{4i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\right.}{a^3f}$$

```
[Out] 1/8*(d*x+c)^(1+m)/a^3/d/(1+m)+3*I*2^(-4-m)*exp(2*I*(e-c*f/d))*(d*x+c)^m*GAM
MA(1+m,-2*I*f*(d*x+c)/d)/a^3/f/((-I*f*(d*x+c)/d)^m)-3*I*2^(-5-2*m)*exp(4*I*
(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-4*I*f*(d*x+c)/d)/a^3/f/((-I*f*(d*x+c)/d)^m)
+I*2^(-4-m)*3^(-1-m)*exp(6*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-6*I*f*(d*x+c)/
d)/a^3/f/((-I*f*(d*x+c)/d)^m)
```

Rubi [A] time = 0.24, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3729, 2181}

$$\frac{3i2^{-m-4}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2if(c+dx)}{d}\right)}{a^3f}-\frac{3i2^{-2m-5}e^{4i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\text{Ga}}{a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (c + d*x)^(1 + m)/(8*a^3*d*(1 + m)) + ((3*I)*2^(-4 - m)*E^((2*I)*(e - (c*f)
/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(a^3*f*(((-I)*f*(c +
 d*x))/d)^m) - ((3*I)*2^(-5 - 2*m)*E^((4*I)*(e - (c*f)/d))*(c + d*x)^m*Gamm
a[1 + m, ((-4*I)*f*(c + d*x))/d])/(a^3*f*(((-I)*f*(c + d*x))/d)^m) + (I*2^(
-4 - m)*3^(-1 - m)*E^((6*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-6*I)
*f*(c + d*x))/d])/(a^3*f*(((-I)*f*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int\frac{(c+dx)^m}{(a+ia\cot(e+fx))^3}\,dx &= \int\left(\frac{(c+dx)^m}{8a^3}-\frac{3e^{2ie+2ifx}(c+dx)^m}{8a^3}+\frac{3e^{4ie+4ifx}(c+dx)^m}{8a^3}-\frac{e^{6ie+6ifx}(c+dx)^m}{8a^3}\right)dx\\ &= \frac{(c+dx)^{1+m}}{8a^3d(1+m)}-\frac{\int e^{6ie+6ifx}(c+dx)^m\,dx}{8a^3}-\frac{3\int e^{2ie+2ifx}(c+dx)^m\,dx}{8a^3}+\frac{3\int e^{4ie+4ifx}(c+\\ &= \frac{(c+dx)^{1+m}}{8a^3d(1+m)}+\frac{3i2^{-4-m}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{a^3f}-\frac{3i2^{-}}{}\end{aligned}$$

Mathematica [A] time = 4.10, size = 238, normalized size = 0.95

$$\frac{2^{-2m-5}3^{-m-1}e^{-\frac{6icf}{d}}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\left(id2^{m+1}3^{m+2}(m+1)e^{\frac{4icf}{d}+2ie}\Gamma\left(m+1,-\frac{2if(c+dx)}{d}\right)-id3^{m+2}(m+1)e^{2i(}\right.}{a^3df(m+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Cot[e + f*x])^3,x]
```

```
[Out] (2^(-5 - 2*m)*3^(-1 - m)*(c + d*x)^m*(12^(1 + m)*E^(((6*I)*c*f)/d)*f*(c + d
*x)*(((-I)*f*(c + d*x))/d)^m + I*2^(1 + m)*3^(2 + m)*d*E^((2*I)*e + ((4*I)*
c*f)/d)*(1 + m)*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d] - I*3^(2 + m)*d*E^((2*
I)*(2*e + (c*f)/d))*(1 + m)*Gamma[1 + m, ((-4*I)*f*(c + d*x))/d] + I*2^(1 +
 m)*d*E^((6*I)*e)*(1 + m)*Gamma[1 + m, ((-6*I)*f*(c + d*x))/d]))/(a^3*d*E^(
((6*I)*c*f)/d)*f*(1 + m)*(((-I)*f*(c + d*x))/d)^m)
```

fricas [A] time = 0.55, size = 192, normalized size = 0.76

$$\frac{(18i\,dm+18i\,d)e^{\left(-\frac{dm\log\left(-\frac{2if}{d}\right)-2ide+2icf}{d}\right)}\Gamma\left(m+1,\frac{-2idfx-2icf}{d}\right)+(-9i\,dm-9i\,d)e^{\left(-\frac{dm\log\left(-\frac{4if}{d}\right)-4ide+4icf}{d}\right)}\Gamma\left(m+1,\frac{-4i}{}\right.}{96\left(a^3dfm+a^3dj\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/96*((18*I*d*m + 18*I*d)*e^(-(d*m*log(-2*I*f/d) - 2*I*d*e + 2*I*c*f)/d)*ga
mma(m + 1, (-2*I*d*f*x - 2*I*c*f)/d) + (-9*I*d*m - 9*I*d)*e^(-(d*m*log(-4*I
*f/d) - 4*I*d*e + 4*I*c*f)/d)*gamma(m + 1, (-4*I*d*f*x - 4*I*c*f)/d) + (2*I
*d*m + 2*I*d)*e^(-(d*m*log(-6*I*f/d) - 6*I*d*e + 6*I*c*f)/d)*gamma(m + 1, (
-6*I*d*f*x - 6*I*c*f)/d) + 12*(d*f*x + c*f)*(d*x + c)^m)/(a^3*d*f*m + a^3*d
*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(i\,a\cot\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*cot(f*x + e) + a)^3, x)
```

maple [F] time = 2.34, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(a+ia\cot\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(dm+d)\int(dx+c)^m\cos\left(6\,fx+6\,e\right)\,dx-3\,(dm+d)\int(dx+c)^m\cos\left(4\,fx+4\,e\right)\,dx+3\,(dm+d)\int(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((d*m + d)*integrate((d*x + c)^m*cos(6*f*x + 6*e), x) - 3*(d*m + d)*in
tegrate((d*x + c)^m*cos(4*f*x + 4*e), x) + 3*(d*m + d)*integrate((d*x + c)^
m*cos(2*f*x + 2*e), x) + (I*d*m + I*d)*integrate((d*x + c)^m*sin(6*f*x + 6*
e), x) + (-3*I*d*m - 3*I*d)*integrate((d*x + c)^m*sin(4*f*x + 4*e), x) + (3
*I*d*m + 3*I*d)*integrate((d*x + c)^m*sin(2*f*x + 2*e), x) - e^(m*log(d*x +
 c) + log(d*x + c)))/(a^3*d*m + a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^m}{\left(a+a\cot\left(e+f\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*cot(e + f*x)*1i)^3,x)
```

```
[Out] int((c + d*x)^m/(a + a*cot(e + f*x)*1i)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{(c+dx)^m}{\cot^3\left(e+fx\right)-3i\cot^2\left(e+fx\right)-3\cot\left(e+fx\right)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] I*Integral((c + d*x)**m/(cot(e + f*x)**3 - 3*I*cot(e + f*x)**2 - 3*cot(e +
f*x) + I), x)/a**3
```

3.37 $\int (c+dx)^3(a+b\cot(e+fx))\,dx$

Optimal. Leaf size=147

$$\frac{a(c+dx)^4}{4d}+\frac{3bd^2(c+dx)\mathrm{Li}_3\left(e^{2i(e+fx)}\right)}{2f^3}-\frac{3ibd(c+dx)^2\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{2f^2}+\frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ib(c+dx)^4}{4d}$$

```
[Out] 1/4*a*(d*x+c)^4/d-1/4*I*b*(d*x+c)^4/d+b*(d*x+c)^3*ln(1-exp(2*I*(f*x+e)))/f-
3/2*I*b*d*(d*x+c)^2*polylog(2,exp(2*I*(f*x+e)))/f^2+3/2*b*d^2*(d*x+c)*polyl
og(3,exp(2*I*(f*x+e)))/f^3+3/4*I*b*d^3*polylog(4,exp(2*I*(f*x+e)))/f^4
```

Rubi [A] time = 0.26, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3722, 3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3bd^2(c+dx)\mathrm{PolyLog}\left(3,e^{2i(e+fx)}\right)}{2f^3}-\frac{3ibd(c+dx)^2\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{2f^2}+\frac{3ibd^3\mathrm{PolyLog}\left(4,e^{2i(e+fx)}\right)}{4f^4}+\frac{a(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Cot[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) - ((I/4)*b*(c + d*x)^4)/d + (b*(c + d*x)^3*Log[1 - E^
((2*I)*(e + f*x))])/f - (((3*I)/2)*b*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(e +
 f*x))])/f^2 + (3*b*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^3)
+ (((3*I)/4)*b*d^3*PolyLog[4, E^((2*I)*(e + f*x))])/f^4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3(a+b\cot(e+fx))\,dx &= \int \left(a(c+dx)^3 + b(c+dx)^3\cot(e+fx)\right)dx \\
&= \frac{a(c+dx)^4}{4d} + b\int (c+dx)^3\cot(e+fx)\,dx \\
&= \frac{a(c+dx)^4}{4d} - \frac{ib(c+dx)^4}{4d} - (2ib)\int \frac{e^{2i(e+fx)}(c+dx)^3}{1-e^{2i(e+fx)}}\,dx \\
&= \frac{a(c+dx)^4}{4d} - \frac{ib(c+dx)^4}{4d} + \frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f} - \frac{(3bd)\int (c+dx)^2 \lo}{} \\
&= \frac{a(c+dx)^4}{4d} - \frac{ib(c+dx)^4}{4d} + \frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f} - \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(e\right.}{2f^2} \\
&= \frac{a(c+dx)^4}{4d} - \frac{ib(c+dx)^4}{4d} + \frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f} - \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(e\right.}{2f^2} \\
&= \frac{a(c+dx)^4}{4d} - \frac{ib(c+dx)^4}{4d} + \frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f} - \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(e\right.}{2f^2} \\
&= \frac{a(c+dx)^4}{4d} - \frac{ib(c+dx)^4}{4d} + \frac{b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f} - \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(e\right.}{2f^2}
\end{aligned}$$

Mathematica [B] time = 7.50, size = 730, normalized size = 4.97

$$\frac{1}{4}x\csc(e)\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)(a\sin(e)+b\cos(e)) + \frac{bc^3\csc(e)(\sin(e)\log(\sin(e)\cos(fx)+\cos(e)\sin(fx)}{f\left(\sin^2(e)+\cos^2(e)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*(a + b*Cot[e + f*x]),x]
```

```
[Out] -1/2*(b*c*d^2*E^(I*e)*Csc[e]*((2*f^3*x^3)/E^((2*I)*e) + (3*I)*(1 - E^((-2*I
)*e))*f^2*x^2*Log[1 - E^((-I)*(e + f*x))] + (3*I)*(1 - E^((-2*I)*e))*f^2*x^
2*Log[1 + E^((-I)*(e + f*x))] - (6*(-1 + E^((2*I)*e))*(f*x*PolyLog[2, -E^((
-I)*(e + f*x))] - I*PolyLog[3, -E^((-I)*(e + f*x))]))/E^((2*I)*e) - (6*(-1
```

```
+ E^((2*I)*e))*(f*x*PolyLog[2, E^((-I)*(e + f*x))] - I*PolyLog[3, E^((-I)*(
e + f*x))]))/E^((2*I)*e)))/f^3 - (b*d^3*E^(I*e)*Csc[e]*((f^4*x^4)/E^((2*I)*
e) + (2*I)*(1 - E^((-2*I)*e))*f^3*x^3*Log[1 - E^((-I)*(e + f*x))] + (2*I)*(
1 - E^((-2*I)*e))*f^3*x^3*Log[1 + E^((-I)*(e + f*x))] - (6*(-1 + E^((2*I)*e
))*(f^2*x^2*PolyLog[2, -E^((-I)*(e + f*x))] - (2*I)*f*x*PolyLog[3, -E^((-I)
*(e + f*x))] - 2*PolyLog[4, -E^((-I)*(e + f*x))]))/E^((2*I)*e) - (6*(-1 + E
^((2*I)*e))*(f^2*x^2*PolyLog[2, E^((-I)*(e + f*x))] - (2*I)*f*x*PolyLog[3,
E^((-I)*(e + f*x))] - 2*PolyLog[4, E^((-I)*(e + f*x))]))/E^((2*I)*e)))/(4*f
^4) + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Csc[e]*(b*Cos[e] + a*S
in[e]))/4 + (b*c^3*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin
[f*x]]*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) - (3*b*c^2*d*Csc[e]*Sec[e]*(E^(I*
ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^(
(-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[
e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]]
 + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^
2]))/(2*f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)])
```

fricas [C] time = 0.74, size = 626, normalized size = 4.26

$$\frac{2\,ad^3f^4x^4 + 8\,acd^2f^4x^3 + 12\,ac^2df^4x^2 + 8\,ac^3f^4x + 3i\,bd^3\mathrm{polylog}\left(4, \cos\left(2\,fx + 2\,e\right) + i\,\sin\left(2\,fx + 2\,e\right)\right) - 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*d^3*f^4*x^4 + 8*a*c*d^2*f^4*x^3 + 12*a*c^2*d*f^4*x^2 + 8*a*c^3*f^4
*x + 3*I*b*d^3*polylog(4, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) - 3*I*b*d^
3*polylog(4, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) + (-6*I*b*d^3*f^2*x^2 -
 12*I*b*c*d^2*f^2*x - 6*I*b*c^2*d*f^2)*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x
 + 2*e)) + (6*I*b*d^3*f^2*x^2 + 12*I*b*c*d^2*f^2*x + 6*I*b*c^2*d*f^2)*dilog
(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f +
3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x +
2*e) + 1/2) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)
*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2) + 4*(b*d^3*f^3*x
^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*
b*c^2*d*e*f^2)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 4*(b*d^3*f
^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f
+ 3*b*c^2*d*e*f^2)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1) + 6*(b*d
^3*f*x + b*c*d^2*f)*polylog(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) + 6*(
b*d^3*f*x + b*c*d^2*f)*polylog(3, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)))/f
^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3\left(b\cot\left(fx + e\right) + a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*(b*cot(f*x + e) + a), x)
```

maple [B] time = 1.32, size = 857, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*cot(f*x+e)),x)
```

```
[Out] I*b*c^3*x-1/4*I*b*d^3*x^4+1/f^4*b*d^3*ln(1-exp(I*(f*x+e)))*e^3+1/f*b*d^3*ln
(exp(I*(f*x+e))+1)*x^3-3*I/f^2*b*c^2*d*polylog(2,exp(I*(f*x+e)))+1/f*b*c^3*
ln(exp(I*(f*x+e))+1)+1/f*b*c^3*ln(exp(I*(f*x+e))-1)-2/f*b*c^3*ln(exp(I*(f*x
+e)))+3/2*a*c^2*d*x^2-3*I/f^2*b*c^2*d*polylog(2,-exp(I*(f*x+e)))+4*I/f^3*b*
c*d^2*e^3-3*I/f^2*b*c^2*d*e^2-2*I/f^3*b*d^3*e^3*x-3*I/f^2*b*d^3*polylog(2,-
exp(I*(f*x+e)))*x^2-3*I/f^2*b*d^3*polylog(2,exp(I*(f*x+e)))*x^2-I*b*c*d^2*x
^3-3/2*I*b*c^2*d*x^2+a*c*d^2*x^3-6*I/f^2*b*c*d^2*polylog(2,exp(I*(f*x+e)))*
x-6*I/f^2*b*c*d^2*polylog(2,-exp(I*(f*x+e)))*x+6*I/f^2*b*c*d^2*e^2*x-6*I/f*
b*c^2*d*e*x+3/f*b*c*d^2*ln(1-exp(I*(f*x+e)))*x^2-3/f^3*b*c*d^2*ln(1-exp(I*(
f*x+e)))*e^2+3/f*b*c^2*d*ln(1-exp(I*(f*x+e)))*x+3/f*b*c*d^2*ln(exp(I*(f*x+e
))+1)*x^2+3/f^2*b*c^2*d*ln(1-exp(I*(f*x+e)))*e-3/f^2*b*c^2*d*e*ln(exp(I*(f*
x+e))-1)-6/f^3*b*c*d^2*e^2*ln(exp(I*(f*x+e)))+a*c^3*x+1/4*a*d^3*x^4+3/f*b*c
^2*d*ln(exp(I*(f*x+e))+1)*x+1/f*b*d^3*ln(1-exp(I*(f*x+e)))*x^3+6/f^2*b*c^2*
d*e*ln(exp(I*(f*x+e)))+3/f^3*b*c*d^2*e^2*ln(exp(I*(f*x+e))-1)+6/f^3*b*d^3*p
olylog(3,-exp(I*(f*x+e)))*x+6/f^3*b*d^3*polylog(3,exp(I*(f*x+e)))*x+2/f^4*b
*d^3*e^3*ln(exp(I*(f*x+e)))+6/f^3*b*c*d^2*polylog(3,exp(I*(f*x+e)))+6/f^3*b
*c*d^2*polylog(3,-exp(I*(f*x+e)))-1/f^4*b*d^3*e^3*ln(exp(I*(f*x+e))-1)-3/2*
I/f^4*b*d^3*e^4+6*I/f^4*b*d^3*polylog(4,-exp(I*(f*x+e)))+6*I/f^4*b*d^3*poly
log(4,exp(I*(f*x+e)))
```

maxima [B] time = 1.16, size = 966, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 - 4*(f*x + e)^3*a*d^3*e/f^3
+ 6*(f*x + e)^2*a*d^3*e^2/f^3 - 4*(f*x + e)*a*d^3*e^3/f^3 + 4*(f*x + e)^3*a
*c*d^2/f^2 - 12*(f*x + e)^2*a*c*d^2*e/f^2 + 12*(f*x + e)*a*c*d^2*e^2/f^2 +
6*(f*x + e)^2*a*c^2*d/f - 12*(f*x + e)*a*c^2*d*e/f + 4*b*c^3*log(sin(f*x +
e)) - 4*b*d^3*e^3*log(sin(f*x + e))/f^3 + 12*b*c*d^2*e^2*log(sin(f*x + e))/
f^2 - 12*b*c^2*d*e*log(sin(f*x + e))/f + (-I*(f*x + e)^4*b*d^3 + 24*I*b*d^3
*polylog(4, -e^(I*f*x + I*e)) + 24*I*b*d^3*polylog(4, e^(I*f*x + I*e)) + (4
*I*b*d^3*e - 4*I*b*c*d^2*f)*(f*x + e)^3 + (-6*I*b*d^3*e^2 + 12*I*b*c*d^2*e*
f - 6*I*b*c^2*d*f^2)*(f*x + e)^2 + (4*I*(f*x + e)^3*b*d^3 + (-12*I*b*d^3*e
+ 12*I*b*c*d^2*f)*(f*x + e)^2 + (12*I*b*d^3*e^2 - 24*I*b*c*d^2*e*f + 12*I*b
*c^2*d*f^2)*(f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) + (-4*I*(f*x
 + e)^3*b*d^3 + (12*I*b*d^3*e - 12*I*b*c*d^2*f)*(f*x + e)^2 + (-12*I*b*d^3*
e^2 + 24*I*b*c*d^2*e*f - 12*I*b*c^2*d*f^2)*(f*x + e))*arctan2(sin(f*x + e),
 -cos(f*x + e) + 1) + (-12*I*(f*x + e)^2*b*d^3 - 12*I*b*d^3*e^2 + 24*I*b*c*
d^2*e*f - 12*I*b*c^2*d*f^2 + (24*I*b*d^3*e - 24*I*b*c*d^2*f)*(f*x + e))*dil
og(-e^(I*f*x + I*e)) + (-12*I*(f*x + e)^2*b*d^3 - 12*I*b*d^3*e^2 + 24*I*b*c
*d^2*e*f - 12*I*b*c^2*d*f^2 + (24*I*b*d^3*e - 24*I*b*c*d^2*f)*(f*x + e))*di
log(e^(I*f*x + I*e)) + 2*((f*x + e)^3*b*d^3 - 3*(b*d^3*e - b*c*d^2*f)*(f*x
+ e)^2 + 3*(b*d^3*e^2 - 2*b*c*d^2*e*f + b*c^2*d*f^2)*(f*x + e))*log(cos(f*x
 + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*((f*x + e)^3*b*d^3 - 3*(
b*d^3*e - b*c*d^2*f)*(f*x + e)^2 + 3*(b*d^3*e^2 - 2*b*c*d^2*e*f + b*c^2*d*f
^2)*(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) +
24*((f*x + e)*b*d^3 - b*d^3*e + b*c*d^2*f)*polylog(3, -e^(I*f*x + I*e)) + 2
4*((f*x + e)*b*d^3 - b*d^3*e + b*c*d^2*f)*polylog(3, e^(I*f*x + I*e)))/f^3)
/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b\cot\left(e + f\,x\right)\right)\,(c + d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))*(c + d*x)^3,x)
```

```
[Out] int((a + b*cot(e + f*x))*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \cot\left(e + fx\right)\right) (c + dx)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*cot(f*x+e)),x)
[Out] Integral((a + b*cot(e + f*x))*(c + d*x)**3, x)
```

3.38 $\int (c+dx)^2(a+b\cot(e+fx))\,dx$

Optimal. Leaf size=112

$$\frac{a(c+dx)^3}{3d}-\frac{ibd(c+dx)\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{f^2}+\frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ib(c+dx)^3}{3d}+\frac{bd^2\mathrm{Li}_3\left(e^{2i(e+fx)}\right)}{2f^3}$$

```
[Out] 1/3*a*(d*x+c)^3/d-1/3*I*b*(d*x+c)^3/d+b*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f-
I*b*d*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^2+1/2*b*d^2*polylog(3,exp(2*I*(
f*x+e)))/f^3
```

Rubi [A] time = 0.21, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3722, 3717, 2190, 2531, 2282, 6589}

$$-\frac{ibd(c+dx)\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{f^2}+\frac{bd^2\mathrm{PolyLog}\left(3,e^{2i(e+fx)}\right)}{2f^3}+\frac{a(c+dx)^3}{3d}+\frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ib(c+d}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Cot[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) - ((I/3)*b*(c + d*x)^3)/d + (b*(c + d*x)^2*Log[1 - E^
((2*I)*(e + f*x))])/f - (I*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])/f
^2 + (b*d^2*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2(a+b\cot(e+fx))\,dx &= \int \left(a(c+dx)^2 + b(c+dx)^2\cot(e+fx)\right)dx \\
&= \frac{a(c+dx)^3}{3d} + b\int (c+dx)^2\cot(e+fx)\,dx \\
&= \frac{a(c+dx)^3}{3d} - \frac{ib(c+dx)^3}{3d} - (2ib)\int \frac{e^{2i(e+fx)}(c+dx)^2}{1-e^{2i(e+fx)}}\,dx \\
&= \frac{a(c+dx)^3}{3d} - \frac{ib(c+dx)^3}{3d} + \frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f} - \frac{(2bd)\int (c+dx)} \\
&= \frac{a(c+dx)^3}{3d} - \frac{ib(c+dx)^3}{3d} + \frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f} - \frac{ibd(c+dx)\mathrm{Li}_2\left(} \\
&= \frac{a(c+dx)^3}{3d} - \frac{ib(c+dx)^3}{3d} + \frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f} - \frac{ibd(c+dx)\mathrm{Li}_2\left(} \\
&= \frac{a(c+dx)^3}{3d} - \frac{ib(c+dx)^3}{3d} + \frac{b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f} - \frac{ibd(c+dx)\mathrm{Li}_2\left(}
\end{aligned}$$

Mathematica [B] time = 2.57, size = 406, normalized size = 3.62

$$\frac{3ac^2f^3x + 3acdf^3x^2 + ad^2f^3x^3 + 3bc^2f^2\log(\sin(e+fx)) + 3bcdf^3x^2\cot(e) - 3bcdf^3x^2e^{i\tan^{-1}(\tan(e))}\cot(e)\sqrt{\mathrm{s}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*(a + b*Cot[e + f*x]),x]
```

```
[Out] (3*a*c^2*f^3*x + (3*I)*b*c*d*f^2*Pi*x + 3*a*c*d*f^3*x^2 + a*d^2*f^3*x^3 + I
*b*d^2*f^3*x^3 - (6*I)*b*c*d*f^2*x*ArcTan[Tan[e]] + 3*b*c*d*f^3*x^2*Cot[e]
+ 3*b*c*d*f*Pi*Log[1 + E^((-2*I)*f*x)] + 3*b*d^2*f^2*x^2*Log[1 - E^((-I)*(e
 + f*x))] + 3*b*d^2*f^2*x^2*Log[1 + E^((-I)*(e + f*x))] + 6*b*c*d*f^2*x*Log
[1 - E^((2*I)*(f*x + ArcTan[Tan[e]]))] + 6*b*c*d*f*ArcTan[Tan[e]]*Log[1 - E
^((2*I)*(f*x + ArcTan[Tan[e]]))] - 3*b*c*d*f*Pi*Log[Cos[f*x]] + 3*b*c^2*f^2
*Log[Sin[e + f*x]] - 6*b*c*d*f*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]
] + (6*I)*b*d^2*f*x*PolyLog[2, -E^((-I)*(e + f*x))] + (6*I)*b*d^2*f*x*PolyL
og[2, E^((-I)*(e + f*x))] - (3*I)*b*c*d*f*PolyLog[2, E^((2*I)*(f*x + ArcTan
[Tan[e]]))] + 6*b*d^2*PolyLog[3, -E^((-I)*(e + f*x))] + 6*b*d^2*PolyLog[3,
E^((-I)*(e + f*x))] - 3*b*c*d*E^(I*ArcTan[Tan[e]])*f^3*x^2*Cot[e]*Sqrt[Sec[
e]^2])/(3*f^3)
```

fricas [C] time = 0.61, size = 403, normalized size = 3.60

$$\frac{4\,ad^2f^3x^3 + 12\,acdf^3x^2 + 12\,ac^2f^3x + 3\,bd^2\mathrm{polylog}\left(3, \cos\left(2\,fx+2\,e\right) + i\,\sin\left(2\,fx+2\,e\right)\right) + 3\,bd^2\mathrm{polylog}\left(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*d^2*f^3*x^3 + 12*a*c*d*f^3*x^2 + 12*a*c^2*f^3*x + 3*b*d^2*polylog
(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) + 3*b*d^2*polylog(3, cos(2*f*x +
 2*e) - I*sin(2*f*x + 2*e)) + (-6*I*b*d^2*f*x - 6*I*b*c*d*f)*dilog(cos(2*f*
x + 2*e) + I*sin(2*f*x + 2*e)) + (6*I*b*d^2*f*x + 6*I*b*c*d*f)*dilog(cos(2*
f*x + 2*e) - I*sin(2*f*x + 2*e)) + 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*
log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2) + 6*(b*d^2*e^2 -
2*b*c*d*e*f + b*c^2*f^2)*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e)
 + 1/2) + 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(-
cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2
*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) +
1))/f^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \left(b \cot\left(fx + e\right) + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(b*cot(f*x + e) + a), x)
```

maple [B] time = 1.22, size = 516, normalized size = 4.61

$$a\,c^2x+acd\,x^2+\frac{2bcd\ln\left(e^{i\left(fx+e\right)}+1\right)x}{f}+\frac{2bcd\ln\left(1-e^{i\left(fx+e\right)}\right)e}{f^2}+\frac{4bcde\ln\left(e^{i\left(fx+e\right)}\right)}{f^2}-\frac{2ibcd\,\mathrm{polylog}\left(2,e^{i\left(fx+e\right)}\right)}{f^2}-\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*cot(f*x+e)),x)
```

```
[Out] a*c^2*x+a*c*d*x^2+I*b*c^2*x-1/3*I*b*d^2*x^3+2/f^3*b*d^2*polylog(3,-exp(I*(f
*x+e)))+2/f^3*b*d^2*polylog(3,exp(I*(f*x+e)))+1/f*b*c^2*ln(exp(I*(f*x+e))+1
)-2/f*b*c^2*ln(exp(I*(f*x+e)))+1/f*b*c^2*ln(exp(I*(f*x+e))-1)+4/3*I/f^3*b*d
^2*e^3+1/3*a*d^2*x^3-2/f^3*b*d^2*e^2*ln(exp(I*(f*x+e)))+1/f^3*b*d^2*e^2*ln(
exp(I*(f*x+e))-1)+1/f*b*d^2*ln(1-exp(I*(f*x+e)))*x^2-1/f^3*b*d^2*ln(1-exp(I
*(f*x+e)))*e^2+1/f*b*d^2*ln(exp(I*(f*x+e))+1)*x^2-4*I/f*b*c*d*e*x-I*b*c*d*x
^2-2*I/f^2*b*c*d*e^2-2*I/f^2*b*c*d*polylog(2,exp(I*(f*x+e)))+2*I/f^2*b*d^2*
e^2*x-2*I/f^2*b*c*d*polylog(2,-exp(I*(f*x+e)))-2*I/f^2*b*d^2*polylog(2,-exp
(I*(f*x+e)))*x-2*I/f^2*b*d^2*polylog(2,exp(I*(f*x+e)))*x-2/f^2*b*c*d*e*ln(e
xp(I*(f*x+e))-1)+2/f*b*c*d*ln(exp(I*(f*x+e))+1)*x+2/f^2*b*c*d*ln(1-exp(I*(f
*x+e)))*e+4/f^2*b*c*d*e*ln(exp(I*(f*x+e)))+2/f*b*c*d*ln(1-exp(I*(f*x+e)))*x
```

maxima [B] time = 1.75, size = 520, normalized size = 4.64

$$\frac{6\left(fx+e\right)ac^2+\frac{2\left(fx+e\right)^3ad^2}{f^2}-\frac{6\left(fx+e\right)^2ad^2e}{f^2}+\frac{6\left(fx+e\right)ad^2e^2}{f^2}+\frac{6\left(fx+e\right)^2acd}{f}-\frac{12\left(fx+e\right)acde}{f}+6\,bc^2\log\left(\sin\left(fx+e\right)\right)+\frac{6\,bc}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(f*x + e)*a*c^2 + 2*(f*x + e)^3*a*d^2/f^2 - 6*(f*x + e)^2*a*d^2*e/f^
2 + 6*(f*x + e)*a*d^2*e^2/f^2 + 6*(f*x + e)^2*a*c*d/f - 12*(f*x + e)*a*c*d*
e/f + 6*b*c^2*log(sin(f*x + e)) + 6*b*d^2*e^2*log(sin(f*x + e))/f^2 - 12*b*
c*d*e*log(sin(f*x + e))/f + (-2*I*(f*x + e)^3*b*d^2 + 12*b*d^2*polylog(3, -
e^(I*f*x + I*e)) + 12*b*d^2*polylog(3, e^(I*f*x + I*e)) + (6*I*b*d^2*e - 6*
I*b*c*d*f)*(f*x + e)^2 + (6*I*(f*x + e)^2*b*d^2 + (-12*I*b*d^2*e + 12*I*b*c
```

```
*d*f)*(f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) + (-6*I*(f*x + e)^
2*b*d^2 + (12*I*b*d^2*e - 12*I*b*c*d*f)*(f*x + e))*arctan2(sin(f*x + e), -c
os(f*x + e) + 1) + (-12*I*(f*x + e)*b*d^2 + 12*I*b*d^2*e - 12*I*b*c*d*f)*di
log(-e^(I*f*x + I*e)) + (-12*I*(f*x + e)*b*d^2 + 12*I*b*d^2*e - 12*I*b*c*d*
f)*dilog(e^(I*f*x + I*e)) + 3*((f*x + e)^2*b*d^2 - 2*(b*d^2*e - b*c*d*f)*(f
*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 3*((f*
x + e)^2*b*d^2 - 2*(b*d^2*e - b*c*d*f)*(f*x + e))*log(cos(f*x + e)^2 + sin(
f*x + e)^2 - 2*cos(f*x + e) + 1))/f^2)/f
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b\cot\left(e + f\,x\right)\right)\,(c + d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))*(c + d*x)^2,x)
```

```
[Out] int((a + b*cot(e + f*x))*(c + d*x)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot\left(e + f\,x\right)\right)\,(c + dx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*cot(f*x+e)),x)
```

```
[Out] Integral((a + b*cot(e + f*x))*(c + d*x)**2, x)
```

3.39 $\int (c+dx)(a+b\cot(e+fx))\,dx$

Optimal. Leaf size=83

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ib(c+dx)^2}{2d}-\frac{ibd\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{2f^2}$$

```
[Out] 1/2*a*(d*x+c)^2/d-1/2*I*b*(d*x+c)^2/d+b*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f-1/
2*I*b*d*polylog(2,exp(2*I*(f*x+e)))/f^2
```

Rubi [A] time = 0.12, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3722, 3717, 2190, 2279, 2391}

$$-\frac{ibd\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{2f^2}+\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ib(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Cot[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - ((I/2)*b*(c + d*x)^2)/d + (b*(c + d*x)*Log[1 - E^((
2*I)*(e + f*x))])/f - ((I/2)*b*d*PolyLog[2, E^((2*I)*(e + f*x))])/f^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)(a+b\cot(e+fx))\,dx &= \int (a(c+dx)+b(c+dx)\cot(e+fx))\,dx \\ &= \frac{a(c+dx)^2}{2d} + b\int (c+dx)\cot(e+fx)\,dx \\ &= \frac{a(c+dx)^2}{2d} - \frac{ib(c+dx)^2}{2d} - (2ib)\int \frac{e^{2i(e+fx)}(c+dx)}{1-e^{2i(e+fx)}}\,dx \\ &= \frac{a(c+dx)^2}{2d} - \frac{ib(c+dx)^2}{2d} + \frac{b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f} - \frac{(bd)\int \log\left(1-e^{2i}\right.}{f} \\ &= \frac{a(c+dx)^2}{2d} - \frac{ib(c+dx)^2}{2d} + \frac{b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f} + \frac{(ibd)\,\text{Subst}\left(\int \frac{\log}{2}\right.}{} \\ &= \frac{a(c+dx)^2}{2d} - \frac{ib(c+dx)^2}{2d} + \frac{b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f} - \frac{ibd\,\text{Li}_2\left(e^{2i(e+fx)}\right)}{2f^2}\end{aligned}$$

Mathematica [B] time = 5.04, size = 204, normalized size = 2.46

$$acx+\frac{1}{2}adx^2+\frac{bc(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f} - \frac{bd\csc(e)\sec(e)\left(f^2x^2e^{i\tan^{-1}(\tan(e))} + \frac{\tan(e)\left(i\text{Li}_2\left(e^{2i\left(fx+\tan^{-1}\right.\right.\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*(a + b*Cot[e + f*x]),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*d*x^2*Cot[e])/2 + (b*c*(Log[Cos[e + f*x]] + Log[Ta
n[e + f*x]]))/f - (b*d*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*
x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[T
an[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*Ar
cTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x +
ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))/(2*f^2*Sqrt[Sec[e]^2*(Cos[e
]^2 + Sin[e]^2)])
```

fricas [B] time = 0.52, size = 224, normalized size = 2.70

$$\frac{2\,adf^2x^2 + 4\,acf^2x - i\,bd\text{Li}_2\left(\cos\left(2\,fx+2\,e\right)+i\,\sin\left(2\,fx+2\,e\right)\right) + i\,bd\text{Li}_2\left(\cos\left(2\,fx+2\,e\right)-i\,\sin\left(2\,fx+2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d*f^2*x^2 + 4*a*c*f^2*x - I*b*d*dilog(cos(2*f*x + 2*e) + I*sin(2*f
*x + 2*e)) + I*b*d*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) - 2*(b*d*e
- b*c*f)*log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2) - 2*(b*d
*e - b*c*f)*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2) + 2*(
b*d*f*x + b*d*e)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 2*(b*d*f
*x + b*d*e)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1))/f^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\left(b\cot\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*cot(f*x + e) + a), x)
```

maple [B] time = 1.11, size = 240, normalized size = 2.89

$$ibcx-\frac{ibd\,\mathrm{polylog}\left(2,e^{i\left(fx+e\right)}\right)}{f^2}+\frac{ad\,x^2}{2}+cax-\frac{2bc\ln\left(e^{i\left(fx+e\right)}\right)}{f}+\frac{bc\ln\left(e^{i\left(fx+e\right)}-1\right)}{f}+\frac{bc\ln\left(e^{i\left(fx+e\right)}+1\right)}{f}-\frac{ibd\,x^2}{2}-\frac{ibc}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*cot(f*x+e)),x)
```

```
[Out] I*b*c*x-I/f^2*b*d*polylog(2,-exp(I*(f*x+e)))+1/2*a*d*x^2+c*a*x-2/f*b*c*ln(e
xp(I*(f*x+e)))+1/f*b*c*ln(exp(I*(f*x+e))-1)+1/f*b*c*ln(exp(I*(f*x+e))+1)-I/
f^2*b*d*polylog(2,exp(I*(f*x+e)))-1/2*I*b*d*x^2-2*I/f*b*d*e*x+1/f*b*d*ln(1-
exp(I*(f*x+e)))*x+1/f^2*b*d*ln(1-exp(I*(f*x+e)))*e-I/f^2*b*d*e^2+1/f*b*d*ln
(exp(I*(f*x+e))+1)*x+2/f^2*b*d*e*ln(exp(I*(f*x+e)))-1/f^2*b*d*e*ln(exp(I*(f
*x+e))-1)
```

maxima [B] time = 0.93, size = 208, normalized size = 2.51

$$\frac{(a-ib)df^2x^2+2\,(a-ib)cf^2x-2ibdfx\arctan\left(\sin\left(fx+e\right),-\cos\left(fx+e\right)+1\right)+2ibcf\arctan\left(\sin\left(fx+e\right),}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a - I*b)*d*f^2*x^2 + 2*(a - I*b)*c*f^2*x - 2*I*b*d*f*x*arctan2(sin(f*
x + e), -cos(f*x + e) + 1) + 2*I*b*c*f*arctan2(sin(f*x + e), cos(f*x + e) -
 1) - 2*I*b*d*dilog(-e^(I*f*x + I*e)) - 2*I*b*d*dilog(e^(I*f*x + I*e)) + (2
*I*b*d*f*x + 2*I*b*c*f)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + (b*d*f*x
+ b*c*f)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + (b*d*f
*x + b*c*f)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1))/f^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\cot\left(e+f\,x\right)\right)\,\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))*(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\cot\left(e+fx\right)\right)\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x)
```

```
[Out] Integral((a + b*cot(e + f*x))*(c + d*x), x)
```

3.40 $\int \frac{a+b\cot(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{a+b\cot(e+fx)}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))/(d*x+c),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\cot(e+fx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(a + b*Cot[e + f*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{a+b\cot(e+fx)}{c+dx}\,dx = \int \frac{a+b\cot(e+fx)}{c+dx}\,dx$$

Mathematica [A] time = 2.23, size = 0, normalized size = 0.00

$$\int \frac{a+b\cot(e+fx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])/(c + d*x),x]
```

```
[Out] Integrate[(a + b*Cot[e + f*x])/(c + d*x), x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\cot\left(fx+e\right)+a}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cot(f*x + e) + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\cot\left(fx+e\right)+a}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e) + a)/(d*x + c), x)
```

maple [A] time = 1.78, size = 0, normalized size = 0.00

$$\int \frac{a + b\cot\left(fx + e\right)}{dx + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))/(d*x+c),x)
```

```
[Out] int((a+b*cot(f*x+e))/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd\int \frac{\sin\left(fx+e\right)}{(dx+c)\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2+2\cos\left(fx+e\right)+1\right)}\,dx - bd\int \frac{\sin\left(fx+e\right)}{(dx+c)\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2-2\cos\left(fx+e\right)+1\right)}\,dx - a\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] -(b*d*integrate(sin(f*x + e)/((d*x + c)*cos(f*x + e)^2 + (d*x + c)*sin(f*x
+ e)^2 + d*x + 2*(d*x + c)*cos(f*x + e) + c), x) - b*d*integrate(sin(f*x +
e)/((d*x + c)*cos(f*x + e)^2 + (d*x + c)*sin(f*x + e)^2 + d*x - 2*(d*x + c)
*cos(f*x + e) + c), x) - a*log(d*x + c))/d
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a + b\cot\left(e + f\,x\right)}{c + d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\cot\left(e + fx\right)}{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c),x)
```

```
[Out] Integral((a + b*cot(e + f*x))/(c + d*x), x)
```

3.41 $\int \frac{a+b\cot(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{a+b\cot(e+fx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))/(d*x+c)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\cot(e+fx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(a + b*Cot[e + f*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{a+b\cot(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\cot(e+fx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 8.19, size = 0, normalized size = 0.00

$$\int \frac{a+b\cot(e+fx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b*Cot[e + f*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\cot\left(fx+e\right)+a}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*cot(f*x + e) + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\cot\left(fx+e\right)+a}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e) + a)/(d*x + c)^2, x)
```

maple [A] time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{a + b\cot\left(fx + e\right)}{\left(dx + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))/(d*x+c)^2,x)
```

```
[Out] int((a+b*cot(f*x+e))/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(bd^2x + bcd\right)\int \frac{\sin\left(fx+e\right)}{(dx+c)^2\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2+2\cos\left(fx+e\right)+1\right)}\,dx - \left(bd^2x + bcd\right)\int \frac{\sin\left(fx+e\right)}{(dx+c)^2\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2-2\cos\left(fx+e\right)\right.}}{d^2x + cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((b*d^2*x + b*c*d)*integrate(sin(f*x + e)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 +
2*c*d*x + c^2)*cos(f*x + e)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(f*x + e)^2 +
c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(f*x + e)), x) - (b*d^2*x + b*c*d)*int
egrate(sin(f*x + e)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(f*x
+ e)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(f*x + e)^2 + c^2 - 2*(d^2*x^2 + 2*c*
d*x + c^2)*cos(f*x + e)), x) + a)/(d^2*x + c*d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a + b\cot\left(e + f\,x\right)}{\left(c + d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))/(c + d*x)^2,x)
```

```
[Out] int((a + b*cot(e + f*x))/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\cot\left(e + fx\right)}{\left(c + dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cot(e + f*x))/(c + d*x)**2, x)
```

3.42 $\int (c+dx)^3 (a+b\cot(e+fx))^2\,dx$

Optimal. Leaf size=295

$$\frac{a^2(c+dx)^4}{4d}+\frac{3abd^2(c+dx)\mathrm{Li}_3\left(e^{2i(e+fx)}\right)}{f^3}-\frac{3iabd(c+dx)^2\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{iab(c$$

```
[Out] -I*b^2*(d*x+c)^3/f+1/4*a^2*(d*x+c)^4/d-1/2*I*a*b*(d*x+c)^4/d-1/4*b^2*(d*x+c
)^4/d-b^2*(d*x+c)^3*cot(f*x+e)/f+3*b^2*d*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f
^2+2*a*b*(d*x+c)^3*ln(1-exp(2*I*(f*x+e)))/f-3*I*b^2*d^2*(d*x+c)*polylog(2,e
xp(2*I*(f*x+e)))/f^3-3*I*a*b*d*(d*x+c)^2*polylog(2,exp(2*I*(f*x+e)))/f^2+3/
2*b^2*d^3*polylog(3,exp(2*I*(f*x+e)))/f^4+3*a*b*d^2*(d*x+c)*polylog(3,exp(2
*I*(f*x+e)))/f^3+3/2*I*a*b*d^3*polylog(4,exp(2*I*(f*x+e)))/f^4
```

Rubi [A] time = 0.53, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3722, 3717, 2190, 2531, 6609, 2282, 6589, 3720, 32}

$$\frac{3abd^2(c+dx)\mathrm{PolyLog}\left(3,e^{2i(e+fx)}\right)}{f^3}-\frac{3iabd(c+dx)^2\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{f^2}+\frac{3iabd^3\mathrm{PolyLog}\left(4,e^{2i(e+fx)}\right)}{2f^4}-\frac{3ib^2d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((-I)*b^2*(c + d*x)^3)/f + (a^2*(c + d*x)^4)/(4*d) - ((I/2)*a*b*(c + d*x)^4
)/d - (b^2*(c + d*x)^4)/(4*d) - (b^2*(c + d*x)^3*Cot[e + f*x])/f + (3*b^2*d
*(c + d*x)^2*Log[1 - E^((2*I)*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^3*Log[1 -
 E^((2*I)*(e + f*x))])/f - ((3*I)*b^2*d^2*(c + d*x)*PolyLog[2, E^((2*I)*(e
+ f*x))])/f^3 - ((3*I)*a*b*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(e + f*x))])/f
^2 + (3*b^2*d^3*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^4) + (3*a*b*d^2*(c +
d*x)*PolyLog[3, E^((2*I)*(e + f*x))])/f^3 + (((3*I)/2)*a*b*d^3*PolyLog[4, E
^((2*I)*(e + f*x))])/f^4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3(a+b\cot(e+fx))^2\,dx = \int \left(a^2(c+dx)^3 + 2ab(c+dx)^3\cot(e+fx) + b^2(c+dx)^3\cot^2(e+fx)\right)dx$$

$$= \frac{a^2(c+dx)^4}{4d} + (2ab)\int (c+dx)^3\cot(e+fx)\,dx + b^2\int (c+dx)^3\cot^2(e+f$$

$$= \frac{a^2(c+dx)^4}{4d} - \frac{iab(c+dx)^4}{2d} - \frac{b^2(c+dx)^3\cot(e+fx)}{f} - (4iab)\int \frac{e^{2i(e+fx)}(}{1-e^2}$$

$$= -\frac{ib^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{iab(c+dx)^4}{2d} - \frac{b^2(c+dx)^4}{4d} - \frac{b^2(c+dx)^3\,c}{f}$$

$$= -\frac{ib^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{iab(c+dx)^4}{2d} - \frac{b^2(c+dx)^4}{4d} - \frac{b^2(c+dx)^3\,c}{f}$$

$$= -\frac{ib^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{iab(c+dx)^4}{2d} - \frac{b^2(c+dx)^4}{4d} - \frac{b^2(c+dx)^3\,c}{f}$$

$$= -\frac{ib^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{iab(c+dx)^4}{2d} - \frac{b^2(c+dx)^4}{4d} - \frac{b^2(c+dx)^3\,c}{f}$$

$$= -\frac{ib^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{iab(c+dx)^4}{2d} - \frac{b^2(c+dx)^4}{4d} - \frac{b^2(c+dx)^3\,c}{f}$$

Mathematica [B] time = 7.43, size = 1611, normalized size = 5.46

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] -1/2*(b^2*d^3*E^(I*e)*Csc[e]*((2*f^3*x^3)/E^((2*I)*e) + (3*I)*(1 - E^((-2*I
)*e))*f^2*x^2*Log[1 - E^((-I)*(e + f*x))] + (3*I)*(1 - E^((-2*I)*e))*f^2*x^
2*Log[1 + E^((-I)*(e + f*x))] - (6*(-1 + E^((2*I)*e))*(f*x*PolyLog[2, -E^((
-I)*(e + f*x))] - I*PolyLog[3, -E^((-I)*(e + f*x))]))/E^((2*I)*e) - (6*(-1
 + E^((2*I)*e))*(f*x*PolyLog[2, E^((-I)*(e + f*x))] - I*PolyLog[3, E^((-I)*(
e + f*x))]))/E^((2*I)*e)))/f^4 - (a*b*c*d^2*E^(I*e)*Csc[e]*((2*f^3*x^3)/E^(
(2*I)*e) + (3*I)*(1 - E^((-2*I)*e))*f^2*x^2*Log[1 - E^((-I)*(e + f*x))] + (
3*I)*(1 - E^((-2*I)*e))*f^2*x^2*Log[1 + E^((-I)*(e + f*x))] - (6*(-1 + E^((
2*I)*e))*(f*x*PolyLog[2, -E^((-I)*(e + f*x))] - I*PolyLog[3, -E^((-I)*(e +
f*x))]))/E^((2*I)*e) - (6*(-1 + E^((2*I)*e))*(f*x*PolyLog[2, E^((-I)*(e + f
*x))] - I*PolyLog[3, E^((-I)*(e + f*x))]))/E^((2*I)*e)))/f^3 - (a*b*d^3*E^(
I*e)*Csc[e]*((f^4*x^4)/E^((2*I)*e) + (2*I)*(1 - E^((-2*I)*e))*f^3*x^3*Log[1
 - E^((-I)*(e + f*x))] + (2*I)*(1 - E^((-2*I)*e))*f^3*x^3*Log[1 + E^((-I)*(
e + f*x))] - (6*(-1 + E^((2*I)*e))*(f^2*x^2*PolyLog[2, -E^((-I)*(e + f*x))]
 - (2*I)*f*x*PolyLog[3, -E^((-I)*(e + f*x))] - 2*PolyLog[4, -E^((-I)*(e + f
*x))]))/E^((2*I)*e) - (6*(-1 + E^((2*I)*e))*(f^2*x^2*PolyLog[2, E^((-I)*(e
+ f*x))] - (2*I)*f*x*PolyLog[3, E^((-I)*(e + f*x))] - 2*PolyLog[4, E^((-I)*
(e + f*x))]))/E^((2*I)*e)))/(2*f^4) + (3*b^2*c^2*d*Csc[e]*(-(f*x*Cos[e]) +
Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2))
 + (2*a*b*c^3*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]
]*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) + (Csc[e]*Csc[e + f*x]*(4*a^2*c^3*f*x*
Cos[f*x] - 4*b^2*c^3*f*x*Cos[f*x] + 6*a^2*c^2*d*f*x^2*Cos[f*x] - 6*b^2*c^2*
d*f*x^2*Cos[f*x] + 4*a^2*c*d^2*f*x^3*Cos[f*x] - 4*b^2*c*d^2*f*x^3*Cos[f*x]
+ a^2*d^3*f*x^4*Cos[f*x] - b^2*d^3*f*x^4*Cos[f*x] - 4*a^2*c^3*f*x*Cos[2*e +
 f*x] + 4*b^2*c^3*f*x*Cos[2*e + f*x] - 6*a^2*c^2*d*f*x^2*Cos[2*e + f*x] + 6
*b^2*c^2*d*f*x^2*Cos[2*e + f*x] - 4*a^2*c*d^2*f*x^3*Cos[2*e + f*x] + 4*b^2*
c*d^2*f*x^3*Cos[2*e + f*x] - a^2*d^3*f*x^4*Cos[2*e + f*x] + b^2*d^3*f*x^4*C
os[2*e + f*x] + 8*b^2*c^3*Sin[f*x] + 24*b^2*c^2*d*x*Sin[f*x] + 8*a*b*c^3*f*
```

```
x*Sin[f*x] + 24*b^2*c*d^2*x^2*Sin[f*x] + 12*a*b*c^2*d*f*x^2*Sin[f*x] + 8*b^
2*d^3*x^3*Sin[f*x] + 8*a*b*c*d^2*f*x^3*Sin[f*x] + 2*a*b*d^3*f*x^4*Sin[f*x]
+ 8*a*b*c^3*f*x*Sin[2*e + f*x] + 12*a*b*c^2*d*f*x^2*Sin[2*e + f*x] + 8*a*b*
c*d^2*f*x^3*Sin[2*e + f*x] + 2*a*b*d^3*f*x^4*Sin[2*e + f*x]))/(8*f) - (3*b^
2*c*d^2*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcT
an[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 -
 E^((2*I)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Lo
g[Sin[f*x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]])
)])*Tan[e])/Sqrt[1 + Tan[e]^2]))/(f^3*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)])
 - (3*a*b*c^2*d*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi
+ 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])
*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Ta
n[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[
Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))/(f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin
[e]^2)])
```

fricas **[C]** time = 0.90, size = 1154, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 4*b^2
*c^3*f^3 - 3*I*a*b*d^3*polylog(4, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e))*si
n(2*f*x + 2*e) + 3*I*a*b*d^3*polylog(4, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*
e))*sin(2*f*x + 2*e) - (-6*I*a*b*d^3*f^2*x^2 - 6*I*a*b*c^2*d*f^2 - 6*I*b^2*
c*d^2*f - 6*I*(2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*dilog(cos(2*f*x + 2*e) + I*s
in(2*f*x + 2*e))*sin(2*f*x + 2*e) - (6*I*a*b*d^3*f^2*x^2 + 6*I*a*b*c^2*d*f^
2 + 6*I*b^2*c*d^2*f + 6*I*(2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*dilog(cos(2*f*x
+ 2*e) - I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + 2*(2*a*b*d^3*e^3 - 2*a*b*c^
3*f^3 - 3*b^2*d^3*e^2 + 3*(2*a*b*c^2*d*e - b^2*c^2*d)*f^2 - 6*(a*b*c*d^2*e^
2 - b^2*c*d^2*e)*f)*log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/
2)*sin(2*f*x + 2*e) + 2*(2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3*e^2 + 3*
(2*a*b*c^2*d*e - b^2*c^2*d)*f^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f)*log(-1
/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x + 2*e) - 2*(2
*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*d*e*f^2 - 3*b^2*d^3*e^2 + 3*(2
*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f + 6*(
a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e
) + 1)*sin(2*f*x + 2*e) - 2*(2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*
d*e*f^2 - 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 - 6*(a*b*c*
d^2*e^2 - b^2*c*d^2*e)*f + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*log(-cos(2*
f*x + 2*e) - I*sin(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e) - 3*(2*a*b*d^3*f*x +
2*a*b*c*d^2*f + b^2*d^3)*polylog(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e))*
sin(2*f*x + 2*e) - 3*(2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*polylog(3, c
os(2*f*x + 2*e) - I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + 4*(b^2*d^3*f^3*x^3
 + 3*b^2*c*d^2*f^3*x^2 + 3*b^2*c^2*d*f^3*x + b^2*c^3*f^3)*cos(2*f*x + 2*e)
- ((a^2 - b^2)*d^3*f^4*x^4 + 4*(a^2 - b^2)*c*d^2*f^4*x^3 + 6*(a^2 - b^2)*c^
2*d*f^4*x^2 + 4*(a^2 - b^2)*c^3*f^4*x)*sin(2*f*x + 2*e))/(f^4*sin(2*f*x + 2
*e))
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \left(b\cot\left(fx+e\right)+a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*(b*cot(f*x + e) + a)^2, x)
```

maple [B] time = 1.82, size = 1571, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*cot(f*x+e))^2,x)
```

```
[Out] 3/f^4*b^2*d^3*e^2*ln(exp(I*(f*x+e))-1)+a^2*c^3*x-b^2*c^3*x-2*I*a*b*c*d^2*x^
3-3*I*a*b*c^2*d*x^2-12*I/f^2*b*a*c*d^2*polylog(2,exp(I*(f*x+e)))*x-12*I/f*b
*a*c^2*d*e*x-12*I/f^2*b*a*c*d^2*polylog(2,-exp(I*(f*x+e)))*x+12*I/f^2*b*a*c
*d^2*e^2*x+a^2*c*d^2*x^3-b^2*c*d^2*x^3+3/2*a^2*c^2*d*x^2-3/2*b^2*c^2*d*x^2+
2/f*b*a*c^3*ln(exp(I*(f*x+e))+1)+3/f^2*b^2*c^2*d*ln(exp(I*(f*x+e))+1)+3/f^2
*b^2*c^2*d*ln(exp(I*(f*x+e))-1)-6/f^2*b^2*c^2*d*ln(exp(I*(f*x+e)))-4/f*b*a*
c^3*ln(exp(I*(f*x+e)))+2/f*b*a*c^3*ln(exp(I*(f*x+e))-1)-6/f^4*b^2*d^3*e^2*l
n(exp(I*(f*x+e)))+3/f^2*b^2*d^3*ln(1-exp(I*(f*x+e)))*x^2-3/f^4*b^2*d^3*ln(1
-exp(I*(f*x+e)))*e^2+3/f^2*b^2*d^3*ln(exp(I*(f*x+e))+1)*x^2-2*I/f*b^2*d^3*x
^3+4*I/f^4*b^2*d^3*e^3+2*I*a*b*c^3*x-1/2*I*a*b*d^3*x^4+1/4*a^2*d^3*x^4-1/4*
b^2*d^3*x^4+6/f^4*b^2*d^3*polylog(3,-exp(I*(f*x+e)))+6/f^4*b^2*d^3*polylog(
3,exp(I*(f*x+e)))-2*I*b^2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/(exp(2*I*(f
*x+e))-1)+2/f*b*a*d^3*ln(exp(I*(f*x+e))+1)*x^3+2/f*b*a*d^3*ln(1-exp(I*(f*x+
e)))*x^3+2/f^4*b*a*d^3*ln(1-exp(I*(f*x+e)))*e^3-6*I/f^2*b*a*c^2*d*e^2-6*I/f
^2*b*a*c^2*d*polylog(2,-exp(I*(f*x+e)))-6*I/f^2*b*a*d^3*polylog(2,-exp(I*(f
*x+e)))*x^2-6*I/f^2*b*a*d^3*polylog(2,exp(I*(f*x+e)))*x^2-6*I/f^2*b*a*c^2*d
*polylog(2,exp(I*(f*x+e)))-4*I/f^3*b*a*d^3*e^3*x-12*I/f^2*b^2*c*d^2*e*x+8*I
/f^3*b*a*c*d^2*e^3+6/f^3*b*a*c*d^2*e^2*ln(exp(I*(f*x+e))-1)-12/f^3*b*a*c*d^
2*e^2*ln(exp(I*(f*x+e)))+12/f^2*b*a*c^2*d*e*ln(exp(I*(f*x+e)))-6/f^2*b*a*c^
2*d*e*ln(exp(I*(f*x+e))-1)+6/f^2*b*ln(1-exp(I*(f*x+e)))*a*c^2*d*e+6/f^2*b^2
*c*d^2*ln(1-exp(I*(f*x+e)))*x+6/f^3*b^2*c*d^2*ln(1-exp(I*(f*x+e)))*e-2/f^4*
b*a*d^3*e^3*ln(exp(I*(f*x+e))-1)+12/f^3*b*a*c*d^2*polylog(3,exp(I*(f*x+e)))
+12/f^3*b*a*c*d^2*polylog(3,-exp(I*(f*x+e)))-3*I/f^4*b*e^4*a*d^3+12*I/f^4*b
*a*d^3*polylog(4,-exp(I*(f*x+e)))-6*I/f*b^2*c*d^2*x^2-6*I/f^3*b^2*c*d^2*e^2
+6*I/f^3*b^2*d^3*e^2*x+6/f*b*ln(exp(I*(f*x+e))+1)*a*c^2*d*x+6/f*b*ln(1-exp(
I*(f*x+e)))*a*c^2*d*x+6/f*b*a*c*d^2*ln(1-exp(I*(f*x+e)))*x^2-6/f^3*b*a*c*d^
2*ln(1-exp(I*(f*x+e)))*e^2+6/f*b*a*c*d^2*ln(exp(I*(f*x+e))+1)*x^2-6/f^3*b^2
*c*d^2*e*ln(exp(I*(f*x+e))-1)+4/f^4*b*a*d^3*e^3*ln(exp(I*(f*x+e)))+12/f^3*b
^2*c*d^2*e*ln(exp(I*(f*x+e)))+12/f^3*b*a*d^3*polylog(3,exp(I*(f*x+e)))*x+12
/f^3*b*a*d^3*polylog(3,-exp(I*(f*x+e)))*x+6/f^2*b^2*c*d^2*ln(exp(I*(f*x+e))
+1)*x-6*I/f^3*b^2*c*d^2*polylog(2,-exp(I*(f*x+e)))-6*I/f^3*b^2*c*d^2*polylo
g(2,exp(I*(f*x+e)))-6*I/f^3*b^2*d^3*polylog(2,-exp(I*(f*x+e)))*x-6*I/f^3*b^
2*d^3*polylog(2,exp(I*(f*x+e)))*x+12*I/f^4*b*a*d^3*polylog(4,exp(I*(f*x+e))
)
```

maxima [B] time = 4.35, size = 4012, normalized size = 13.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a^2*c^3 + (f*x + e)^4*a^2*d^3/f^3 - 4*(f*x + e)^3*a^2*d^3*
e/f^3 + 6*(f*x + e)^2*a^2*d^3*e^2/f^3 - 4*(f*x + e)*a^2*d^3*e^3/f^3 + 4*(f*
x + e)^3*a^2*c*d^2/f^2 - 12*(f*x + e)^2*a^2*c*d^2*e/f^2 + 12*(f*x + e)*a^2*
c*d^2*e^2/f^2 + 6*(f*x + e)^2*a^2*c^2*d/f - 12*(f*x + e)*a^2*c^2*d*e/f + 8*
a*b*c^3*log(sin(f*x + e)) - 8*a*b*d^3*e^3*log(sin(f*x + e))/f^3 + 24*a*b*c*
d^2*e^2*log(sin(f*x + e))/f^2 - 24*a*b*c^2*d*e*log(sin(f*x + e))/f + 4*((2*
a*b - I*b^2)*(f*x + e)^4*d^3 + 8*b^2*d^3*e^3 - 24*b^2*c*d^2*e^2*f + 24*b^2*
c^2*d*e*f^2 - 8*b^2*c^3*f^3 - ((8*a*b - 4*I*b^2)*d^3*e - (8*a*b - 4*I*b^2)*
c*d^2*f)*(f*x + e)^3 + ((12*a*b - 6*I*b^2)*d^3*e^2 - (24*a*b - 12*I*b^2)*c*
d^2*e*f + (12*a*b - 6*I*b^2)*c^2*d*f^2)*(f*x + e)^2 + (4*I*b^2*d^3*e^3 - 12
```

```
*I*b^2*c*d^2*e^2*f + 12*I*b^2*c^2*d*e*f^2 - 4*I*b^2*c^3*f^3)*(f*x + e) - (8
*(f*x + e)^3*a*b*d^3 + 12*b^2*d^3*e^2 - 24*b^2*c*d^2*e*f + 12*b^2*c^2*d*f^2
 - 12*(2*a*b*d^3*e - 2*a*b*c*d^2*f - b^2*d^3)*(f*x + e)^2 + 24*(a*b*d^3*e^2
 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*c*d^2*e - b^2*c*d^2)*f)*(f*x + e) - 4
*(2*(f*x + e)^3*a*b*d^3 + 3*b^2*d^3*e^2 - 6*b^2*c*d^2*e*f + 3*b^2*c^2*d*f^2
 - 3*(2*a*b*d^3*e - 2*a*b*c*d^2*f - b^2*d^3)*(f*x + e)^2 + 6*(a*b*d^3*e^2 +
 a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*c*d^2*e - b^2*c*d^2)*f)*(f*x + e))*cos(
2*f*x + 2*e) - (8*I*(f*x + e)^3*a*b*d^3 + 12*I*b^2*d^3*e^2 - 24*I*b^2*c*d^2
*e*f + 12*I*b^2*c^2*d*f^2 + (-24*I*a*b*d^3*e + 24*I*a*b*c*d^2*f + 12*I*b^2*
d^3)*(f*x + e)^2 + (24*I*a*b*d^3*e^2 + 24*I*a*b*c^2*d*f^2 - 24*I*b^2*d^3*e
+ (-48*I*a*b*c*d^2*e + 24*I*b^2*c*d^2)*f)*(f*x + e))*sin(2*f*x + 2*e))*arct
an2(sin(f*x + e), cos(f*x + e) + 1) - (12*b^2*d^3*e^2 - 24*b^2*c*d^2*e*f +
12*b^2*c^2*d*f^2 - 12*(b^2*d^3*e^2 - 2*b^2*c*d^2*e*f + b^2*c^2*d*f^2)*cos(2
*f*x + 2*e) - (12*I*b^2*d^3*e^2 - 24*I*b^2*c*d^2*e*f + 12*I*b^2*c^2*d*f^2)*
sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) - 1) + (8*(f*x + e)^3*
a*b*d^3 - 12*(2*a*b*d^3*e - 2*a*b*c*d^2*f - b^2*d^3)*(f*x + e)^2 + 24*(a*b*
d^3*e^2 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*c*d^2*e - b^2*c*d^2)*f)*(f*x +
 e) - 4*(2*(f*x + e)^3*a*b*d^3 - 3*(2*a*b*d^3*e - 2*a*b*c*d^2*f - b^2*d^3)*
(f*x + e)^2 + 6*(a*b*d^3*e^2 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*c*d^2*e -
 b^2*c*d^2)*f)*(f*x + e))*cos(2*f*x + 2*e) + (-8*I*(f*x + e)^3*a*b*d^3 + (2
4*I*a*b*d^3*e - 24*I*a*b*c*d^2*f - 12*I*b^2*d^3)*(f*x + e)^2 + (-24*I*a*b*d
^3*e^2 - 24*I*a*b*c^2*d*f^2 + 24*I*b^2*d^3*e + (48*I*a*b*c*d^2*e - 24*I*b^2
*c*d^2)*f)*(f*x + e))*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), -cos(f*x + e)
 + 1) - ((2*a*b - I*b^2)*(f*x + e)^4*d^3 + (8*b^2*d^3 - (8*a*b - 4*I*b^2)*d
^3*e + (8*a*b - 4*I*b^2)*c*d^2*f)*(f*x + e)^3 - (24*b^2*d^3*e - (12*a*b - 6
*I*b^2)*d^3*e^2 - (12*a*b - 6*I*b^2)*c^2*d*f^2 - (24*b^2*c*d^2 - (24*a*b -
12*I*b^2)*c*d^2*e)*f)*(f*x + e)^2 - (-4*I*b^2*d^3*e^3 + 4*I*b^2*c^3*f^3 - 2
4*b^2*d^3*e^2 - 12*(I*b^2*c^2*d*e + 2*b^2*c^2*d)*f^2 - 12*(-I*b^2*c*d^2*e^2
 - 4*b^2*c*d^2*e)*f)*(f*x + e))*cos(2*f*x + 2*e) + (24*(f*x + e)^2*a*b*d^3
+ 24*a*b*d^3*e^2 + 24*a*b*c^2*d*f^2 - 24*b^2*d^3*e - 24*(2*a*b*d^3*e - 2*a*
b*c*d^2*f - b^2*d^3)*(f*x + e) - 24*(2*a*b*c*d^2*e - b^2*c*d^2)*f - 24*((f*
x + e)^2*a*b*d^3 + a*b*d^3*e^2 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*d^3*e -
 2*a*b*c*d^2*f - b^2*d^3)*(f*x + e) - (2*a*b*c*d^2*e - b^2*c*d^2)*f)*cos(2*
f*x + 2*e) + (-24*I*(f*x + e)^2*a*b*d^3 - 24*I*a*b*d^3*e^2 - 24*I*a*b*c^2*d
*f^2 + 24*I*b^2*d^3*e + (48*I*a*b*d^3*e - 48*I*a*b*c*d^2*f - 24*I*b^2*d^3)*
(f*x + e) + (48*I*a*b*c*d^2*e - 24*I*b^2*c*d^2)*f)*sin(2*f*x + 2*e))*dilog(
-e^(I*f*x + I*e)) + (24*(f*x + e)^2*a*b*d^3 + 24*a*b*d^3*e^2 + 24*a*b*c^2*d
*f^2 - 24*b^2*d^3*e - 24*(2*a*b*d^3*e - 2*a*b*c*d^2*f - b^2*d^3)*(f*x + e)
- 24*(2*a*b*c*d^2*e - b^2*c*d^2)*f - 24*((f*x + e)^2*a*b*d^3 + a*b*d^3*e^2
+ a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*d^3*e - 2*a*b*c*d^2*f - b^2*d^3)*(f*x
+ e) - (2*a*b*c*d^2*e - b^2*c*d^2)*f)*cos(2*f*x + 2*e) + (-24*I*(f*x + e)^2
*a*b*d^3 - 24*I*a*b*d^3*e^2 - 24*I*a*b*c^2*d*f^2 + 24*I*b^2*d^3*e + (48*I*a
*b*d^3*e - 48*I*a*b*c*d^2*f - 24*I*b^2*d^3)*(f*x + e) + (48*I*a*b*c*d^2*e -
 24*I*b^2*c*d^2)*f)*sin(2*f*x + 2*e))*dilog(e^(I*f*x + I*e)) + (4*I*(f*x +
e)^3*a*b*d^3 + 6*I*b^2*d^3*e^2 - 12*I*b^2*c*d^2*e*f + 6*I*b^2*c^2*d*f^2 + (
-12*I*a*b*d^3*e + 12*I*a*b*c*d^2*f + 6*I*b^2*d^3)*(f*x + e)^2 + (12*I*a*b*d
^3*e^2 + 12*I*a*b*c^2*d*f^2 - 12*I*b^2*d^3*e + (-24*I*a*b*c*d^2*e + 12*I*b^
2*c*d^2)*f)*(f*x + e) + (-4*I*(f*x + e)^3*a*b*d^3 - 6*I*b^2*d^3*e^2 + 12*I*
b^2*c*d^2*e*f - 6*I*b^2*c^2*d*f^2 + (12*I*a*b*d^3*e - 12*I*a*b*c*d^2*f - 6*
I*b^2*d^3)*(f*x + e)^2 + (-12*I*a*b*d^3*e^2 - 12*I*a*b*c^2*d*f^2 + 12*I*b^2
*d^3*e + (24*I*a*b*c*d^2*e - 12*I*b^2*c*d^2)*f)*(f*x + e))*cos(2*f*x + 2*e)
 + 2*(2*(f*x + e)^3*a*b*d^3 + 3*b^2*d^3*e^2 - 6*b^2*c*d^2*e*f + 3*b^2*c^2*d
*f^2 - 3*(2*a*b*d^3*e - 2*a*b*c*d^2*f - b^2*d^3)*(f*x + e)^2 + 6*(a*b*d^3*e
^2 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*c*d^2*e - b^2*c*d^2)*f)*(f*x + e))*
sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)
 + (4*I*(f*x + e)^3*a*b*d^3 + 6*I*b^2*d^3*e^2 - 12*I*b^2*c*d^2*e*f + 6*I*b^
2*c^2*d*f^2 + (-12*I*a*b*d^3*e + 12*I*a*b*c*d^2*f + 6*I*b^2*d^3)*(f*x + e)^
2 + (12*I*a*b*d^3*e^2 + 12*I*a*b*c^2*d*f^2 - 12*I*b^2*d^3*e + (-24*I*a*b*c*
d^2*e + 12*I*b^2*c*d^2)*f)*(f*x + e) + (-4*I*(f*x + e)^3*a*b*d^3 - 6*I*b^2*
```

```
d^3*e^2 + 12*I*b^2*c*d^2*e*f - 6*I*b^2*c^2*d*f^2 + (12*I*a*b*d^3*e - 12*I*a
*b*c*d^2*f - 6*I*b^2*d^3)*(f*x + e)^2 + (-12*I*a*b*d^3*e^2 - 12*I*a*b*c^2*d
*f^2 + 12*I*b^2*d^3*e + (24*I*a*b*c*d^2*e - 12*I*b^2*c*d^2)*f)*(f*x + e))*c
os(2*f*x + 2*e) + 2*(2*(f*x + e)^3*a*b*d^3 + 3*b^2*d^3*e^2 - 6*b^2*c*d^2*e*
f + 3*b^2*c^2*d*f^2 - 3*(2*a*b*d^3*e - 2*a*b*c*d^2*f - b^2*d^3)*(f*x + e)^2
 + 6*(a*b*d^3*e^2 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*c*d^2*e - b^2*c*d^2)
*f)*(f*x + e))*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*co
s(f*x + e) + 1) + 48*(a*b*d^3*cos(2*f*x + 2*e) + I*a*b*d^3*sin(2*f*x + 2*e)
 - a*b*d^3)*polylog(4, -e^(I*f*x + I*e)) + 48*(a*b*d^3*cos(2*f*x + 2*e) + I
*a*b*d^3*sin(2*f*x + 2*e) - a*b*d^3)*polylog(4, e^(I*f*x + I*e)) + (48*I*(f
*x + e)*a*b*d^3 - 48*I*a*b*d^3*e + 48*I*a*b*c*d^2*f + 24*I*b^2*d^3 + (-48*I
*(f*x + e)*a*b*d^3 + 48*I*a*b*d^3*e - 48*I*a*b*c*d^2*f - 24*I*b^2*d^3)*cos(
2*f*x + 2*e) + 24*(2*(f*x + e)*a*b*d^3 - 2*a*b*d^3*e + 2*a*b*c*d^2*f + b^2*
d^3)*sin(2*f*x + 2*e))*polylog(3, -e^(I*f*x + I*e)) + (48*I*(f*x + e)*a*b*d
^3 - 48*I*a*b*d^3*e + 48*I*a*b*c*d^2*f + 24*I*b^2*d^3 + (-48*I*(f*x + e)*a*
b*d^3 + 48*I*a*b*d^3*e - 48*I*a*b*c*d^2*f - 24*I*b^2*d^3)*cos(2*f*x + 2*e)
+ 24*(2*(f*x + e)*a*b*d^3 - 2*a*b*d^3*e + 2*a*b*c*d^2*f + b^2*d^3)*sin(2*f*
x + 2*e))*polylog(3, e^(I*f*x + I*e)) + ((-2*I*a*b - b^2)*(f*x + e)^4*d^3 +
 (-8*I*b^2*d^3 - 4*(-2*I*a*b - b^2)*d^3*e - 4*(2*I*a*b + b^2)*c*d^2*f)*(f*x
 + e)^3 + (24*I*b^2*d^3*e - 6*(2*I*a*b + b^2)*d^3*e^2 - 6*(2*I*a*b + b^2)*c
^2*d*f^2 + (-24*I*b^2*c*d^2 - 12*(-2*I*a*b - b^2)*c*d^2*e)*f)*(f*x + e)^2 +
 (4*b^2*d^3*e^3 - 4*b^2*c^3*f^3 - 24*I*b^2*d^3*e^2 + (12*b^2*c^2*d*e - 24*I
*b^2*c^2*d)*f^2 - (12*b^2*c*d^2*e^2 - 48*I*b^2*c*d^2*e)*f)*(f*x + e))*sin(2
*f*x + 2*e))/(-4*I*f^3*cos(2*f*x + 2*e) + 4*f^3*sin(2*f*x + 2*e) + 4*I*f^3)
)/f
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\cot\left(e + f\,x\right)\right)^2 (c + d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^2*(c + d*x)^3,x)
```

```
[Out] int((a + b*cot(e + f*x))^2*(c + d*x)^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot\left(e + f\,x\right)\right)^2 (c + dx)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**2*(c + d*x)**3, x)
```

3.43 $\int (c+dx)^2(a+b\cot(e+fx))^2\,dx$

Optimal. Leaf size=227

$$\frac{a^2(c+dx)^3}{3d}-\frac{2iabd(c+dx)\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{2iab(c+dx)^3}{3d}+\frac{abd^2\mathrm{Li}_3\left(e^{2i(e+fx)}\right)}{f^3}+\frac{2b^2}{}$$

```
[Out] -I*b^2*(d*x+c)^2/f+1/3*a^2*(d*x+c)^3/d-2/3*I*a*b*(d*x+c)^3/d-1/3*b^2*(d*x+c
)^3/d-b^2*(d*x+c)^2*cot(f*x+e)/f+2*b^2*d*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f^2
+2*a*b*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f-I*b^2*d^2*polylog(2,exp(2*I*(f*x+
e)))/f^3-2*I*a*b*d*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^2+a*b*d^2*polylog(
3,exp(2*I*(f*x+e)))/f^3
```

Rubi [A] time = 0.38, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3722, 3717, 2190, 2531, 2282, 6589, 3720, 2279, 2391, 32}

$$-\frac{2iabd(c+dx)\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{f^2}+\frac{abd^2\mathrm{PolyLog}\left(3,e^{2i(e+fx)}\right)}{f^3}-\frac{ib^2d^2\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{f^3}+\frac{a^2(c+dx)^3}{3d}+\frac{2ab(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((-I)*b^2*(c + d*x)^2)/f + (a^2*(c + d*x)^3)/(3*d) - (((2*I)/3)*a*b*(c + d*
x)^3)/d - (b^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^2*Cot[e + f*x])/f + (2*b
^2*d*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^2*Log[1
 - E^((2*I)*(e + f*x))])/f - (I*b^2*d^2*PolyLog[2, E^((2*I)*(e + f*x))])/f^
3 - ((2*I)*a*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (a*b*d^2*
PolyLog[3, E^((2*I)*(e + f*x))])/f^3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2(a+b\cot(e+fx))^2\,dx = \int \left(a^2(c+dx)^2 + 2ab(c+dx)^2\cot(e+fx) + b^2(c+dx)^2\cot^2(e+fx)\right)dx$$

$$= \frac{a^2(c+dx)^3}{3d} + (2ab)\int (c+dx)^2\cot(e+fx)\,dx + b^2\int (c+dx)^2\cot^2(e+fx)$$

$$= \frac{a^2(c+dx)^3}{3d} - \frac{2iab(c+dx)^3}{3d} - \frac{b^2(c+dx)^2\cot(e+fx)}{f} - (4iab)\int \frac{e^{2i(e+fx)}(c$$

$$= -\frac{ib^2(c+dx)^2}{f} + \frac{a^2(c+dx)^3}{3d} - \frac{2iab(c+dx)^3}{3d} - \frac{b^2(c+dx)^3}{3d} - \frac{b^2(c+dx)^2\cot}{f}$$

$$= -\frac{ib^2(c+dx)^2}{f} + \frac{a^2(c+dx)^3}{3d} - \frac{2iab(c+dx)^3}{3d} - \frac{b^2(c+dx)^3}{3d} - \frac{b^2(c+dx)^2\cot}{f}$$

$$= -\frac{ib^2(c+dx)^2}{f} + \frac{a^2(c+dx)^3}{3d} - \frac{2iab(c+dx)^3}{3d} - \frac{b^2(c+dx)^3}{3d} - \frac{b^2(c+dx)^2\cot}{f}$$

$$= -\frac{ib^2(c+dx)^2}{f} + \frac{a^2(c+dx)^3}{3d} - \frac{2iab(c+dx)^3}{3d} - \frac{b^2(c+dx)^3}{3d} - \frac{b^2(c+dx)^2\cot}{f}$$

Mathematica [B] time = 7.22, size = 729, normalized size = 3.21

$$\frac{1}{3}x\csc(e)\left(3c^2+3cdx+d^2x^2\right)\left(a^2\sin(e)+2ab\cos(e)-b^2\sin(e)\right)+\frac{2abc^2\csc(e)(\sin(e)\log(\sin(e)\cos(fx)+\cos(e)}{f\left(\sin^2(e)+\cos^2(e)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] -1/3*(a*b*d^2*E^(I*e)*Csc[e]*((2*f^3*x^3)/E^((2*I)*e) + (3*I)*(1 - E^((-2*I
)*e))*f^2*x^2*Log[1 - E^((-I)*(e + f*x))] + (3*I)*(1 - E^((-2*I)*e))*f^2*x^
2*Log[1 + E^((-I)*(e + f*x))] - (6*(-1 + E^((2*I)*e))*(f*x*PolyLog[2, -E^((
-I)*(e + f*x))] - I*PolyLog[3, -E^((-I)*(e + f*x))]))/E^((2*I)*e) - (6*(-1
+ E^((2*I)*e))*(f*x*PolyLog[2, E^((-I)*(e + f*x))] - I*PolyLog[3, E^((-I)*(
e + f*x))]))/E^((2*I)*e)))/f^3 + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csc[e]*(2*a
*b*Cos[e] + a^2*Sin[e] - b^2*Sin[e]))/3 + (2*b^2*c*d*Csc[e]*(-(f*x*Cos[e])
+ Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2
)) + (2*a*b*c^2*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*
x]]*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) + (Csc[e]*Csc[e + f*x]*(b^2*c^2*Sin[
f*x] + 2*b^2*c*d*x*Sin[f*x] + b^2*d^2*x^2*Sin[f*x]))/f - (b^2*d^2*Csc[e]*Se
c[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*
Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x +
ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTa
n[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[
1 + Tan[e]^2]))/(f^3*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)]) - (2*a*b*c*d*Csc
[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]])
 - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(
f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x +
 ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e])
/Sqrt[1 + Tan[e]^2]))/(f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)])
```

fricas [C] time = 0.83, size = 715, normalized size = 3.15

$$-\frac{6\,b^2d^2f^2x^2 + 12\,b^2cdf^2x + 6\,b^2c^2f^2 - 3\,abd^2\mathrm{polylog}\left(3,\cos\left(2\,fx+2\,e\right)+i\,\sin\left(2\,fx+2\,e\right)\right)\sin\left(2\,fx+2\,e\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^2*d^2*f^2*x^2 + 12*b^2*c*d*f^2*x + 6*b^2*c^2*f^2 - 3*a*b*d^2*poly
log(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) - 3*a*b*d^2*
polylog(3, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) - (-6*I*
a*b*d^2*f*x - 6*I*a*b*c*d*f - 3*I*b^2*d^2)*dilog(cos(2*f*x + 2*e) + I*sin(2
*f*x + 2*e))*sin(2*f*x + 2*e) - (6*I*a*b*d^2*f*x + 6*I*a*b*c*d*f + 3*I*b^2*
d^2)*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) - 6*(a*b
*d^2*e^2 + a*b*c^2*f^2 - b^2*d^2*e - (2*a*b*c*d*e - b^2*c*d)*f)*log(-1/2*co
s(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x + 2*e) - 6*(a*b*d^
2*e^2 + a*b*c^2*f^2 - b^2*d^2*e - (2*a*b*c*d*e - b^2*c*d)*f)*log(-1/2*cos(2
*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x + 2*e) - 6*(a*b*d^2*f
^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*d*e*f + b^2*d^2*e + (2*a*b*c*d*f^2 + b^2*d^2
*f)*x)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e) - 6
*(a*b*d^2*f^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*d*e*f + b^2*d^2*e + (2*a*b*c*d*f^
2 + b^2*d^2*f)*x)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1)*sin(2*f*x
 + 2*e) + 6*(b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*cos(2*f*x + 2
*e) - 2*((a^2 - b^2)*d^2*f^3*x^3 + 3*(a^2 - b^2)*c*d*f^3*x^2 + 3*(a^2 - b^2
)*c^2*f^3*x)*sin(2*f*x + 2*e))/(f^3*sin(2*f*x + 2*e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \left(b \cot\left(fx + e\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(b*cot(f*x + e) + a)^2, x)
```

maple [B] time = 1.64, size = 899, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*cot(f*x+e))^2,x)
```

```
[Out] -8*I/f*b*a*c*d*e*x-2*I*a*b*c*d*x^2-b^2*c*d*x^2+a^2*c*d*x^2+4/f*b*ln(exp(I*(
f*x+e))+1)*a*c*d*x-4/f^2*b*a*c*d*e*ln(exp(I*(f*x+e))-1)+8/f^2*b*a*c*d*e*ln(
exp(I*(f*x+e)))+4*I/f^2*b*a*d^2*e^2*x-4*I/f^2*b*a*c*d*polylog(2,exp(I*(f*x+
e)))-4*I/f^2*b*a*c*d*polylog(2,-exp(I*(f*x+e)))-4*I/f^2*b*a*d^2*polylog(2,-
exp(I*(f*x+e)))*x-4*I/f^2*b*a*d^2*polylog(2,exp(I*(f*x+e)))*x-4*I/f^2*b*a*c
*d*e^2+4/f^2*b*ln(1-exp(I*(f*x+e)))*a*c*d*e+4/f*b*ln(1-exp(I*(f*x+e)))*a*c*
d*x+2/f*b*a*d^2*ln(1-exp(I*(f*x+e)))*x^2+2/f*b*a*d^2*ln(exp(I*(f*x+e))+1)*x
^2-2/f^3*b*a*d^2*e^2*ln(1-exp(I*(f*x+e)))+2/f^3*b*a*d^2*e^2*ln(exp(I*(f*x+e
))-1)-4/f^3*b*a*d^2*e^2*ln(exp(I*(f*x+e)))-4*I/f^2*b^2*d^2*e*x+8/3*I/f^3*b*
a*d^2*e^3-2/3*I*a*b*d^2*x^3+2/f^2*b^2*d^2*ln(exp(I*(f*x+e))+1)*x+4/f^3*b*a*
d^2*polylog(3,-exp(I*(f*x+e)))+2/f*b*a*c^2*ln(exp(I*(f*x+e))+1)-4/f*b*a*c^2
*ln(exp(I*(f*x+e)))+2/f^2*b^2*c*d*ln(exp(I*(f*x+e))+1)+2/f*b*a*c^2*ln(exp(I
*(f*x+e))-1)+2/f^2*b^2*c*d*ln(exp(I*(f*x+e))-1)+2/f^2*b^2*d^2*ln(1-exp(I*(f
*x+e)))*x+2/f^3*b^2*d^2*ln(1-exp(I*(f*x+e)))*e-b^2*c^2*x+1/3*a^2*d^2*x^3-1/
3*b^2*d^2*x^3+a^2*c^2*x+2*I*a*b*c^2*x-2*I/f^3*b^2*d^2*polylog(2,-exp(I*(f*x
+e)))-2*I/f^3*b^2*d^2*polylog(2,exp(I*(f*x+e)))+4/f^3*b^2*d^2*e*ln(exp(I*(f
*x+e)))-2/f^3*b^2*d^2*e*ln(exp(I*(f*x+e))-1)+4/f^3*b*a*d^2*polylog(3,exp(I*
(f*x+e)))-4/f^2*b^2*c*d*ln(exp(I*(f*x+e)))-2*I/f*b^2*d^2*x^2-2*I/f^3*b^2*d^
2*e^2-2*I*b^2*(d^2*x^2+2*c*d*x+c^2)/f/(exp(2*I*(f*x+e))-1)
```

maxima [B] time = 1.99, size = 1945, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^2*c^2 + (f*x + e)^3*a^2*d^2/f^2 - 3*(f*x + e)^2*a^2*d^2*
e/f^2 + 3*(f*x + e)*a^2*d^2*e^2/f^2 + 3*(f*x + e)^2*a^2*c*d/f - 6*(f*x + e)
*a^2*c*d*e/f + 6*a*b*c^2*log(sin(f*x + e)) + 6*a*b*d^2*e^2*log(sin(f*x + e)
)/f^2 - 12*a*b*c*d*e*log(sin(f*x + e))/f + 3*((2*a*b - I*b^2)*(f*x + e)^3*d
^2 - 6*b^2*d^2*e^2 + 12*b^2*c*d*e*f - 6*b^2*c^2*f^2 - ((6*a*b - 3*I*b^2)*d^
2*e - (6*a*b - 3*I*b^2)*c*d*f)*(f*x + e)^2 + (-3*I*b^2*d^2*e^2 + 6*I*b^2*c*
d*e*f - 3*I*b^2*c^2*f^2)*(f*x + e) - (6*(f*x + e)^2*a*b*d^2 - 6*b^2*d^2*e +
 6*b^2*c*d*f - 6*(2*a*b*d^2*e - 2*a*b*c*d*f - b^2*d^2)*(f*x + e) - 6*((f*x
+ e)^2*a*b*d^2 - b^2*d^2*e + b^2*c*d*f - (2*a*b*d^2*e - 2*a*b*c*d*f - b^2*d
^2)*(f*x + e))*cos(2*f*x + 2*e) - (6*I*(f*x + e)^2*a*b*d^2 - 6*I*b^2*d^2*e
+ 6*I*b^2*c*d*f + (-12*I*a*b*d^2*e + 12*I*a*b*c*d*f + 6*I*b^2*d^2)*(f*x + e
))*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) + (6*b^2*d^2*e
 - 6*b^2*c*d*f - 6*(b^2*d^2*e - b^2*c*d*f)*cos(2*f*x + 2*e) + (-6*I*b^2*d^2
*e + 6*I*b^2*c*d*f)*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) -
1) + (6*(f*x + e)^2*a*b*d^2 - 6*(2*a*b*d^2*e - 2*a*b*c*d*f - b^2*d^2)*(f*x
+ e) - 6*((f*x + e)^2*a*b*d^2 - (2*a*b*d^2*e - 2*a*b*c*d*f - b^2*d^2)*(f*x
+ e))*cos(2*f*x + 2*e) + (-6*I*(f*x + e)^2*a*b*d^2 + (12*I*a*b*d^2*e - 12*I
*a*b*c*d*f - 6*I*b^2*d^2)*(f*x + e))*sin(2*f*x + 2*e))*arctan2(sin(f*x + e)
, -cos(f*x + e) + 1) - ((2*a*b - I*b^2)*(f*x + e)^3*d^2 + (6*b^2*d^2 - (6*a
*b - 3*I*b^2)*d^2*e + (6*a*b - 3*I*b^2)*c*d*f)*(f*x + e)^2 - (3*I*b^2*d^2*e
^2 + 3*I*b^2*c^2*f^2 + 12*b^2*d^2*e - 6*(I*b^2*c*d*e + 2*b^2*c*d)*f)*(f*x +
 e))*cos(2*f*x + 2*e) + (12*(f*x + e)*a*b*d^2 - 12*a*b*d^2*e + 12*a*b*c*d*f
 + 6*b^2*d^2 - 6*(2*(f*x + e)*a*b*d^2 - 2*a*b*d^2*e + 2*a*b*c*d*f + b^2*d^2
)*cos(2*f*x + 2*e) + (-12*I*(f*x + e)*a*b*d^2 + 12*I*a*b*d^2*e - 12*I*a*b*c
*d*f - 6*I*b^2*d^2)*sin(2*f*x + 2*e))*dilog(-e^(I*f*x + I*e)) + (12*(f*x +
e)*a*b*d^2 - 12*a*b*d^2*e + 12*a*b*c*d*f + 6*b^2*d^2 - 6*(2*(f*x + e)*a*b*d
^2 - 2*a*b*d^2*e + 2*a*b*c*d*f + b^2*d^2)*cos(2*f*x + 2*e) + (-12*I*(f*x +
e)*a*b*d^2 + 12*I*a*b*d^2*e - 12*I*a*b*c*d*f - 6*I*b^2*d^2)*sin(2*f*x + 2*e
))*dilog(e^(I*f*x + I*e)) + (3*I*(f*x + e)^2*a*b*d^2 - 3*I*b^2*d^2*e + 3*I*
b^2*c*d*f + (-6*I*a*b*d^2*e + 6*I*a*b*c*d*f + 3*I*b^2*d^2)*(f*x + e) + (-3*
I*(f*x + e)^2*a*b*d^2 + 3*I*b^2*d^2*e - 3*I*b^2*c*d*f + (6*I*a*b*d^2*e - 6*
I*a*b*c*d*f - 3*I*b^2*d^2)*(f*x + e))*cos(2*f*x + 2*e) + 3*((f*x + e)^2*a*b
*d^2 - b^2*d^2*e + b^2*c*d*f - (2*a*b*d^2*e - 2*a*b*c*d*f - b^2*d^2)*(f*x +
 e))*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e)
 + 1) + (3*I*(f*x + e)^2*a*b*d^2 - 3*I*b^2*d^2*e + 3*I*b^2*c*d*f + (-6*I*a*
b*d^2*e + 6*I*a*b*c*d*f + 3*I*b^2*d^2)*(f*x + e) + (-3*I*(f*x + e)^2*a*b*d^
2 + 3*I*b^2*d^2*e - 3*I*b^2*c*d*f + (6*I*a*b*d^2*e - 6*I*a*b*c*d*f - 3*I*b^
2*d^2)*(f*x + e))*cos(2*f*x + 2*e) + 3*((f*x + e)^2*a*b*d^2 - b^2*d^2*e + b
^2*c*d*f - (2*a*b*d^2*e - 2*a*b*c*d*f - b^2*d^2)*(f*x + e))*sin(2*f*x + 2*e
))*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) + (-12*I*a*b*d
^2*cos(2*f*x + 2*e) + 12*a*b*d^2*sin(2*f*x + 2*e) + 12*I*a*b*d^2)*polylog(3
, -e^(I*f*x + I*e)) + (-12*I*a*b*d^2*cos(2*f*x + 2*e) + 12*a*b*d^2*sin(2*f*
x + 2*e) + 12*I*a*b*d^2)*polylog(3, e^(I*f*x + I*e)) + ((-2*I*a*b - b^2)*(f
*x + e)^3*d^2 + (-6*I*b^2*d^2 - 3*(-2*I*a*b - b^2)*d^2*e - 3*(2*I*a*b + b^2
)*c*d*f)*(f*x + e)^2 - (3*b^2*d^2*e^2 + 3*b^2*c^2*f^2 - 12*I*b^2*d^2*e - (6
*b^2*c*d*e - 12*I*b^2*c*d)*f)*(f*x + e))*sin(2*f*x + 2*e))/(-3*I*f^2*cos(2*
f*x + 2*e) + 3*f^2*sin(2*f*x + 2*e) + 3*I*f^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\cot\left(e + f\,x\right)\right)^2 (c + d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^2*(c + d*x)^2,x)
```

```
[Out] int((a + b*cot(e + f*x))^2*(c + d*x)^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot\left(e + fx\right)\right)^2 (c + dx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*cot(f*x+e))**2,x)
[Out] Integral((a + b*cot(e + f*x))**2*(c + d*x)**2, x)
```

3.44 $\int (c+dx)(a+b\cot(e+fx))^2\,dx$

Optimal. Leaf size=137

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{iab(c+dx)^2}{d}-\frac{iabd\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{f^2}-\frac{b^2(c+dx)\cot(e+fx)}{f}-b^2cx+\frac{b^2d\,l}{}$$

```
[Out] -b^2*c*x-1/2*b^2*d*x^2+1/2*a^2*(d*x+c)^2/d-I*a*b*(d*x+c)^2/d-b^2*(d*x+c)*co
t(f*x+e)/f+2*a*b*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f+b^2*d*ln(sin(f*x+e))/f^2-
I*a*b*d*polylog(2,exp(2*I*(f*x+e)))/f^2
```

Rubi [A] time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3722, 3717, 2190, 2279, 2391, 3720, 3475}

$$-\frac{iabd\,\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{f^2}+\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{iab(c+dx)^2}{d}-\frac{b^2(c+dx)\cot(e+fx)}{f}-b$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] -(b^2*c*x) - (b^2*d*x^2)/2 + (a^2*(c + d*x)^2)/(2*d) - (I*a*b*(c + d*x)^2)/
d - (b^2*(c + d*x)*Cot[e + f*x])/f + (2*a*b*(c + d*x)*Log[1 - E^((2*I)*(e +
 f*x))])/f + (b^2*d*Log[Sin[e + f*x]])/f^2 - (I*a*b*d*PolyLog[2, E^((2*I)*(
e + f*x))])/f^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)(a+b\cot(e+fx))^2\,dx &= \int \left(a^2(c+dx)+2ab(c+dx)\cot(e+fx)+b^2(c+dx)\cot^2(e+fx)\right)dx \\
&= \frac{a^2(c+dx)^2}{2d}+(2ab)\int (c+dx)\cot(e+fx)\,dx+b^2\int (c+dx)\cot^2(e+fx) \\
&= \frac{a^2(c+dx)^2}{2d}-\frac{iab(c+dx)^2}{d}-\frac{b^2(c+dx)\cot(e+fx)}{f}-(4iab)\int \frac{e^{2i(e+fx)}(c \\
&= -b^2cx-\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{iab(c+dx)^2}{d}-\frac{b^2(c+dx)\cot(e+fx)}{f}+ \\
&= -b^2cx-\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{iab(c+dx)^2}{d}-\frac{b^2(c+dx)\cot(e+fx)}{f}+ \\
&= -b^2cx-\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{iab(c+dx)^2}{d}-\frac{b^2(c+dx)\cot(e+fx)}{f}+
\end{aligned}$$

Mathematica [A] time = 2.25, size = 200, normalized size = 1.46

$$\frac{\sin(e+fx)(a+b\cot(e+fx))^2\left(\sin(e+fx)\left(-\left((e+fx)\left(a^2(-2cf+de-dfx)+2iabd(e+fx)+b^2(2cf-de-\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((a + b*Cot[e + f*x])^2*Sin[e + f*x]*(-2*b^2*f*(c + d*x)*Cos[e + f*x] + (-(
(e + f*x)*((2*I)*a*b*d*(e + f*x) + a^2*(d*e - 2*c*f - d*f*x) + b^2*(-(d*e)
+ 2*c*f + d*f*x))) + 4*a*b*d*(e + f*x)*Log[1 - E^((2*I)*(e + f*x))] + 2*b*(
b*d - 2*a*d*e + 2*a*c*f)*Log[Sin[e + f*x]])*Sin[e + f*x] - (2*I)*a*b*d*Poly
Log[2, E^((2*I)*(e + f*x))]*Sin[e + f*x]))/(2*f^2*(b*Cos[e + f*x] + a*Sin[e
 + f*x])^2)
```

fricas [B] time = 0.88, size = 378, normalized size = 2.76

$$-\frac{2\,b^2dfx+i\,abd\mathrm{Li}_2\left(\cos\left(2\,fx+2\,e\right)+i\,\sin\left(2\,fx+2\,e\right)\right)\sin\left(2\,fx+2\,e\right)-i\,abd\mathrm{Li}_2\left(\cos\left(2\,fx+2\,e\right)-i\,\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*d*f*x + I*a*b*d*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e))*si
n(2*f*x + 2*e) - I*a*b*d*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e))*sin(2
```

```
*f*x + 2*e) + 2*b^2*c*f + (2*a*b*d*e - 2*a*b*c*f - b^2*d)*log(-1/2*cos(2*f*
x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x + 2*e) + (2*a*b*d*e - 2*
a*b*c*f - b^2*d)*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2)*
sin(2*f*x + 2*e) - 2*(a*b*d*f*x + a*b*d*e)*log(-cos(2*f*x + 2*e) + I*sin(2*
f*x + 2*e) + 1)*sin(2*f*x + 2*e) - 2*(a*b*d*f*x + a*b*d*e)*log(-cos(2*f*x +
 2*e) - I*sin(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e) + 2*(b^2*d*f*x + b^2*c*f)*
cos(2*f*x + 2*e) - ((a^2 - b^2)*d*f^2*x^2 + 2*(a^2 - b^2)*c*f^2*x)*sin(2*f*
x + 2*e))/(f^2*sin(2*f*x + 2*e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\left(b\cot\left(fx+e\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*cot(f*x + e) + a)^2, x)
```

maple [B] time = 1.35, size = 365, normalized size = 2.66

$$-\frac{4ibadex}{f}-\frac{2ibad\,\mathrm{polylog}\left(2,\mathrm{e}^{i\left(fx+e\right)}\right)}{f^2}+\frac{a^2d\,x^2}{2}-\frac{b^2d\,x^2}{2}+a^2cx-b^2cx-\frac{2ib^2\,(dx+c)}{f\left(\mathrm{e}^{2i\left(fx+e\right)}-1\right)}+\frac{b^2d\ln\left(\mathrm{e}^{i\left(fx+e\right)}+1\right)}{f^2}+\frac{b^2d\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*cot(f*x+e))^2,x)
```

```
[Out] -4*I/f*b*a*d*e*x-2*I*b^2*(d*x+c)/f/(exp(2*I*(f*x+e))-1)+1/2*a^2*d*x^2-1/2*b
^2*d*x^2+a^2*c*x-b^2*c*x-2*I/f^2*b*a*d*polylog(2,-exp(I*(f*x+e)))+1/f^2*b^2
*d*ln(exp(I*(f*x+e))+1)+1/f^2*b^2*d*ln(exp(I*(f*x+e))-1)-2/f^2*b^2*d*ln(exp
(I*(f*x+e)))-4/f*b*a*c*ln(exp(I*(f*x+e)))+2/f*b*a*c*ln(exp(I*(f*x+e))+1)+2/
f*b*a*c*ln(exp(I*(f*x+e))-1)+4/f^2*b*a*d*e*ln(exp(I*(f*x+e)))-2/f^2*b*a*d*e
*ln(exp(I*(f*x+e))-1)-2*I/f^2*b*a*d*e^2+2/f*b*ln(1-exp(I*(f*x+e)))*a*d*x+2/
f^2*b*ln(1-exp(I*(f*x+e)))*a*d*e-I*a*b*d*x^2+2/f*b*ln(exp(I*(f*x+e))+1)*a*d
*x+2*I*a*b*c*x-2*I/f^2*b*a*d*polylog(2,exp(I*(f*x+e)))
```

maxima [B] time = 1.45, size = 776, normalized size = 5.66

$$\frac{2\left(fx+e\right)a^2c+\frac{\left(fx+e\right)^2a^2d}{f}-\frac{2\left(fx+e\right)a^2de}{f}+4\,abc\log\left(\sin\left(fx+e\right)\right)-\frac{4\,abde\log\left(\sin\left(fx+e\right)\right)}{f}+\frac{2\left(\left(2\,ab-i\,b^2\right)\left(fx+e\right)^2d+4\,b^2de-4\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a^2*c + (f*x + e)^2*a^2*d/f - 2*(f*x + e)*a^2*d*e/f + 4*a*
b*c*log(sin(f*x + e)) - 4*a*b*d*e*log(sin(f*x + e))/f + 2*((2*a*b - I*b^2)*
(f*x + e)^2*d + 4*b^2*d*e - 4*b^2*c*f + (2*I*b^2*d*e - 2*I*b^2*c*f)*(f*x +
e) - (4*(f*x + e)*a*b*d + 2*b^2*d - 2*(2*(f*x + e)*a*b*d + b^2*d)*cos(2*f*x
 + 2*e) - (4*I*(f*x + e)*a*b*d + 2*I*b^2*d)*sin(2*f*x + 2*e))*arctan2(sin(f
*x + e), cos(f*x + e) + 1) + 2*(b^2*d*cos(2*f*x + 2*e) + I*b^2*d*sin(2*f*x
+ 2*e) - b^2*d)*arctan2(sin(f*x + e), cos(f*x + e) - 1) - (4*(f*x + e)*a*b*
d*cos(2*f*x + 2*e) + 4*I*(f*x + e)*a*b*d*sin(2*f*x + 2*e) - 4*(f*x + e)*a*b
*d)*arctan2(sin(f*x + e), -cos(f*x + e) + 1) - ((2*a*b - I*b^2)*(f*x + e)^2
*d - (-2*I*b^2*d*e + 2*I*b^2*c*f - 4*b^2*d)*(f*x + e))*cos(2*f*x + 2*e) - 4
*(a*b*d*cos(2*f*x + 2*e) + I*a*b*d*sin(2*f*x + 2*e) - a*b*d)*dilog(-e^(I*f*
x + I*e)) - 4*(a*b*d*cos(2*f*x + 2*e) + I*a*b*d*sin(2*f*x + 2*e) - a*b*d)*d
```

```
ilog(e^(I*f*x + I*e)) + (2*I*(f*x + e)*a*b*d + I*b^2*d + (-2*I*(f*x + e)*a*
b*d - I*b^2*d)*cos(2*f*x + 2*e) + (2*(f*x + e)*a*b*d + b^2*d)*sin(2*f*x + 2
*e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + (2*I*(f*x
+ e)*a*b*d + I*b^2*d + (-2*I*(f*x + e)*a*b*d - I*b^2*d)*cos(2*f*x + 2*e) +
(2*(f*x + e)*a*b*d + b^2*d)*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f*x
+ e)^2 - 2*cos(f*x + e) + 1) + ((-2*I*a*b - b^2)*(f*x + e)^2*d + (2*b^2*d*e
 - 2*b^2*c*f - 4*I*b^2*d)*(f*x + e))*sin(2*f*x + 2*e))/(-2*I*f*cos(2*f*x +
2*e) + 2*f*sin(2*f*x + 2*e) + 2*I*f))/f
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b\cot\left(e + f\,x\right)\right)^2\,(c + d\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^2*(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))^2*(c + d*x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot\left(e + f\,x\right)\right)^2\,(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**2*(c + d*x), x)
```

3.45 $\int \frac{(a+b\cot(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\cot(e+fx))^2}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))^2/(d*x+c),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cot(e+fx))^2}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])^2/(c + d*x),x]
```

```
[Out] Defer[Int][(a + b*Cot[e + f*x])^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{(a+b\cot(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\cot(e+fx))^2}{c+dx}\,dx$$

Mathematica [A] time = 19.85, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(e+fx))^2}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x),x]
```

```
[Out] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x), x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\cot\left(fx+e\right)^2+2\,ab\cot\left(fx+e\right)+a^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*cot(f*x + e)^2 + 2*a*b*cot(f*x + e) + a^2)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\cot\left(fx+e\right)+a\right)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e) + a)^2/(d*x + c), x)
```

maple [A] time = 5.79, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\cot\left(fx + e\right)\right)^2}{dx + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))^2/(d*x+c),x)
[Out] int((a+b*cot(f*x+e))^2/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\cot\left(e + f\,x\right)\right)^2}{c + d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^2/(c + d*x),x)
[Out] int((a + b*cot(e + f*x))^2/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\cot\left(e + fx\right)\right)^2}{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))**2/(d*x+c),x)
[Out] Integral((a + b*cot(e + f*x))**2/(c + d*x), x)
```

3.46 $\int \frac{(a+b\cot(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\cot(e+fx))^2}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))^2/(d*x+c)^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cot(e+fx))^2}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(a + b*Cot[e + f*x])^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{(a+b\cot(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\cot(e+fx))^2}{(c+dx)^2}\,dx$$

Mathematica [A] time = 16.28, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(e+fx))^2}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x)^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\cot\left(fx+e\right)^2+2\,ab\cot\left(fx+e\right)+a^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cot(f*x + e)^2 + 2*a*b*cot(f*x + e) + a^2)/(d^2*x^2 + 2*c*d*x
 + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\cot\left(fx+e\right)+a\right)^2}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e) + a)^2/(d*x + c)^2, x)
```

maple [A] time = 7.24, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\cot\left(fx + e\right)\right)^2}{\left(dx + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] int((a+b*cot(f*x+e))^2/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(a^2 - b^2\right)df x + 2\,b^2 d\sin\left(2\,fx + 2\,e\right) + \left(a^2 - b^2\right)cf + \left(\left(a^2 - b^2\right)df x + \left(a^2 - b^2\right)cf\right)\cos\left(2\,fx + 2\,e\right)^2 + \left(\left(a^2 -\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((a^2 - b^2)*d*f*x + 2*b^2*d*sin(2*f*x + 2*e) + (a^2 - b^2)*c*f + ((a^2 -
b^2)*d*f*x + (a^2 - b^2)*c*f)*cos(2*f*x + 2*e)^2 + ((a^2 - b^2)*d*f*x + (a^
2 - b^2)*c*f)*sin(2*f*x + 2*e)^2 - 2*((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*
cos(2*f*x + 2*e) + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2 + 2*c*d^
2*f*x + c^2*d*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*s
in(2*f*x + 2*e)^2 - 2*(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e))
*integrate(2*(a*b*d*f*x + a*b*c*f - b^2*d)*sin(f*x + e)/(d^3*f*x^3 + 3*c*d^
2*f*x^2 + 3*c^2*d*f*x + c^3*f + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x +
c^3*f)*cos(f*x + e)^2 + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*s
in(f*x + e)^2 + 2*(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*cos(f*x
 + e)), x) - (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2 + 2*c*d^2*f*x
+ c^2*d*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*sin(2*f
*x + 2*e)^2 - 2*(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e))*integ
rate(2*(a*b*d*f*x + a*b*c*f - b^2*d)*sin(f*x + e)/(d^3*f*x^3 + 3*c*d^2*f*x^
2 + 3*c^2*d*f*x + c^3*f + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)
*cos(f*x + e)^2 + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*sin(f*x
 + e)^2 - 2*(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*cos(f*x + e))
, x))/(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d
*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*sin(2*f*x + 2*
e)^2 - 2*(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\cot\left(e + f\,x\right)\right)^2}{\left(c + d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^2/(c + d*x)^2,x)
```

```
[Out] int((a + b*cot(e + f*x))^2/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\cot\left(e + fx\right)\right)^2}{\left(c + dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))**2/(d*x+c)**2,x)
[Out] Integral((a + b*cot(e + f*x))**2/(c + d*x)**2, x)
```

3.47 $\int (c+dx)^3(a+b\cot(e+fx))^3\,dx$

Optimal. Leaf size=603

$$\frac{a^3(c+dx)^4}{4d}+\frac{9a^2bd^2(c+dx)\mathrm{Li}_3\left(e^{2i(e+fx)}\right)}{2f^3}-\frac{9ia^2bd(c+dx)^2\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{2f^2}+\frac{3a^2b(c+dx)^3\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{3i}{}$$

```
[Out] 1/4*I*b^3*(d*x+c)^4/d-3*I*a*b^2*(d*x+c)^3/f-1/2*b^3*(d*x+c)^3/f+1/4*a^3*(d*
x+c)^4/d-3/2*I*b^3*d^3*polylog(2,exp(2*I*(f*x+e)))/f^4-3/4*a*b^2*(d*x+c)^4/
d-9/2*I*a^2*b*d*(d*x+c)^2*polylog(2,exp(2*I*(f*x+e)))/f^2-3/2*b^3*d*(d*x+c)
^2*cot(f*x+e)/f^2-3*a*b^2*(d*x+c)^3*cot(f*x+e)/f-1/2*b^3*(d*x+c)^3*cot(f*x+
e)^2/f+3*b^3*d^2*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f^3+9*a*b^2*d*(d*x+c)^2*ln(
1-exp(2*I*(f*x+e)))/f^2+3*a^2*b*(d*x+c)^3*ln(1-exp(2*I*(f*x+e)))/f-b^3*(d*x
+c)^3*ln(1-exp(2*I*(f*x+e)))/f+3/2*I*b^3*d*(d*x+c)^2*polylog(2,exp(2*I*(f*x
+e)))/f^2-3/2*I*b^3*d*(d*x+c)^2/f^2-3/4*I*a^2*b*(d*x+c)^4/d-3/4*I*b^3*d^3*p
olylog(4,exp(2*I*(f*x+e)))/f^4+9/2*a*b^2*d^3*polylog(3,exp(2*I*(f*x+e)))/f^
4+9/2*a^2*b*d^2*(d*x+c)*polylog(3,exp(2*I*(f*x+e)))/f^3-3/2*b^3*d^2*(d*x+c)
*polylog(3,exp(2*I*(f*x+e)))/f^3+9/4*I*a^2*b*d^3*polylog(4,exp(2*I*(f*x+e))
)/f^4-9*I*a*b^2*d^2*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^3
```

Rubi [A] time = 0.98, antiderivative size = 603, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3722, 3717, 2190, 2531, 6609, 2282, 6589, 3720, 32, 2279, 2391}

$$\frac{9a^2bd^2(c+dx)\mathrm{PolyLog}\left(3,e^{2i(e+fx)}\right)}{2f^3}-\frac{9ia^2bd(c+dx)^2\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{2f^2}+\frac{9ia^2bd^3\mathrm{PolyLog}\left(4,e^{2i(e+fx)}\right)}{4f^4}-\frac{9i}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Cot[e + f*x])^3,x]
```

```
[Out] (((-3*I)/2)*b^3*d*(c + d*x)^2)/f^2 - ((3*I)*a*b^2*(c + d*x)^3)/f - (b^3*(c
+ d*x)^3)/(2*f) + (a^3*(c + d*x)^4)/(4*d) - (((3*I)/4)*a^2*b*(c + d*x)^4)/d
 - (3*a*b^2*(c + d*x)^4)/(4*d) + ((I/4)*b^3*(c + d*x)^4)/d - (3*b^3*d*(c +
d*x)^2*Cot[e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)^3*Cot[e + f*x])/f - (b^3*
(c + d*x)^3*Cot[e + f*x]^2)/(2*f) + (3*b^3*d^2*(c + d*x)*Log[1 - E^((2*I)*(
e + f*x))])/f^3 + (9*a*b^2*d*(c + d*x)^2*Log[1 - E^((2*I)*(e + f*x))])/f^2
+ (3*a^2*b*(c + d*x)^3*Log[1 - E^((2*I)*(e + f*x))])/f - (b^3*(c + d*x)^3*L
og[1 - E^((2*I)*(e + f*x))])/f - (((3*I)/2)*b^3*d^3*PolyLog[2, E^((2*I)*(e
+ f*x))])/f^4 - ((9*I)*a*b^2*d^2*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])
/f^3 - (((9*I)/2)*a^2*b*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(e + f*x))])/f^2
+ (((3*I)/2)*b^3*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (9*a*
b^2*d^3*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^4) + (9*a^2*b*d^2*(c + d*x)*P
olyLog[3, E^((2*I)*(e + f*x))])/(2*f^3) - (3*b^3*d^2*(c + d*x)*PolyLog[3, E
^((2*I)*(e + f*x))])/(2*f^3) + (((9*I)/4)*a^2*b*d^3*PolyLog[4, E^((2*I)*(e
+ f*x))])/f^4 - (((3*I)/4)*b^3*d^3*PolyLog[4, E^((2*I)*(e + f*x))])/f^4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3(a+b\cot(e+fx))^3\,dx = \int \left(a^3(c+dx)^3 + 3a^2b(c+dx)^3\cot(e+fx) + 3ab^2(c+dx)^3\cot^2(e+fx)\right.$$

$$= \frac{a^3(c+dx)^4}{4d} + \left(3a^2b\right)\int (c+dx)^3\cot(e+fx)\,dx + \left(3ab^2\right)\int (c+dx)^3\cot^2$$

$$= \frac{a^3(c+dx)^4}{4d} - \frac{3ia^2b(c+dx)^4}{4d} - \frac{3ab^2(c+dx)^3\cot(e+fx)}{f} - \frac{b^3(c+dx)^3\,c}{2f}$$

$$= -\frac{3iab^2(c+dx)^3}{f} + \frac{a^3(c+dx)^4}{4d} - \frac{3ia^2b(c+dx)^4}{4d} - \frac{3ab^2(c+dx)^4}{4d} + \frac{ib^3(c$$

$$= -\frac{3ib^3d(c+dx)^2}{2f^2} - \frac{3iab^2(c+dx)^3}{f} - \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} - \frac{3ia^2b(c$$

$$= -\frac{3ib^3d(c+dx)^2}{2f^2} - \frac{3iab^2(c+dx)^3}{f} - \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} - \frac{3ia^2b(c$$

$$= -\frac{3ib^3d(c+dx)^2}{2f^2} - \frac{3iab^2(c+dx)^3}{f} - \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} - \frac{3ia^2b(c$$

$$= -\frac{3ib^3d(c+dx)^2}{2f^2} - \frac{3iab^2(c+dx)^3}{f} - \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} - \frac{3ia^2b(c$$

$$= -\frac{3ib^3d(c+dx)^2}{2f^2} - \frac{3iab^2(c+dx)^3}{f} - \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} - \frac{3ia^2b(c$$

Mathematica [B] time = 8.54, size = 3045, normalized size = 5.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*(a + b*Cot[e + f*x])^3,x]
```

```
[Out] ((-(b^3*c^3) - 3*b^3*c^2*d*x - 3*b^3*c*d^2*x^2 - b^3*d^3*x^3)*Csc[e + f*x]^
2)/(2*f) - (3*a*b^2*d^3*E^(I*e)*Csc[e]*((2*f^3*x^3)/E^((2*I)*e) + (3*I)*(1
 - E^((-2*I)*e))*f^2*x^2*Log[1 - E^((-I)*(e + f*x))] + (3*I)*(1 - E^((-2*I)*
e))*f^2*x^2*Log[1 + E^((-I)*(e + f*x))] - (6*(-1 + E^((2*I)*e))*(f*x*PolyLo
g[2, -E^((-I)*(e + f*x))] - I*PolyLog[3, -E^((-I)*(e + f*x))]))/E^((2*I)*e)
 - (6*(-1 + E^((2*I)*e))*(f*x*PolyLog[2, E^((-I)*(e + f*x))] - I*PolyLog[3,
 E^((-I)*(e + f*x))]))/E^((2*I)*e)))/(2*f^4) - (3*a^2*b*c*d^2*E^(I*e)*Csc[e
]*((2*f^3*x^3)/E^((2*I)*e) + (3*I)*(1 - E^((-2*I)*e))*f^2*x^2*Log[1 - E^((-
I)*(e + f*x))] + (3*I)*(1 - E^((-2*I)*e))*f^2*x^2*Log[1 + E^((-I)*(e + f*x)
)] - (6*(-1 + E^((2*I)*e))*(f*x*PolyLog[2, -E^((-I)*(e + f*x))] - I*PolyLog
[3, -E^((-I)*(e + f*x))]))/E^((2*I)*e) - (6*(-1 + E^((2*I)*e))*(f*x*PolyLog
[2, E^((-I)*(e + f*x))] - I*PolyLog[3, E^((-I)*(e + f*x))]))/E^((2*I)*e)))/
(2*f^3) + (b^3*c*d^2*E^(I*e)*Csc[e]*((2*f^3*x^3)/E^((2*I)*e) + (3*I)*(1 - E
^((-2*I)*e))*f^2*x^2*Log[1 - E^((-I)*(e + f*x))] + (3*I)*(1 - E^((-2*I)*e))
*f^2*x^2*Log[1 + E^((-I)*(e + f*x))] - (6*(-1 + E^((2*I)*e))*(f*x*PolyLog[2
, -E^((-I)*(e + f*x))] - I*PolyLog[3, -E^((-I)*(e + f*x))]))/E^((2*I)*e) -
(6*(-1 + E^((2*I)*e))*(f*x*PolyLog[2, E^((-I)*(e + f*x))] - I*PolyLog[3, E^
((-I)*(e + f*x))]))/E^((2*I)*e)))/(2*f^3) - (3*a^2*b*d^3*E^(I*e)*Csc[e]*((f
^4*x^4)/E^((2*I)*e) + (2*I)*(1 - E^((-2*I)*e))*f^3*x^3*Log[1 - E^((-I)*(e +
 f*x))] + (2*I)*(1 - E^((-2*I)*e))*f^3*x^3*Log[1 + E^((-I)*(e + f*x))] - (6
```

```
*(-1 + E^((2*I)*e))*(f^2*x^2*PolyLog[2, -E^((-I)*(e + f*x))] - (2*I)*f*x*Po
lyLog[3, -E^((-I)*(e + f*x))] - 2*PolyLog[4, -E^((-I)*(e + f*x))]))/E^((2*I
)*e) - (6*(-1 + E^((2*I)*e))*(f^2*x^2*PolyLog[2, E^((-I)*(e + f*x))] - (2*I
)*f*x*PolyLog[3, E^((-I)*(e + f*x))] - 2*PolyLog[4, E^((-I)*(e + f*x))]))/E
^((2*I)*e)))/(4*f^4) + (b^3*d^3*E^(I*e)*Csc[e]*((f^4*x^4)/E^((2*I)*e) + (2*
I)*(1 - E^((-2*I)*e))*f^3*x^3*Log[1 - E^((-I)*(e + f*x))] + (2*I)*(1 - E^((
-2*I)*e))*f^3*x^3*Log[1 + E^((-I)*(e + f*x))] - (6*(-1 + E^((2*I)*e))*(f^2*
x^2*PolyLog[2, -E^((-I)*(e + f*x))] - (2*I)*f*x*PolyLog[3, -E^((-I)*(e + f*
x))] - 2*PolyLog[4, -E^((-I)*(e + f*x))]))/E^((2*I)*e) - (6*(-1 + E^((2*I)*
e))*(f^2*x^2*PolyLog[2, E^((-I)*(e + f*x))] - (2*I)*f*x*PolyLog[3, E^((-I)*
(e + f*x))] - 2*PolyLog[4, E^((-I)*(e + f*x))]))/E^((2*I)*e)))/(4*f^4) + (3
*b^3*c*d^2*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*S
in[e]))/(f^3*(Cos[e]^2 + Sin[e]^2)) + (9*a*b^2*c^2*d*Csc[e]*(-(f*x*Cos[e])
+ Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2
)) + (3*a^2*b*c^3*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[
f*x]]*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) - (b^3*c^3*Csc[e]*(-(f*x*Cos[e]) +
 Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2))
+ (3*x^2*(-(a^3*c^2*d) + (3*I)*a^2*b*c^2*d + 3*a*b^2*c^2*d - I*b^3*c^2*d +
a^3*c^2*d*Cos[2*e] + (3*I)*a^2*b*c^2*d*Cos[2*e] - 3*a*b^2*c^2*d*Cos[2*e] -
I*b^3*c^2*d*Cos[2*e] + I*a^3*c^2*d*Sin[2*e] - 3*a^2*b*c^2*d*Sin[2*e] - (3*I
)*a*b^2*c^2*d*Sin[2*e] + b^3*c^2*d*Sin[2*e]))/(2*(-1 + Cos[2*e] + I*Sin[2*e
])) + (x^3*(-(a^3*c*d^2) + (3*I)*a^2*b*c*d^2 + 3*a*b^2*c*d^2 - I*b^3*c*d^2
+ a^3*c*d^2*Cos[2*e] + (3*I)*a^2*b*c*d^2*Cos[2*e] - 3*a*b^2*c*d^2*Cos[2*e]
- I*b^3*c*d^2*Cos[2*e] + I*a^3*c*d^2*Sin[2*e] - 3*a^2*b*c*d^2*Sin[2*e] - (3
*I)*a*b^2*c*d^2*Sin[2*e] + b^3*c*d^2*Sin[2*e]))/(-1 + Cos[2*e] + I*Sin[2*e]
) + (x^4*(-(a^3*d^3) + (3*I)*a^2*b*d^3 + 3*a*b^2*d^3 - I*b^3*d^3 + a^3*d^3*
Cos[2*e] + (3*I)*a^2*b*d^3*Cos[2*e] - 3*a*b^2*d^3*Cos[2*e] - I*b^3*d^3*Cos[
2*e] + I*a^3*d^3*Sin[2*e] - 3*a^2*b*d^3*Sin[2*e] - (3*I)*a*b^2*d^3*Sin[2*e]
 + b^3*d^3*Sin[2*e]))/(4*(-1 + Cos[2*e] + I*Sin[2*e])) + x*(a^3*c^3 - 3*a*b
^2*c^3 + ((3*I)*a^2*b*c^3)/(-1 + Cos[2*e] + I*Sin[2*e]) + ((3*I)*a^2*b*c^3*
Cos[2*e] - 3*a^2*b*c^3*Sin[2*e])/(-1 + Cos[2*e] + I*Sin[2*e]) + ((-2*I)*b^3
*c^3*Cos[2*e] + 2*b^3*c^3*Sin[2*e])/((-1 + Cos[2*e] + I*Sin[2*e])*(1 + Cos[
2*e] + Cos[4*e] + I*Sin[2*e] + I*Sin[4*e])) + ((-2*I)*b^3*c^3*Cos[4*e] + 2*
b^3*c^3*Sin[4*e])/((-1 + Cos[2*e] + I*Sin[2*e])*(1 + Cos[2*e] + Cos[4*e] +
I*Sin[2*e] + I*Sin[4*e])) - (I*b^3*c^3)/(-1 + Cos[6*e] + I*Sin[6*e]) + ((-I
)*b^3*c^3*Cos[6*e] + b^3*c^3*Sin[6*e])/(-1 + Cos[6*e] + I*Sin[6*e])) + (3*C
sc[e]*Csc[e + f*x]*(b^3*c^2*d*Sin[f*x] + 2*a*b^2*c^3*f*Sin[f*x] + 2*b^3*c*d
^2*x*Sin[f*x] + 6*a*b^2*c^2*d*f*x*Sin[f*x] + b^3*d^3*x^2*Sin[f*x] + 6*a*b^2
*c*d^2*f*x^2*Sin[f*x] + 2*a*b^2*d^3*f*x^3*Sin[f*x]))/(2*f^2) - (3*b^3*d^3*C
sc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]
]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)
*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x
 + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])*Tan[e
])/Sqrt[1 + Tan[e]^2]))/(2*f^4*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)]) - (9*a
*b^2*c*d^2*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*A
rcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[
1 - E^((2*I)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]
*Log[Sin[f*x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e
]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))/(f^3*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2
)]) - (9*a^2*b*c^2*d*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*
(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan
[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcT
an[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + Ar
cTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))/(2*f^2*Sqrt[Sec[e]^2*(Cos[e]^
2 + Sin[e]^2)]) + (3*b^3*c^2*d*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2
+ ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x +
ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]
] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)
*(f*x + ArcTan[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))/(2*f^2*Sqrt[Sec[e]^
```

```
2*(Cos[e]^2 + Sin[e]^2)])
```

fricas [C] time = 0.71, size = 2749, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(a^3 - 3*a*b^2)*d^3*f^4*x^4 - 8*b^3*c^3*f^3 - 8*(b^3*d^3*f^3 - (a^3
 - 3*a*b^2)*c*d^2*f^4)*x^3 - 12*(2*b^3*c*d^2*f^3 - (a^3 - 3*a*b^2)*c^2*d*f^
4)*x^2 - 8*(3*b^3*c^2*d*f^3 - (a^3 - 3*a*b^2)*c^3*f^4)*x - 2*((a^3 - 3*a*b^
2)*d^3*f^4*x^4 + 4*(a^3 - 3*a*b^2)*c*d^2*f^4*x^3 + 6*(a^3 - 3*a*b^2)*c^2*d*
f^4*x^2 + 4*(a^3 - 3*a*b^2)*c^3*f^4*x)*cos(2*f*x + 2*e) - (6*I*(3*a^2*b - b
^3)*d^3*f^2*x^2 + 36*I*a*b^2*c*d^2*f + 6*I*b^3*d^3 + 6*I*(3*a^2*b - b^3)*c^
2*d*f^2 + 12*I*(3*a*b^2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x + (-6*I*(3*a^2
*b - b^3)*d^3*f^2*x^2 - 36*I*a*b^2*c*d^2*f - 6*I*b^3*d^3 - 6*I*(3*a^2*b - b
^3)*c^2*d*f^2 - 12*I*(3*a*b^2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x)*cos(2*f
*x + 2*e))*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) - (-6*I*(3*a^2*b -
b^3)*d^3*f^2*x^2 - 36*I*a*b^2*c*d^2*f - 6*I*b^3*d^3 - 6*I*(3*a^2*b - b^3)*c
^2*d*f^2 - 12*I*(3*a*b^2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x + (6*I*(3*a^2
*b - b^3)*d^3*f^2*x^2 + 36*I*a*b^2*c*d^2*f + 6*I*b^3*d^3 + 6*I*(3*a^2*b - b
^3)*c^2*d*f^2 + 12*I*(3*a*b^2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x)*cos(2*f
*x + 2*e))*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) + 4*(9*a*b^2*d^3*e^
2 - 3*b^3*d^3*e - (3*a^2*b - b^3)*d^3*e^3 + (3*a^2*b - b^3)*c^3*f^3 + 3*(3*
a*b^2*c^2*d - (3*a^2*b - b^3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2
 - (3*a^2*b - b^3)*c*d^2*e^2)*f - (9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b
 - b^3)*d^3*e^3 + (3*a^2*b - b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b - b
^3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b - b^3)*c*d^2*e
^2)*f)*cos(2*f*x + 2*e))*log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e)
 + 1/2) + 4*(9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b - b^3)*d^3*e^3 + (3*a
^2*b - b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b - b^3)*c^2*d*e)*f^2 - 3*(
6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b - b^3)*c*d^2*e^2)*f - (9*a*b^2*d^3*e
^2 - 3*b^3*d^3*e - (3*a^2*b - b^3)*d^3*e^3 + (3*a^2*b - b^3)*c^3*f^3 + 3*(3
*a*b^2*c^2*d - (3*a^2*b - b^3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^
2 - (3*a^2*b - b^3)*c*d^2*e^2)*f)*cos(2*f*x + 2*e))*log(-1/2*cos(2*f*x + 2*
e) - 1/2*I*sin(2*f*x + 2*e) + 1/2) + 4*((3*a^2*b - b^3)*d^3*f^3*x^3 - 9*a*b
^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b - b^3)*d^3*e^3 + 3*(3*a^2*b - b^3)*c^2*
d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b - b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*
c*d^2*e - (3*a^2*b - b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f +
 (3*a^2*b - b^3)*c^2*d*f^3)*x - ((3*a^2*b - b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*
e^2 + 3*b^3*d^3*e + (3*a^2*b - b^3)*d^3*e^3 + 3*(3*a^2*b - b^3)*c^2*d*e*f^2
 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b - b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e
 - (3*a^2*b - b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2
*b - b^3)*c^2*d*f^3)*x)*cos(2*f*x + 2*e))*log(-cos(2*f*x + 2*e) + I*sin(2*f
*x + 2*e) + 1) + 4*((3*a^2*b - b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d
^3*e + (3*a^2*b - b^3)*d^3*e^3 + 3*(3*a^2*b - b^3)*c^2*d*e*f^2 + 3*(3*a*b^2
*d^3*f^2 + (3*a^2*b - b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b -
 b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b - b^3)*c^2
*d*f^3)*x - ((3*a^2*b - b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e +
(3*a^2*b - b^3)*d^3*e^3 + 3*(3*a^2*b - b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^
2 + (3*a^2*b - b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b - b^3)*c
*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b - b^3)*c^2*d*f^3)
*x)*cos(2*f*x + 2*e))*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1) - (3*
I*(3*a^2*b - b^3)*d^3*cos(2*f*x + 2*e) - 3*I*(3*a^2*b - b^3)*d^3)*polylog(4
, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) - (-3*I*(3*a^2*b - b^3)*d^3*cos(2*
f*x + 2*e) + 3*I*(3*a^2*b - b^3)*d^3)*polylog(4, cos(2*f*x + 2*e) - I*sin(2
*f*x + 2*e)) + 6*(3*a*b^2*d^3 + (3*a^2*b - b^3)*d^3*f*x + (3*a^2*b - b^3)*c
*d^2*f - (3*a*b^2*d^3 + (3*a^2*b - b^3)*d^3*f*x + (3*a^2*b - b^3)*c*d^2*f)*
cos(2*f*x + 2*e))*polylog(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) + 6*(3*
```

```
a*b^2*d^3 + (3*a^2*b - b^3)*d^3*f*x + (3*a^2*b - b^3)*c*d^2*f - (3*a*b^2*d^
3 + (3*a^2*b - b^3)*d^3*f*x + (3*a^2*b - b^3)*c*d^2*f)*cos(2*f*x + 2*e))*po
lylog(3, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) - 12*(2*a*b^2*d^3*f^3*x^3 +
 2*a*b^2*c^3*f^3 + b^3*c^2*d*f^2 + (6*a*b^2*c*d^2*f^3 + b^3*d^3*f^2)*x^2 +
2*(3*a*b^2*c^2*d*f^3 + b^3*c*d^2*f^2)*x)*sin(2*f*x + 2*e))/(f^4*cos(2*f*x +
 2*e) - f^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3\left(b\cot\left(fx+e\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*(b*cot(f*x + e) + a)^3, x)
```

maple [B] time = 2.40, size = 3113, normalized size = 5.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*cot(f*x+e))^3,x)
```

```
[Out] -3*I*a^2*b*c*d^2*x^3-9/2*I*a^2*b*c^2*d*x^2-6/f^2*b^3*c^2*d*e*ln(exp(I*(f*x+
e)))-18/f^2*b^2*a*c^2*d*ln(exp(I*(f*x+e)))+9/f^4*b^2*a*d^3*e^2*ln(exp(I*(f*
x+e))-1)+9/f^2*b^2*a*c^2*d*ln(exp(I*(f*x+e))-1)-1/f*b^3*d^3*ln(exp(I*(f*x+e
))+1)*x^3-18*I/f^2*b*polylog(2,exp(I*(f*x+e)))*a^2*c*d^2*x-18*I/f^2*b*polyl
og(2,-exp(I*(f*x+e)))*a^2*c*d^2*x-18*I/f*b*a^2*c^2*d*e*x+a^3*c*d^2*x^3+18*I
/f^2*b*a^2*c*d^2*e^2*x-36*I/f^2*b^2*a*c*d^2*e*x-3/4*a*b^2*d^3*x^4+3/2*a^3*c
^2*d*x^2-3*b^2*a*c^3*x+b^2*(3*I*d^3*x^2*b+3*I*b*c^2*d+2*b*d^3*f*x^3*exp(2*I
*(f*x+e))-18*I*a*c^2*d*f*x*exp(2*I*(f*x+e))-6*I*b*c*d^2*x*exp(2*I*(f*x+e))-
3*I*b*d^3*x^2*exp(2*I*(f*x+e))+6*b*c*d^2*f*x^2*exp(2*I*(f*x+e))+6*I*a*d^3*f
*x^3+6*I*b*c*d^2*x-6*I*a*d^3*f*x^3*exp(2*I*(f*x+e))+6*b*c^2*d*f*x*exp(2*I*(
f*x+e))+18*I*a*c*d^2*f*x^2-3*I*b*c^2*d*exp(2*I*(f*x+e))+6*I*a*c^3*f+2*b*c^3
*f*exp(2*I*(f*x+e))+18*I*a*c^2*d*f*x-18*I*a*c*d^2*f*x^2*exp(2*I*(f*x+e))-6*
I*a*c^3*f*exp(2*I*(f*x+e)))/f^2/(exp(2*I*(f*x+e))-1)^2-I*b^3*c^3*x+2/f*b^3*
c^3*ln(exp(I*(f*x+e)))-1/f*b^3*c^3*ln(exp(I*(f*x+e))-1)-1/f*b^3*c^3*ln(exp(
I*(f*x+e))+1)-9/f^4*b^2*a*d^3*e^2*ln(1-exp(I*(f*x+e)))-3/f*b^3*ln(exp(I*(f*
x+e))+1)*c*d^2*x^2-3/f*b^3*ln(exp(I*(f*x+e))+1)*c^2*d*x-1/f*b^3*d^3*ln(1-ex
p(I*(f*x+e)))*x^3-1/f^4*b^3*d^3*ln(1-exp(I*(f*x+e)))*e^3+9/f^2*b^2*a*d^3*ln
(1-exp(I*(f*x+e)))*x^2+9/f^2*b^2*a*d^3*ln(exp(I*(f*x+e))+1)*x^2-3/f*b^3*ln(
1-exp(I*(f*x+e)))*c^2*d*x-3/f^2*b^3*ln(1-exp(I*(f*x+e)))*c^2*d*e+3/f*b*a^2*
d^3*ln(exp(I*(f*x+e))+1)*x^3-6*I/f^3*b*a^2*d^3*e^3*x-6*I/f^2*b^3*c*d^2*e^2*
x+12*I/f^3*b*a^2*c*d^2*e^3-9*I/f^2*b*a^2*c^2*d*polylog(2,exp(I*(f*x+e)))+6*
I/f^2*b^3*polylog(2,-exp(I*(f*x+e)))*c*d^2*x-18*I/f^3*b^2*a*c*d^2*e^2-18*I/
f^3*b^2*a*d^3*polylog(2,exp(I*(f*x+e)))*x-18*I/f^3*b^2*a*d^3*polylog(2,-exp
(I*(f*x+e)))*x+1/4*a^3*d^3*x^4+a^3*c^3*x-3*a*b^2*c*d^2*x^3-9/2*a*b^2*c^2*d*
x^2+1/4*I*b^3*d^3*x^4+I*b^3*c*d^2*x^3+9/f*b*ln(exp(I*(f*x+e))+1)*a^2*c*d^2*
x^2+9/f*b*ln(1-exp(I*(f*x+e)))*a^2*c^2*d*x+9/f^2*b*ln(1-exp(I*(f*x+e)))*a^2
*c^2*d*e-9/f^3*b*ln(1-exp(I*(f*x+e)))*a^2*c*d^2*e^2-18/f^3*b*a^2*c*d^2*e^2*
ln(exp(I*(f*x+e)))-18/f^4*b^2*a*d^3*e^2*ln(exp(I*(f*x+e)))+18/f^3*b*a^2*c*d
^2*polylog(3,exp(I*(f*x+e)))+18/f^3*b*a^2*c*d^2*polylog(3,-exp(I*(f*x+e)))-
3/f^3*b^3*c*d^2*e^2*ln(exp(I*(f*x+e))-1)+9/f^2*b^2*a*c^2*d*ln(exp(I*(f*x+e)
)+1)-6/f^3*b^3*d^3*polylog(3,-exp(I*(f*x+e)))*x+6/f^4*b^3*d^3*e*ln(exp(I*(f
*x+e)))+18/f^4*b^2*a*d^3*polylog(3,exp(I*(f*x+e)))+3/f*b*a^2*c^3*ln(exp(I*(
f*x+e))-1)+3/f*b*a^2*c^3*ln(exp(I*(f*x+e))+1)+3/f^3*b^3*c*d^2*ln(exp(I*(f*x
+e))-1)-6/f^3*b^3*c*d^2*polylog(3,-exp(I*(f*x+e)))-6/f*b*a^2*c^3*ln(exp(I*(
f*x+e)))+3/f^3*b^3*c*d^2*ln(exp(I*(f*x+e))+1)-6/f^3*b^3*c*d^2*polylog(3,exp
(I*(f*x+e)))-6/f^3*b^3*c*d^2*ln(exp(I*(f*x+e)))+18/f^4*b^2*a*d^3*polylog(3,
```

```
-exp(I*(f*x+e)))-3/f^4*b^3*d^3*e*ln(exp(I*(f*x+e))-1)-2/f^4*b^3*d^3*e^3*ln(
exp(I*(f*x+e)))+1/f^4*b^3*d^3*e^3*ln(exp(I*(f*x+e))-1)-3*I/f^4*b^3*d^3*poly
log(2,-exp(I*(f*x+e)))-3*I/f^4*b^3*d^3*e^2-3*I/f^2*b^3*d^3*x^2+3/2*I/f^4*b^
3*d^3*e^4+9/f^3*b*a^2*c*d^2*e^2*ln(exp(I*(f*x+e))-1)+18/f^2*b*a^2*c^2*d*e*l
n(exp(I*(f*x+e)))+36/f^3*b^2*a*c*d^2*e*ln(exp(I*(f*x+e)))-18/f^3*b^2*a*c*d^
2*e*ln(exp(I*(f*x+e))-1)-9/f^2*b*a^2*c^2*d*e*ln(exp(I*(f*x+e))-1)+3/f*b*a^2
*d^3*ln(1-exp(I*(f*x+e)))*x^3+3/f^4*b*a^2*d^3*ln(1-exp(I*(f*x+e)))*e^3-3/4*
I*a^2*b*d^3*x^4+3/2*I*b^3*c^2*d*x^2-3/f*b^3*ln(1-exp(I*(f*x+e)))*c*d^2*x^2-
6*I/f^4*b^3*d^3*polylog(4,exp(I*(f*x+e)))-6*I/f^4*b^3*d^3*polylog(4,-exp(I*
(f*x+e)))-3*I/f^4*b^3*d^3*polylog(2,exp(I*(f*x+e)))+6*I/f^2*b^3*polylog(2,e
xp(I*(f*x+e)))*c*d^2*x-18*I/f^3*b^2*a*c*d^2*polylog(2,exp(I*(f*x+e)))-18*I/
f^3*b^2*a*c*d^2*polylog(2,-exp(I*(f*x+e)))+6*I/f*b^3*c^2*d*e*x+18*I/f^3*b^2
*a*d^3*e^2*x-9*I/f^2*b*a^2*c^2*d*polylog(2,-exp(I*(f*x+e)))-18*I/f*b^2*a*c*
d^2*x^2-9*I/f^2*b*a^2*d^3*polylog(2,exp(I*(f*x+e)))*x^2-9*I/f^2*b*a^2*c^2*d
*e^2-9*I/f^2*b*a^2*d^3*polylog(2,-exp(I*(f*x+e)))*x^2+3*I*a^2*b*c^3*x+3/f^3
*b^3*d^3*ln(1-exp(I*(f*x+e)))*x+3/f^4*b^3*d^3*ln(1-exp(I*(f*x+e)))*e+3/f^3*
b^3*d^3*ln(exp(I*(f*x+e))+1)*x-6/f^3*b^3*d^3*polylog(3,exp(I*(f*x+e)))*x+18
/f^3*b*a^2*d^3*polylog(3,exp(I*(f*x+e)))*x+18/f^3*b*a^2*d^3*polylog(3,-exp(
I*(f*x+e)))*x+3/f^3*b^3*ln(1-exp(I*(f*x+e)))*c*d^2*e^2-3/f^4*b*a^2*d^3*e^3*
ln(exp(I*(f*x+e))-1)+6/f^4*b*a^2*d^3*e^3*ln(exp(I*(f*x+e)))+6/f^3*b^3*c*d^2
*e^2*ln(exp(I*(f*x+e)))+3/f^2*b^3*c^2*d*e*ln(exp(I*(f*x+e))-1)+12*I/f^4*b^2
*a*d^3*e^3+3*I/f^2*b^3*c^2*d*e^2+3*I/f^2*b^3*c^2*d*polylog(2,exp(I*(f*x+e))
)+3*I/f^2*b^3*c^2*d*polylog(2,-exp(I*(f*x+e)))-6*I/f^3*b^3*d^3*e*x-4*I/f^3*
b^3*c*d^2*e^3-9/2*I/f^4*b*a^2*d^3*e^4+18*I/f^4*b*a^2*d^3*polylog(4,exp(I*(f
*x+e)))+18*I/f^4*b*a^2*d^3*polylog(4,-exp(I*(f*x+e)))-6*I/f*b^2*a*d^3*x^3+3
*I/f^2*b^3*d^3*polylog(2,-exp(I*(f*x+e)))*x^2+3*I/f^2*b^3*d^3*polylog(2,exp
(I*(f*x+e)))*x^2+2*I/f^3*b^3*d^3*e^3*x+18/f^2*b^2*ln(1-exp(I*(f*x+e)))*a*c*
d^2*x+18/f^2*b^2*ln(exp(I*(f*x+e))+1)*a*c*d^2*x+9/f*b*ln(exp(I*(f*x+e))+1)*
a^2*c^2*d*x+18/f^3*b^2*ln(1-exp(I*(f*x+e)))*a*c*d^2*e+9/f*b*ln(1-exp(I*(f*x
+e)))*a^2*c*d^2*x^2
```

maxima [B] time = 35.80, size = 11204, normalized size = 18.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a^3*c^3 + (f*x + e)^4*a^3*d^3/f^3 - 4*(f*x + e)^3*a^3*d^3*
e/f^3 + 6*(f*x + e)^2*a^3*d^3*e^2/f^3 - 4*(f*x + e)*a^3*d^3*e^3/f^3 + 4*(f*
x + e)^3*a^3*c*d^2/f^2 - 12*(f*x + e)^2*a^3*c*d^2*e/f^2 + 12*(f*x + e)*a^3*
c*d^2*e^2/f^2 + 6*(f*x + e)^2*a^3*c^2*d/f - 12*(f*x + e)*a^3*c^2*d*e/f + 12
*a^2*b*c^3*log(sin(f*x + e)) - 12*a^2*b*d^3*e^3*log(sin(f*x + e))/f^3 + 36*
a^2*b*c*d^2*e^2*log(sin(f*x + e))/f^2 - 36*a^2*b*c^2*d*e*log(sin(f*x + e))/
f - 4*(24*a*b^2*d^3*e^3 - 24*a*b^2*c^3*f^3 + (3*a^2*b - 3*I*a*b^2 - b^3)*(f
*x + e)^4*d^3 - 12*b^3*d^3*e^2 - 4*((3*a^2*b - 3*I*a*b^2 - b^3)*d^3*e - (3*
a^2*b - 3*I*a*b^2 - b^3)*c*d^2*f)*(f*x + e)^3 + 6*((3*a^2*b - 3*I*a*b^2 - b
^3)*d^3*e^2 - 2*(3*a^2*b - 3*I*a*b^2 - b^3)*c*d^2*e*f + (3*a^2*b - 3*I*a*b^
2 - b^3)*c^2*d*f^2)*(f*x + e)^2 + 12*(6*a*b^2*c^2*d*e - b^3*c^2*d)*f^2 - 4*
((-3*I*a*b^2 - b^3)*d^3*e^3 + 3*(3*I*a*b^2 + b^3)*c*d^2*e^2*f + 3*(-3*I*a*b
^2 - b^3)*c^2*d*e*f^2 + (3*I*a*b^2 + b^3)*c^3*f^3)*(f*x + e) - 24*(3*a*b^2*
c*d^2*e^2 - b^3*c*d^2*e)*f - (4*b^3*d^3*e^3 - 4*b^3*c^3*f^3 + 36*a*b^2*d^3*
e^2 + 4*(3*a^2*b - b^3)*(f*x + e)^3*d^3 - 12*b^3*d^3*e + 12*(3*a*b^2*d^3 -
(3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e)^2 + 12*(b^3*c^2*
d*e + 3*a*b^2*c^2*d)*f^2 - 12*(6*a*b^2*d^3*e - b^3*d^3 - (3*a^2*b - b^3)*d^
3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^
2*e)*f)*(f*x + e) - 12*(b^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*d^2)*f + 4*
(b^3*d^3*e^3 - b^3*c^3*f^3 + 9*a*b^2*d^3*e^2 + (3*a^2*b - b^3)*(f*x + e)^3*
d^3 - 3*b^3*d^3*e + 3*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3
)*c*d^2*f)*(f*x + e)^2 + 3*(b^3*c^2*d*e + 3*a*b^2*c^2*d)*f^2 - 3*(6*a*b^2*d
```

```
^3*e - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3
*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*(f*x + e) - 3*(b^3*c*d^2*e^2 + 6
*a*b^2*c*d^2*e - b^3*c*d^2)*f)*cos(4*f*x + 4*e) - 8*(b^3*d^3*e^3 - b^3*c^3*
f^3 + 9*a*b^2*d^3*e^2 + (3*a^2*b - b^3)*(f*x + e)^3*d^3 - 3*b^3*d^3*e + 3*(
3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e)^2
+ 3*(b^3*c^2*d*e + 3*a*b^2*c^2*d)*f^2 - 3*(6*a*b^2*d^3*e - b^3*d^3 - (3*a^2
*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b
 - b^3)*c*d^2*e)*f)*(f*x + e) - 3*(b^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*
d^2)*f)*cos(2*f*x + 2*e) - (-4*I*b^3*d^3*e^3 + 4*I*b^3*c^3*f^3 - 36*I*a*b^2
*d^3*e^2 + (-12*I*a^2*b + 4*I*b^3)*(f*x + e)^3*d^3 + 12*I*b^3*d^3*e + (-36*
I*a*b^2*d^3 + (36*I*a^2*b - 12*I*b^3)*d^3*e + (-36*I*a^2*b + 12*I*b^3)*c*d^
2*f)*(f*x + e)^2 + (-12*I*b^3*c^2*d*e - 36*I*a*b^2*c^2*d)*f^2 + (72*I*a*b^2
*d^3*e - 12*I*b^3*d^3 + (-36*I*a^2*b + 12*I*b^3)*d^3*e^2 + (-36*I*a^2*b + 1
2*I*b^3)*c^2*d*f^2 + (-72*I*a*b^2*c*d^2 + (72*I*a^2*b - 24*I*b^3)*c*d^2*e)*
f)*(f*x + e) + (12*I*b^3*c*d^2*e^2 + 72*I*a*b^2*c*d^2*e - 12*I*b^3*c*d^2)*f
)*sin(4*f*x + 4*e) - (8*I*b^3*d^3*e^3 - 8*I*b^3*c^3*f^3 + 72*I*a*b^2*d^3*e^
2 + (24*I*a^2*b - 8*I*b^3)*(f*x + e)^3*d^3 - 24*I*b^3*d^3*e + (72*I*a*b^2*d
^3 + (-72*I*a^2*b + 24*I*b^3)*d^3*e + (72*I*a^2*b - 24*I*b^3)*c*d^2*f)*(f*x
 + e)^2 + (24*I*b^3*c^2*d*e + 72*I*a*b^2*c^2*d)*f^2 + (-144*I*a*b^2*d^3*e +
 24*I*b^3*d^3 + (72*I*a^2*b - 24*I*b^3)*d^3*e^2 + (72*I*a^2*b - 24*I*b^3)*c
^2*d*f^2 + (144*I*a*b^2*c*d^2 + (-144*I*a^2*b + 48*I*b^3)*c*d^2*e)*f)*(f*x
+ e) + (-24*I*b^3*c*d^2*e^2 - 144*I*a*b^2*c*d^2*e + 24*I*b^3*c*d^2)*f)*sin(
2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - (4*b^3*d^3*e^3 - 4*
b^3*c^3*f^3 + 36*a*b^2*d^3*e^2 - 12*b^3*d^3*e + 12*(b^3*c^2*d*e + 3*a*b^2*c
^2*d)*f^2 - 12*(b^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*d^2)*f + 4*(b^3*d^3
*e^3 - b^3*c^3*f^3 + 9*a*b^2*d^3*e^2 - 3*b^3*d^3*e + 3*(b^3*c^2*d*e + 3*a*b
^2*c^2*d)*f^2 - 3*(b^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*d^2)*f)*cos(4*f*
x + 4*e) - 8*(b^3*d^3*e^3 - b^3*c^3*f^3 + 9*a*b^2*d^3*e^2 - 3*b^3*d^3*e + 3
*(b^3*c^2*d*e + 3*a*b^2*c^2*d)*f^2 - 3*(b^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b
^3*c*d^2)*f)*cos(2*f*x + 2*e) - (-4*I*b^3*d^3*e^3 + 4*I*b^3*c^3*f^3 - 36*I*
a*b^2*d^3*e^2 + 12*I*b^3*d^3*e + (-12*I*b^3*c^2*d*e - 36*I*a*b^2*c^2*d)*f^2
 + (12*I*b^3*c*d^2*e^2 + 72*I*a*b^2*c*d^2*e - 12*I*b^3*c*d^2)*f)*sin(4*f*x
+ 4*e) - (8*I*b^3*d^3*e^3 - 8*I*b^3*c^3*f^3 + 72*I*a*b^2*d^3*e^2 - 24*I*b^3
*d^3*e + (24*I*b^3*c^2*d*e + 72*I*a*b^2*c^2*d)*f^2 + (-24*I*b^3*c*d^2*e^2 -
 144*I*a*b^2*c*d^2*e + 24*I*b^3*c*d^2)*f)*sin(2*f*x + 2*e))*arctan2(sin(f*x
 + e), cos(f*x + e) - 1) + (4*(3*a^2*b - b^3)*(f*x + e)^3*d^3 + 12*(3*a*b^2
*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e)^2 - 12*(6
*a*b^2*d^3*e - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^
2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*(f*x + e) + 4*((3*a^2*b
- b^3)*(f*x + e)^3*d^3 + 3*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b
- b^3)*c*d^2*f)*(f*x + e)^2 - 3*(6*a*b^2*d^3*e - b^3*d^3 - (3*a^2*b - b^3)*
d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*
d^2*e)*f)*(f*x + e))*cos(4*f*x + 4*e) - 8*((3*a^2*b - b^3)*(f*x + e)^3*d^3
+ 3*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x +
e)^2 - 3*(6*a*b^2*d^3*e - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^
3)*c^2*d*f^2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*(f*x + e))*co
s(2*f*x + 2*e) + ((12*I*a^2*b - 4*I*b^3)*(f*x + e)^3*d^3 + (36*I*a*b^2*d^3
+ (-36*I*a^2*b + 12*I*b^3)*d^3*e + (36*I*a^2*b - 12*I*b^3)*c*d^2*f)*(f*x +
e)^2 + (-72*I*a*b^2*d^3*e + 12*I*b^3*d^3 + (36*I*a^2*b - 12*I*b^3)*d^3*e^2
+ (36*I*a^2*b - 12*I*b^3)*c^2*d*f^2 + (72*I*a*b^2*c*d^2 + (-72*I*a^2*b + 24
*I*b^3)*c*d^2*e)*f)*(f*x + e))*sin(4*f*x + 4*e) + ((-24*I*a^2*b + 8*I*b^3)*
(f*x + e)^3*d^3 + (-72*I*a*b^2*d^3 + (72*I*a^2*b - 24*I*b^3)*d^3*e + (-72*I
*a^2*b + 24*I*b^3)*c*d^2*f)*(f*x + e)^2 + (144*I*a*b^2*d^3*e - 24*I*b^3*d^3
 + (-72*I*a^2*b + 24*I*b^3)*d^3*e^2 + (-72*I*a^2*b + 24*I*b^3)*c^2*d*f^2 +
(-144*I*a*b^2*c*d^2 + (144*I*a^2*b - 48*I*b^3)*c*d^2*e)*f)*(f*x + e))*sin(2
*f*x + 2*e))*arctan2(sin(f*x + e), -cos(f*x + e) + 1) + ((3*a^2*b - 3*I*a*b
^2 - b^3)*(f*x + e)^4*d^3 + 4*(6*a*b^2*d^3 - (3*a^2*b - 3*I*a*b^2 - b^3)*d^
3*e + (3*a^2*b - 3*I*a*b^2 - b^3)*c*d^2*f)*(f*x + e)^3 - 6*(12*a*b^2*d^3*e
- 2*b^3*d^3 - (3*a^2*b - 3*I*a*b^2 - b^3)*d^3*e^2 - (3*a^2*b - 3*I*a*b^2 -
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b^3)*c^2*d*f^2 - 2*(6*a*b^2*c*d^2 - (3*a^2*b - 3*I*a*b^2 - b^3)*c*d^2*e)*f)
*(f*x + e)^2 + 4*(18*a*b^2*d^3*e^2 - 6*b^3*d^3*e - (-3*I*a*b^2 - b^3)*d^3*e
^3 - (3*I*a*b^2 + b^3)*c^3*f^3 + 3*(6*a*b^2*c^2*d - (-3*I*a*b^2 - b^3)*c^2*
d*e)*f^2 - 3*(12*a*b^2*c*d^2*e - 2*b^3*c*d^2 + (3*I*a*b^2 + b^3)*c*d^2*e^2)
*f)*(f*x + e))*cos(4*f*x + 4*e) - (2*(3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^
4*d^3 - 12*b^3*d^3*e^2 + (24*a*b^2 + 8*I*b^3)*d^3*e^3 - (24*a*b^2 + 8*I*b^3
)*c^3*f^3 - (8*(3*a^2*b - 3*I*a*b^2 - b^3)*d^3*e - 8*(3*a^2*b - 3*I*a*b^2 -
 b^3)*c*d^2*f - (24*a*b^2 - 8*I*b^3)*d^3)*(f*x + e)^3 + (12*b^3*d^3 + 12*(3
*a^2*b - 3*I*a*b^2 - b^3)*d^3*e^2 + 12*(3*a^2*b - 3*I*a*b^2 - b^3)*c^2*d*f^
2 - (72*a*b^2 - 24*I*b^3)*d^3*e - (24*(3*a^2*b - 3*I*a*b^2 - b^3)*c*d^2*e -
 (72*a*b^2 - 24*I*b^3)*c*d^2)*f)*(f*x + e)^2 - (12*b^3*c^2*d - (72*a*b^2 +
24*I*b^3)*c^2*d*e)*f^2 - (24*b^3*d^3*e - 8*(3*I*a*b^2 + b^3)*d^3*e^3 - 8*(-
3*I*a*b^2 - b^3)*c^3*f^3 - (72*a*b^2 - 24*I*b^3)*d^3*e^2 - (24*(3*I*a*b^2 +
 b^3)*c^2*d*e + (72*a*b^2 - 24*I*b^3)*c^2*d)*f^2 - (24*b^3*c*d^2 + 24*(-3*I
*a*b^2 - b^3)*c*d^2*e^2 - (144*a*b^2 - 48*I*b^3)*c*d^2*e)*f)*(f*x + e) + (2
4*b^3*c*d^2*e - (72*a*b^2 + 24*I*b^3)*c*d^2*e^2)*f)*cos(2*f*x + 2*e) - (72*
a*b^2*d^3*e - 12*(3*a^2*b - b^3)*(f*x + e)^2*d^3 - 12*b^3*d^3 - 12*(3*a^2*b
 - b^3)*d^3*e^2 - 12*(3*a^2*b - b^3)*c^2*d*f^2 - 24*(3*a*b^2*d^3 - (3*a^2*b
 - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e) - 24*(3*a*b^2*c*d^2 - (3
*a^2*b - b^3)*c*d^2*e)*f + 12*(6*a*b^2*d^3*e - (3*a^2*b - b^3)*(f*x + e)^2*
d^3 - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*
a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e) - 2*
(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*cos(4*f*x + 4*e) - 24*(6*a*b^2
*d^3*e - (3*a^2*b - b^3)*(f*x + e)^2*d^3 - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^
2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3
*a^2*b - b^3)*c*d^2*f)*(f*x + e) - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2
*e)*f)*cos(2*f*x + 2*e) - (-72*I*a*b^2*d^3*e + (36*I*a^2*b - 12*I*b^3)*(f*x
 + e)^2*d^3 + 12*I*b^3*d^3 + (36*I*a^2*b - 12*I*b^3)*d^3*e^2 + (36*I*a^2*b
- 12*I*b^3)*c^2*d*f^2 + (72*I*a*b^2*d^3 + (-72*I*a^2*b + 24*I*b^3)*d^3*e +
(72*I*a^2*b - 24*I*b^3)*c*d^2*f)*(f*x + e) + (72*I*a*b^2*c*d^2 + (-72*I*a^2
*b + 24*I*b^3)*c*d^2*e)*f)*sin(4*f*x + 4*e) - (144*I*a*b^2*d^3*e + (-72*I*a
^2*b + 24*I*b^3)*(f*x + e)^2*d^3 - 24*I*b^3*d^3 + (-72*I*a^2*b + 24*I*b^3)*
d^3*e^2 + (-72*I*a^2*b + 24*I*b^3)*c^2*d*f^2 + (-144*I*a*b^2*d^3 + (144*I*a
^2*b - 48*I*b^3)*d^3*e + (-144*I*a^2*b + 48*I*b^3)*c*d^2*f)*(f*x + e) + (-1
44*I*a*b^2*c*d^2 + (144*I*a^2*b - 48*I*b^3)*c*d^2*e)*f)*sin(2*f*x + 2*e))*d
ilog(-e^(I*f*x + I*e)) - (72*a*b^2*d^3*e - 12*(3*a^2*b - b^3)*(f*x + e)^2*d
^3 - 12*b^3*d^3 - 12*(3*a^2*b - b^3)*d^3*e^2 - 12*(3*a^2*b - b^3)*c^2*d*f^2
 - 24*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x
+ e) - 24*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f + 12*(6*a*b^2*d^3*e -
 (3*a^2*b - b^3)*(f*x + e)^2*d^3 - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a
^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b -
 b^3)*c*d^2*f)*(f*x + e) - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*c
os(4*f*x + 4*e) - 24*(6*a*b^2*d^3*e - (3*a^2*b - b^3)*(f*x + e)^2*d^3 - b^3
*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*d^3
 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e) - 2*(3*a*b^2*
c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*cos(2*f*x + 2*e) - (-72*I*a*b^2*d^3*e +
 (36*I*a^2*b - 12*I*b^3)*(f*x + e)^2*d^3 + 12*I*b^3*d^3 + (36*I*a^2*b - 12*
I*b^3)*d^3*e^2 + (36*I*a^2*b - 12*I*b^3)*c^2*d*f^2 + (72*I*a*b^2*d^3 + (-72
*I*a^2*b + 24*I*b^3)*d^3*e + (72*I*a^2*b - 24*I*b^3)*c*d^2*f)*(f*x + e) + (
72*I*a*b^2*c*d^2 + (-72*I*a^2*b + 24*I*b^3)*c*d^2*e)*f)*sin(4*f*x + 4*e) -
(144*I*a*b^2*d^3*e + (-72*I*a^2*b + 24*I*b^3)*(f*x + e)^2*d^3 - 24*I*b^3*d^
3 + (-72*I*a^2*b + 24*I*b^3)*d^3*e^2 + (-72*I*a^2*b + 24*I*b^3)*c^2*d*f^2 +
 (-144*I*a*b^2*d^3 + (144*I*a^2*b - 48*I*b^3)*d^3*e + (-144*I*a^2*b + 48*I*
b^3)*c*d^2*f)*(f*x + e) + (-144*I*a*b^2*c*d^2 + (144*I*a^2*b - 48*I*b^3)*c*
d^2*e)*f)*sin(2*f*x + 2*e))*dilog(e^(I*f*x + I*e)) + (2*I*b^3*d^3*e^3 - 2*I
*b^3*c^3*f^3 + 18*I*a*b^2*d^3*e^2 + (6*I*a^2*b - 2*I*b^3)*(f*x + e)^3*d^3 -
 6*I*b^3*d^3*e + (18*I*a*b^2*d^3 + (-18*I*a^2*b + 6*I*b^3)*d^3*e + (18*I*a^
2*b - 6*I*b^3)*c*d^2*f)*(f*x + e)^2 + (6*I*b^3*c^2*d*e + 18*I*a*b^2*c^2*d)*
f^2 + (-36*I*a*b^2*d^3*e + 6*I*b^3*d^3 + (18*I*a^2*b - 6*I*b^3)*d^3*e^2 + (
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18*I*a^2*b - 6*I*b^3)*c^2*d*f^2 + (36*I*a*b^2*c*d^2 + (-36*I*a^2*b + 12*I*b
^3)*c*d^2*e)*f)*(f*x + e) + (-6*I*b^3*c*d^2*e^2 - 36*I*a*b^2*c*d^2*e + 6*I*
b^3*c*d^2)*f + (2*I*b^3*d^3*e^3 - 2*I*b^3*c^3*f^3 + 18*I*a*b^2*d^3*e^2 + (6
*I*a^2*b - 2*I*b^3)*(f*x + e)^3*d^3 - 6*I*b^3*d^3*e + (18*I*a*b^2*d^3 + (-1
8*I*a^2*b + 6*I*b^3)*d^3*e + (18*I*a^2*b - 6*I*b^3)*c*d^2*f)*(f*x + e)^2 +
(6*I*b^3*c^2*d*e + 18*I*a*b^2*c^2*d)*f^2 + (-36*I*a*b^2*d^3*e + 6*I*b^3*d^3
 + (18*I*a^2*b - 6*I*b^3)*d^3*e^2 + (18*I*a^2*b - 6*I*b^3)*c^2*d*f^2 + (36*
I*a*b^2*c*d^2 + (-36*I*a^2*b + 12*I*b^3)*c*d^2*e)*f)*(f*x + e) + (-6*I*b^3*
c*d^2*e^2 - 36*I*a*b^2*c*d^2*e + 6*I*b^3*c*d^2)*f)*cos(4*f*x + 4*e) + (-4*I
*b^3*d^3*e^3 + 4*I*b^3*c^3*f^3 - 36*I*a*b^2*d^3*e^2 + (-12*I*a^2*b + 4*I*b^
3)*(f*x + e)^3*d^3 + 12*I*b^3*d^3*e + (-36*I*a*b^2*d^3 + (36*I*a^2*b - 12*I
*b^3)*d^3*e + (-36*I*a^2*b + 12*I*b^3)*c*d^2*f)*(f*x + e)^2 + (-12*I*b^3*c^
2*d*e - 36*I*a*b^2*c^2*d)*f^2 + (72*I*a*b^2*d^3*e - 12*I*b^3*d^3 + (-36*I*a
^2*b + 12*I*b^3)*d^3*e^2 + (-36*I*a^2*b + 12*I*b^3)*c^2*d*f^2 + (-72*I*a*b^
2*c*d^2 + (72*I*a^2*b - 24*I*b^3)*c*d^2*e)*f)*(f*x + e) + (12*I*b^3*c*d^2*e
^2 + 72*I*a*b^2*c*d^2*e - 12*I*b^3*c*d^2)*f)*cos(2*f*x + 2*e) - 2*(b^3*d^3*
e^3 - b^3*c^3*f^3 + 9*a*b^2*d^3*e^2 + (3*a^2*b - b^3)*(f*x + e)^3*d^3 - 3*b
^3*d^3*e + 3*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f
)*(f*x + e)^2 + 3*(b^3*c^2*d*e + 3*a*b^2*c^2*d)*f^2 - 3*(6*a*b^2*d^3*e - b^
3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*c*
d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*(f*x + e) - 3*(b^3*c*d^2*e^2 + 6*a*b^2*c*
d^2*e - b^3*c*d^2)*f)*sin(4*f*x + 4*e) + 4*(b^3*d^3*e^3 - b^3*c^3*f^3 + 9*a
*b^2*d^3*e^2 + (3*a^2*b - b^3)*(f*x + e)^3*d^3 - 3*b^3*d^3*e + 3*(3*a*b^2*d
^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e)^2 + 3*(b^3*
c^2*d*e + 3*a*b^2*c^2*d)*f^2 - 3*(6*a*b^2*d^3*e - b^3*d^3 - (3*a^2*b - b^3)
*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c
*d^2*e)*f)*(f*x + e) - 3*(b^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*d^2)*f)*s
in(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)
+ (2*I*b^3*d^3*e^3 - 2*I*b^3*c^3*f^3 + 18*I*a*b^2*d^3*e^2 + (6*I*a^2*b - 2*
I*b^3)*(f*x + e)^3*d^3 - 6*I*b^3*d^3*e + (18*I*a*b^2*d^3 + (-18*I*a^2*b + 6
*I*b^3)*d^3*e + (18*I*a^2*b - 6*I*b^3)*c*d^2*f)*(f*x + e)^2 + (6*I*b^3*c^2*
d*e + 18*I*a*b^2*c^2*d)*f^2 + (-36*I*a*b^2*d^3*e + 6*I*b^3*d^3 + (18*I*a^2*
b - 6*I*b^3)*d^3*e^2 + (18*I*a^2*b - 6*I*b^3)*c^2*d*f^2 + (36*I*a*b^2*c*d^2
 + (-36*I*a^2*b + 12*I*b^3)*c*d^2*e)*f)*(f*x + e) + (-6*I*b^3*c*d^2*e^2 - 3
6*I*a*b^2*c*d^2*e + 6*I*b^3*c*d^2)*f + (2*I*b^3*d^3*e^3 - 2*I*b^3*c^3*f^3 +
 18*I*a*b^2*d^3*e^2 + (6*I*a^2*b - 2*I*b^3)*(f*x + e)^3*d^3 - 6*I*b^3*d^3*e
 + (18*I*a*b^2*d^3 + (-18*I*a^2*b + 6*I*b^3)*d^3*e + (18*I*a^2*b - 6*I*b^3)
*c*d^2*f)*(f*x + e)^2 + (6*I*b^3*c^2*d*e + 18*I*a*b^2*c^2*d)*f^2 + (-36*I*a
*b^2*d^3*e + 6*I*b^3*d^3 + (18*I*a^2*b - 6*I*b^3)*d^3*e^2 + (18*I*a^2*b - 6
*I*b^3)*c^2*d*f^2 + (36*I*a*b^2*c*d^2 + (-36*I*a^2*b + 12*I*b^3)*c*d^2*e)*f
)*(f*x + e) + (-6*I*b^3*c*d^2*e^2 - 36*I*a*b^2*c*d^2*e + 6*I*b^3*c*d^2)*f)*
cos(4*f*x + 4*e) + (-4*I*b^3*d^3*e^3 + 4*I*b^3*c^3*f^3 - 36*I*a*b^2*d^3*e^2
 + (-12*I*a^2*b + 4*I*b^3)*(f*x + e)^3*d^3 + 12*I*b^3*d^3*e + (-36*I*a*b^2*
d^3 + (36*I*a^2*b - 12*I*b^3)*d^3*e + (-36*I*a^2*b + 12*I*b^3)*c*d^2*f)*(f*
x + e)^2 + (-12*I*b^3*c^2*d*e - 36*I*a*b^2*c^2*d)*f^2 + (72*I*a*b^2*d^3*e -
 12*I*b^3*d^3 + (-36*I*a^2*b + 12*I*b^3)*d^3*e^2 + (-36*I*a^2*b + 12*I*b^3)
*c^2*d*f^2 + (-72*I*a*b^2*c*d^2 + (72*I*a^2*b - 24*I*b^3)*c*d^2*e)*f)*(f*x
+ e) + (12*I*b^3*c*d^2*e^2 + 72*I*a*b^2*c*d^2*e - 12*I*b^3*c*d^2)*f)*cos(2*
f*x + 2*e) - 2*(b^3*d^3*e^3 - b^3*c^3*f^3 + 9*a*b^2*d^3*e^2 + (3*a^2*b - b^
3)*(f*x + e)^3*d^3 - 3*b^3*d^3*e + 3*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e +
 (3*a^2*b - b^3)*c*d^2*f)*(f*x + e)^2 + 3*(b^3*c^2*d*e + 3*a*b^2*c^2*d)*f^2
 - 3*(6*a*b^2*d^3*e - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c
^2*d*f^2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*(f*x + e) - 3*(b^
3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*d^2)*f)*sin(4*f*x + 4*e) + 4*(b^3*d^3
*e^3 - b^3*c^3*f^3 + 9*a*b^2*d^3*e^2 + (3*a^2*b - b^3)*(f*x + e)^3*d^3 - 3*
b^3*d^3*e + 3*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*
f)*(f*x + e)^2 + 3*(b^3*c^2*d*e + 3*a*b^2*c^2*d)*f^2 - 3*(6*a*b^2*d^3*e - b
^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*c
*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*(f*x + e) - 3*(b^3*c*d^2*e^2 + 6*a*b^2*c
```

```
*d^2*e - b^3*c*d^2)*f)*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f*x + e)^
2 - 2*cos(f*x + e) + 1) - (24*(3*a^2*b - b^3)*d^3*cos(4*f*x + 4*e) - 48*(3*
a^2*b - b^3)*d^3*cos(2*f*x + 2*e) - (-72*I*a^2*b + 24*I*b^3)*d^3*sin(4*f*x
+ 4*e) - (144*I*a^2*b - 48*I*b^3)*d^3*sin(2*f*x + 2*e) + 24*(3*a^2*b - b^3)
*d^3)*polylog(4, -e^(I*f*x + I*e)) - (24*(3*a^2*b - b^3)*d^3*cos(4*f*x + 4*
e) - 48*(3*a^2*b - b^3)*d^3*cos(2*f*x + 2*e) - (-72*I*a^2*b + 24*I*b^3)*d^3
*sin(4*f*x + 4*e) - (144*I*a^2*b - 48*I*b^3)*d^3*sin(2*f*x + 2*e) + 24*(3*a
^2*b - b^3)*d^3)*polylog(4, e^(I*f*x + I*e)) + (72*I*a*b^2*d^3 + (72*I*a^2*
b - 24*I*b^3)*(f*x + e)*d^3 + (-72*I*a^2*b + 24*I*b^3)*d^3*e + (72*I*a^2*b
- 24*I*b^3)*c*d^2*f + (72*I*a*b^2*d^3 + (72*I*a^2*b - 24*I*b^3)*(f*x + e)*d
^3 + (-72*I*a^2*b + 24*I*b^3)*d^3*e + (72*I*a^2*b - 24*I*b^3)*c*d^2*f)*cos(
4*f*x + 4*e) + (-144*I*a*b^2*d^3 + (-144*I*a^2*b + 48*I*b^3)*(f*x + e)*d^3
+ (144*I*a^2*b - 48*I*b^3)*d^3*e + (-144*I*a^2*b + 48*I*b^3)*c*d^2*f)*cos(2
*f*x + 2*e) - 24*(3*a*b^2*d^3 + (3*a^2*b - b^3)*(f*x + e)*d^3 - (3*a^2*b -
b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*sin(4*f*x + 4*e) + 48*(3*a*b^2*d^3 +
(3*a^2*b - b^3)*(f*x + e)*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d
^2*f)*sin(2*f*x + 2*e))*polylog(3, -e^(I*f*x + I*e)) + (72*I*a*b^2*d^3 + (7
2*I*a^2*b - 24*I*b^3)*(f*x + e)*d^3 + (-72*I*a^2*b + 24*I*b^3)*d^3*e + (72*
I*a^2*b - 24*I*b^3)*c*d^2*f + (72*I*a*b^2*d^3 + (72*I*a^2*b - 24*I*b^3)*(f*
x + e)*d^3 + (-72*I*a^2*b + 24*I*b^3)*d^3*e + (72*I*a^2*b - 24*I*b^3)*c*d^2
*f)*cos(4*f*x + 4*e) + (-144*I*a*b^2*d^3 + (-144*I*a^2*b + 48*I*b^3)*(f*x +
 e)*d^3 + (144*I*a^2*b - 48*I*b^3)*d^3*e + (-144*I*a^2*b + 48*I*b^3)*c*d^2*
f)*cos(2*f*x + 2*e) - 24*(3*a*b^2*d^3 + (3*a^2*b - b^3)*(f*x + e)*d^3 - (3*
a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*sin(4*f*x + 4*e) + 48*(3*a*b^
2*d^3 + (3*a^2*b - b^3)*(f*x + e)*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b -
b^3)*c*d^2*f)*sin(2*f*x + 2*e))*polylog(3, e^(I*f*x + I*e)) + ((3*I*a^2*b +
 3*a*b^2 - I*b^3)*(f*x + e)^4*d^3 + (24*I*a*b^2*d^3 + (-12*I*a^2*b - 12*a*b
^2 + 4*I*b^3)*d^3*e + (12*I*a^2*b + 12*a*b^2 - 4*I*b^3)*c*d^2*f)*(f*x + e)^
3 + (-72*I*a*b^2*d^3*e + 12*I*b^3*d^3 + (18*I*a^2*b + 18*a*b^2 - 6*I*b^3)*d
^3*e^2 + (18*I*a^2*b + 18*a*b^2 - 6*I*b^3)*c^2*d*f^2 + (72*I*a*b^2*c*d^2 +
(-36*I*a^2*b - 36*a*b^2 + 12*I*b^3)*c*d^2*e)*f)*(f*x + e)^2 + (72*I*a*b^2*d
^3*e^2 - 24*I*b^3*d^3*e - (12*a*b^2 - 4*I*b^3)*d^3*e^3 + (12*a*b^2 - 4*I*b^
3)*c^3*f^3 + (72*I*a*b^2*c^2*d - (36*a*b^2 - 12*I*b^3)*c^2*d*e)*f^2 + (-144
*I*a*b^2*c*d^2*e + 24*I*b^3*c*d^2 + (36*a*b^2 - 12*I*b^3)*c*d^2*e^2)*f)*(f*
x + e))*sin(4*f*x + 4*e) + ((-6*I*a^2*b - 6*a*b^2 + 2*I*b^3)*(f*x + e)^4*d^
3 + 12*I*b^3*d^3*e^2 - 8*(3*I*a*b^2 - b^3)*d^3*e^3 - 8*(-3*I*a*b^2 + b^3)*c
^3*f^3 + ((24*I*a^2*b + 24*a*b^2 - 8*I*b^3)*d^3*e + (-24*I*a^2*b - 24*a*b^2
 + 8*I*b^3)*c*d^2*f - 8*(3*I*a*b^2 + b^3)*d^3)*(f*x + e)^3 + (-12*I*b^3*d^3
 + (-36*I*a^2*b - 36*a*b^2 + 12*I*b^3)*d^3*e^2 + (-36*I*a^2*b - 36*a*b^2 +
12*I*b^3)*c^2*d*f^2 - 24*(-3*I*a*b^2 - b^3)*d^3*e + ((72*I*a^2*b + 72*a*b^2
 - 24*I*b^3)*c*d^2*e - 24*(3*I*a*b^2 + b^3)*c*d^2)*f)*(f*x + e)^2 - 12*(-I*
b^3*c^2*d + 2*(3*I*a*b^2 - b^3)*c^2*d*e)*f^2 + (24*I*b^3*d^3*e + (24*a*b^2
- 8*I*b^3)*d^3*e^3 - (24*a*b^2 - 8*I*b^3)*c^3*f^3 - 24*(3*I*a*b^2 + b^3)*d^
3*e^2 + ((72*a*b^2 - 24*I*b^3)*c^2*d*e - 24*(3*I*a*b^2 + b^3)*c^2*d)*f^2 +
(-24*I*b^3*c*d^2 - (72*a*b^2 - 24*I*b^3)*c*d^2*e^2 - 48*(-3*I*a*b^2 - b^3)*
c*d^2*e)*f)*(f*x + e) + (-24*I*b^3*c*d^2*e - 24*(-3*I*a*b^2 + b^3)*c*d^2*e^
2)*f)*sin(2*f*x + 2*e))/(-4*I*f^3*cos(4*f*x + 4*e) + 8*I*f^3*cos(2*f*x + 2*
e) + 4*f^3*sin(4*f*x + 4*e) - 8*f^3*sin(2*f*x + 2*e) - 4*I*f^3))/f
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b \cot\left(e + f\,x\right)\right)^3 (c + d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^3*(c + d*x)^3,x)
```

```
[Out] int((a + b*cot(e + f*x))^3*(c + d*x)^3, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot\left(e + fx\right)\right)^3 (c + dx)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*cot(f*x+e))**3,x)
[Out] Integral((a + b*cot(e + f*x))**3*(c + d*x)**3, x)
```

3.48 $\int (c+dx)^2(a+b\cot(e+fx))^3\,dx$

Optimal. Leaf size=433

$$\frac{a^3(c+dx)^3}{3d}-\frac{3ia^2bd(c+dx)\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{f^2}+\frac{3a^2b(c+dx)^2\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{ia^2b(c+dx)^3}{d}+\frac{3a^2bd^2\mathrm{Li}_3\left(e^{2i(e+fx}\right.}{2f^3}$$

```
[Out] -b^3*c*d*x/f-1/2*b^3*d^2*x^2/f+I*b^3*d*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/
f^2+1/3*a^3*(d*x+c)^3/d-3*I*a*b^2*d^2*polylog(2,exp(2*I*(f*x+e)))/f^3-a*b^2
*(d*x+c)^3/d-3*I*a*b^2*(d*x+c)^2/f-b^3*d*(d*x+c)*cot(f*x+e)/f^2-3*a*b^2*(d*
x+c)^2*cot(f*x+e)/f-1/2*b^3*(d*x+c)^2*cot(f*x+e)^2/f+6*a*b^2*d*(d*x+c)*ln(1
-exp(2*I*(f*x+e)))/f^2+3*a^2*b*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f-b^3*(d*x+
c)^2*ln(1-exp(2*I*(f*x+e)))/f+b^3*d^2*ln(sin(f*x+e))/f^3-I*a^2*b*(d*x+c)^3/
d-3*I*a^2*b*d*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^2+1/3*I*b^3*(d*x+c)^3/d
+3/2*a^2*b*d^2*polylog(3,exp(2*I*(f*x+e)))/f^3-1/2*b^3*d^2*polylog(3,exp(2*
I*(f*x+e)))/f^3
```

Rubi [A] time = 0.65, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3722, 3717, 2190, 2531, 2282, 6589, 3720, 2279, 2391, 32, 3475}

$$-\frac{3ia^2bd(c+dx)\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{f^2}+\frac{3a^2bd^2\mathrm{PolyLog}\left(3,e^{2i(e+fx)}\right)}{2f^3}-\frac{3iab^2d^2\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{f^3}+\frac{ib^3d(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Cot[e + f*x])^3,x]
```

```
[Out] -((b^3*c*d*x)/f) - (b^3*d^2*x^2)/(2*f) - ((3*I)*a*b^2*(c + d*x)^2)/f + (a^3
*(c + d*x)^3)/(3*d) - (I*a^2*b*(c + d*x)^3)/d - (a*b^2*(c + d*x)^3)/d + ((I
/3)*b^3*(c + d*x)^3)/d - (b^3*d*(c + d*x)*Cot[e + f*x])/f^2 - (3*a*b^2*(c +
 d*x)^2*Cot[e + f*x])/f - (b^3*(c + d*x)^2*Cot[e + f*x]^2)/(2*f) + (6*a*b^2
*d*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f^2 + (3*a^2*b*(c + d*x)^2*Log[1
 - E^((2*I)*(e + f*x))])/f - (b^3*(c + d*x)^2*Log[1 - E^((2*I)*(e + f*x))])
/f + (b^3*d^2*Log[Sin[e + f*x]])/f^3 - ((3*I)*a*b^2*d^2*PolyLog[2, E^((2*I)
*(e + f*x))])/f^3 - ((3*I)*a^2*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))
])/f^2 + (I*b^3*d*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (3*a^2*b
*d^2*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^3) - (b^3*d^2*PolyLog[3, E^((2*I
)*(e + f*x))])/(2*f^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2(a+b\cot(e+fx))^3\,dx = \int \left(a^3(c+dx)^2 + 3a^2b(c+dx)^2\cot(e+fx) + 3ab^2(c+dx)^2\cot^2(e+fx)\right.$$

$$= \frac{a^3(c+dx)^3}{3d} + \left(3a^2b\right)\int (c+dx)^2\cot(e+fx)\,dx + \left(3ab^2\right)\int (c+dx)^2\cot^2$$

$$= \frac{a^3(c+dx)^3}{3d} - \frac{ia^2b(c+dx)^3}{d} - \frac{3ab^2(c+dx)^2\cot(e+fx)}{f} - \frac{b^3(c+dx)^2\cot}{2f}$$

$$= -\frac{3iab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{ia^2b(c+dx)^3}{d} - \frac{ab^2(c+dx)^3}{d} + \frac{ib^3(c+$$

$$= -\frac{b^3cdx}{f} - \frac{b^3d^2x^2}{2f} - \frac{3iab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{ia^2b(c+dx)^3}{d} - \frac{ab^2(c$$

$$= -\frac{b^3cdx}{f} - \frac{b^3d^2x^2}{2f} - \frac{3iab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{ia^2b(c+dx)^3}{d} - \frac{ab^2(c$$

$$= -\frac{b^3cdx}{f} - \frac{b^3d^2x^2}{2f} - \frac{3iab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{ia^2b(c+dx)^3}{d} - \frac{ab^2(c$$

$$= -\frac{b^3cdx}{f} - \frac{b^3d^2x^2}{2f} - \frac{3iab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{ia^2b(c+dx)^3}{d} - \frac{ab^2(c$$

Mathematica [B] time = 7.90, size = 2013, normalized size = 4.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*(a + b*Cot[e + f*x])^3,x]
```

```
[Out] -1/2*(a^2*b*d^2*E^(I*e)*Csc[e]*((2*f^3*x^3)/E^((2*I)*e) + (3*I)*(1 - E^((-2
*I)*e))*f^2*x^2*Log[1 - E^((-I)*(e + f*x))] + (3*I)*(1 - E^((-2*I)*e))*f^2*
x^2*Log[1 + E^((-I)*(e + f*x))] - (6*(-1 + E^((2*I)*e))*(f*x*PolyLog[2, -E^
((-I)*(e + f*x))] - I*PolyLog[3, -E^((-I)*(e + f*x))]))/E^((2*I)*e) - (6*(-
1 + E^((2*I)*e))*(f*x*PolyLog[2, E^((-I)*(e + f*x))] - I*PolyLog[3, E^((-I)
*(e + f*x))]))/E^((2*I)*e)))/f^3 + (b^3*d^2*E^(I*e)*Csc[e]*((2*f^3*x^3)/E^(
(2*I)*e) + (3*I)*(1 - E^((-2*I)*e))*f^2*x^2*Log[1 - E^((-I)*(e + f*x))] + (
3*I)*(1 - E^((-2*I)*e))*f^2*x^2*Log[1 + E^((-I)*(e + f*x))] - (6*(-1 + E^((
2*I)*e))*(f*x*PolyLog[2, -E^((-I)*(e + f*x))] - I*PolyLog[3, -E^((-I)*(e +
f*x))]))/E^((2*I)*e) - (6*(-1 + E^((2*I)*e))*(f*x*PolyLog[2, E^((-I)*(e + f
*x))] - I*PolyLog[3, E^((-I)*(e + f*x))]))/E^((2*I)*e)))/(6*f^3) + (b^3*d^2
*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f
^3*(Cos[e]^2 + Sin[e]^2)) + (6*a*b^2*c*d*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*
x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2)) + (3*a^2*
b*c^2*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]
))/(f*(Cos[e]^2 + Sin[e]^2)) - (b^3*c^2*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x
]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) + (Csc[e]*Cs
c[e + f*x]^2*(6*b^3*c*d*Cos[e] + 18*a*b^2*c^2*f*Cos[e] + 6*b^3*d^2*x*Cos[e]
 + 36*a*b^2*c*d*f*x*Cos[e] + 18*a^2*b*c^2*f^2*x*Cos[e] - 6*b^3*c^2*f^2*x*Co
s[e] + 18*a*b^2*d^2*f*x^2*Cos[e] + 18*a^2*b*c*d*f^2*x^2*Cos[e] - 6*b^3*c*d*
f^2*x^2*Cos[e] + 6*a^2*b*d^2*f^2*x^3*Cos[e] - 2*b^3*d^2*f^2*x^3*Cos[e] - 6*
b^3*c*d*Cos[e + 2*f*x] - 18*a*b^2*c^2*f*Cos[e + 2*f*x] - 6*b^3*d^2*x*Cos[e
+ 2*f*x] - 36*a*b^2*c*d*f*x*Cos[e + 2*f*x] - 9*a^2*b*c^2*f^2*x*Cos[e + 2*f*
x] + 3*b^3*c^2*f^2*x*Cos[e + 2*f*x] - 18*a*b^2*d^2*f*x^2*Cos[e + 2*f*x] - 9
*a^2*b*c*d*f^2*x^2*Cos[e + 2*f*x] + 3*b^3*c*d*f^2*x^2*Cos[e + 2*f*x] - 3*a^
2*b*d^2*f^2*x^3*Cos[e + 2*f*x] + b^3*d^2*f^2*x^3*Cos[e + 2*f*x] - 9*a^2*b*c
^2*f^2*x*Cos[3*e + 2*f*x] + 3*b^3*c^2*f^2*x*Cos[3*e + 2*f*x] - 9*a^2*b*c*d*
```

```
f^2*x^2*Cos[3*e + 2*f*x] + 3*b^3*c*d*f^2*x^2*Cos[3*e + 2*f*x] - 3*a^2*b*d^2
*f^2*x^3*Cos[3*e + 2*f*x] + b^3*d^2*f^2*x^3*Cos[3*e + 2*f*x] - 6*b^3*c^2*f*
Sin[e] - 12*b^3*c*d*f*x*Sin[e] + 6*a^3*c^2*f^2*x*Sin[e] - 18*a*b^2*c^2*f^2*
x*Sin[e] - 6*b^3*d^2*f*x^2*Sin[e] + 6*a^3*c*d*f^2*x^2*Sin[e] - 18*a*b^2*c*d
*f^2*x^2*Sin[e] + 2*a^3*d^2*f^2*x^3*Sin[e] - 6*a*b^2*d^2*f^2*x^3*Sin[e] + 3
*a^3*c^2*f^2*x*Sin[e + 2*f*x] - 9*a*b^2*c^2*f^2*x*Sin[e + 2*f*x] + 3*a^3*c*
d*f^2*x^2*Sin[e + 2*f*x] - 9*a*b^2*c*d*f^2*x^2*Sin[e + 2*f*x] + a^3*d^2*f^2
*x^3*Sin[e + 2*f*x] - 3*a*b^2*d^2*f^2*x^3*Sin[e + 2*f*x] - 3*a^3*c^2*f^2*x*
Sin[3*e + 2*f*x] + 9*a*b^2*c^2*f^2*x*Sin[3*e + 2*f*x] - 3*a^3*c*d*f^2*x^2*S
in[3*e + 2*f*x] + 9*a*b^2*c*d*f^2*x^2*Sin[3*e + 2*f*x] - a^3*d^2*f^2*x^3*Si
n[3*e + 2*f*x] + 3*a*b^2*d^2*f^2*x^3*Sin[3*e + 2*f*x]))/(12*f^2) - (3*a*b^2
*d^2*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[
Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^
((2*I)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[S
in[f*x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]]))])
*Tan[e])/Sqrt[1 + Tan[e]^2]))/(f^3*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)]) -
(3*a^2*b*c*d*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2
*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Lo
g[1 - E^((2*I)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e
]]*Log[Sin[f*x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan
[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))/(f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]
^2)]) + (b^3*c*d*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi
 + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]]
)*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]]))] + Pi*Log[Cos[f*x]] + 2*ArcTan[T
an[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]] + I*PolyLog[2, E^((2*I)*(f*x + ArcTan
[Tan[e]]))])*Tan[e])/Sqrt[1 + Tan[e]^2]))/(f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Si
n[e]^2)])
```

fricas [C] time = 1.18, size = 1564, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(4*(a^3 - 3*a*b^2)*d^2*f^3*x^3 - 12*b^3*c^2*f^2 - 12*(b^3*d^2*f^2 - (
a^3 - 3*a*b^2)*c*d*f^3)*x^2 - 12*(2*b^3*c*d*f^2 - (a^3 - 3*a*b^2)*c^2*f^3)*
x - 4*((a^3 - 3*a*b^2)*d^2*f^3*x^3 + 3*(a^3 - 3*a*b^2)*c*d*f^3*x^2 + 3*(a^3
 - 3*a*b^2)*c^2*f^3*x)*cos(2*f*x + 2*e) - (18*I*a*b^2*d^2 + 6*I*(3*a^2*b -
b^3)*d^2*f*x + 6*I*(3*a^2*b - b^3)*c*d*f + (-18*I*a*b^2*d^2 - 6*I*(3*a^2*b
- b^3)*d^2*f*x - 6*I*(3*a^2*b - b^3)*c*d*f)*cos(2*f*x + 2*e))*dilog(cos(2*f
*x + 2*e) + I*sin(2*f*x + 2*e)) - (-18*I*a*b^2*d^2 - 6*I*(3*a^2*b - b^3)*d^
2*f*x - 6*I*(3*a^2*b - b^3)*c*d*f + (18*I*a*b^2*d^2 + 6*I*(3*a^2*b - b^3)*d
^2*f*x + 6*I*(3*a^2*b - b^3)*c*d*f)*cos(2*f*x + 2*e))*dilog(cos(2*f*x + 2*e
) - I*sin(2*f*x + 2*e)) - 6*(6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b - b^3)*d^2*
e^2 - (3*a^2*b - b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b - b^3)*c*d*e)*f -
 (6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b - b^3)*d^2*e^2 - (3*a^2*b - b^3)*c^2*f
^2 - 2*(3*a*b^2*c*d - (3*a^2*b - b^3)*c*d*e)*f)*cos(2*f*x + 2*e))*log(-1/2*
cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2) - 6*(6*a*b^2*d^2*e - b^3*d
^2 - (3*a^2*b - b^3)*d^2*e^2 - (3*a^2*b - b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (
3*a^2*b - b^3)*c*d*e)*f - (6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b - b^3)*d^2*e^
2 - (3*a^2*b - b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b - b^3)*c*d*e)*f)*co
s(2*f*x + 2*e))*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2) +
 6*((3*a^2*b - b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*d^2*e^2 +
 2*(3*a^2*b - b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b - b^3)*c*d*f^2)*x
- ((3*a^2*b - b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*d^2*e^2 +
2*(3*a^2*b - b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b - b^3)*c*d*f^2)*x)*
cos(2*f*x + 2*e))*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 6*((3*a
^2*b - b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*d^2*e^2 + 2*(3*a^
2*b - b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b - b^3)*c*d*f^2)*x - ((3*a^
```

```
2*b - b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*d^2*e^2 + 2*(3*a^2
*b - b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b - b^3)*c*d*f^2)*x)*cos(2*f*
x + 2*e))*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1) - 3*((3*a^2*b - b
^3)*d^2*cos(2*f*x + 2*e) - (3*a^2*b - b^3)*d^2)*polylog(3, cos(2*f*x + 2*e)
 + I*sin(2*f*x + 2*e)) - 3*((3*a^2*b - b^3)*d^2*cos(2*f*x + 2*e) - (3*a^2*b
 - b^3)*d^2)*polylog(3, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) - 12*(3*a*b^
2*d^2*f^2*x^2 + 3*a*b^2*c^2*f^2 + b^3*c*d*f + (6*a*b^2*c*d*f^2 + b^3*d^2*f)
*x)*sin(2*f*x + 2*e))/(f^3*cos(2*f*x + 2*e) - f^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \left(b \cot\left(fx + e\right) + a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(b*cot(f*x + e) + a)^3, x)
```

maple [B] time = 2.07, size = 1740, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*cot(f*x+e))^3,x)
```

```
[Out] -3*a*b^2*c*d*x^2-12*I/f*b*a^2*c*d*e*x+a^3*c*d*x^2+1/3*I*b^3*d^2*x^3+I*b^3*c
*d*x^2-1/f*b^3*c^2*ln(exp(I*(f*x+e))-1)-1/f*b^3*c^2*ln(exp(I*(f*x+e))+1)+1/
f^3*b^3*d^2*ln(exp(I*(f*x+e))+1)+1/f^3*b^3*d^2*ln(exp(I*(f*x+e))-1)-2/f^3*b
^3*d^2*polylog(3,exp(I*(f*x+e)))-2/f^3*b^3*d^2*polylog(3,-exp(I*(f*x+e)))-2
/f^3*b^3*d^2*ln(exp(I*(f*x+e)))-3*I*a^2*b*c*d*x^2-a*b^2*d^2*x^3-3*b^2*a*c^2
*x+2*b^2*(-3*I*a*d^2*f*x^2*exp(2*I*(f*x+e))-6*I*a*c*d*f*x*exp(2*I*(f*x+e))+
b*d^2*f*x^2*exp(2*I*(f*x+e))-3*I*a*c^2*f*exp(2*I*(f*x+e))+3*I*a*d^2*f*x^2-I
*b*d^2*x*exp(2*I*(f*x+e))+2*b*c*d*f*x*exp(2*I*(f*x+e))+6*I*a*c*d*f*x-I*b*c*
d*exp(2*I*(f*x+e))+b*c^2*f*exp(2*I*(f*x+e))+3*I*a*c^2*f+I*d^2*x*b+I*b*c*d)/
f^2/(exp(2*I*(f*x+e))-1)^2+6/f*b*ln(exp(I*(f*x+e))+1)*a^2*c*d*x+6/f*b*ln(1-
exp(I*(f*x+e)))*a^2*c*d*x+6/f^2*b*ln(1-exp(I*(f*x+e)))*a^2*c*d*e-6/f^2*b*a^
2*c*d*e*ln(exp(I*(f*x+e))-1)+12/f^2*b*a^2*c*d*e*ln(exp(I*(f*x+e)))+2/f*b^3*
c^2*ln(exp(I*(f*x+e)))+1/3*a^3*d^2*x^3+a^3*c^2*x-I*a^2*b*d^2*x^3-12*I/f^2*b
^2*a*d^2*e*x+4*I/f*b^3*c*d*e*x+6*I/f^2*b*a^2*d^2*e^2*x-6*I/f^2*b*a^2*c*d*e^
2-6*I/f^2*b*polylog(2,-exp(I*(f*x+e)))*a^2*d^2*x-6*I/f^2*b*a^2*c*d*polylog(
2,-exp(I*(f*x+e)))-6*I/f^2*b*polylog(2,exp(I*(f*x+e)))*a^2*d^2*x-6*I/f^2*b*
a^2*c*d*polylog(2,exp(I*(f*x+e)))+6/f^3*b*a^2*d^2*polylog(3,exp(I*(f*x+e)))
+1/f^3*b^3*ln(1-exp(I*(f*x+e)))*d^2*e^2-4/3*I/f^3*b^3*d^2*e^3-I*b^3*c^2*x+3
/f*b*ln(1-exp(I*(f*x+e)))*a^2*d^2*x^2+6/f^3*b^2*ln(1-exp(I*(f*x+e)))*a*d^2*
e-3/f^3*b*ln(1-exp(I*(f*x+e)))*a^2*d^2*e^2+6/f^2*b^2*a*c*d*ln(exp(I*(f*x+e)
)-1)+6/f^2*b^2*a*c*d*ln(exp(I*(f*x+e))+1)-12/f^2*b^2*a*c*d*ln(exp(I*(f*x+e)
))-4/f^2*b^3*c*d*e*ln(exp(I*(f*x+e)))-6/f^3*b^2*a*d^2*e*ln(exp(I*(f*x+e))-1
)+3*I*a^2*b*c^2*x-1/f*b^3*ln(1-exp(I*(f*x+e)))*d^2*x^2-1/f*b^3*ln(exp(I*(f*
x+e))+1)*d^2*x^2+3/f*b*a^2*c^2*ln(exp(I*(f*x+e))+1)-1/f^3*b^3*d^2*e^2*ln(ex
p(I*(f*x+e))-1)+2/f^3*b^3*d^2*e^2*ln(exp(I*(f*x+e)))-6/f*b*a^2*c^2*ln(exp(I
*(f*x+e)))+3/f*b*a^2*c^2*ln(exp(I*(f*x+e))-1)+6/f^3*b*a^2*d^2*polylog(3,-ex
p(I*(f*x+e)))-6/f^3*b*a^2*d^2*e^2*ln(exp(I*(f*x+e)))+3/f^3*b*a^2*d^2*e^2*ln
(exp(I*(f*x+e))-1)+12/f^3*b^2*a*d^2*e*ln(exp(I*(f*x+e)))-2/f*b^3*ln(1-exp(I
*(f*x+e)))*c*d*x-2/f^2*b^3*ln(1-exp(I*(f*x+e)))*c*d*e-2/f*b^3*ln(exp(I*(f*x
+e))+1)*c*d*x+6/f^2*b^2*ln(1-exp(I*(f*x+e)))*a*d^2*x+6/f^2*b^2*ln(exp(I*(f*
x+e))+1)*a*d^2*x+3/f*b*ln(exp(I*(f*x+e))+1)*a^2*d^2*x^2+2/f^2*b^3*c*d*e*ln(
exp(I*(f*x+e))-1)-6*I/f^3*b^2*a*d^2*polylog(2,exp(I*(f*x+e)))-6*I/f^3*b^2*a
*d^2*e^2+4*I/f^3*b*a^2*d^2*e^3-6*I/f*b^2*a*d^2*x^2+2*I/f^2*b^3*polylog(2,ex
p(I*(f*x+e)))*d^2*x+2*I/f^2*b^3*polylog(2,-exp(I*(f*x+e)))*d^2*x+2*I/f^2*b^
```

```
3*c*d*polylog(2,exp(I*(f*x+e)))+2*I/f^2*b^3*c*d*polylog(2,-exp(I*(f*x+e)))-
6*I/f^3*b^2*a*d^2*polylog(2,-exp(I*(f*x+e)))+2*I/f^2*b^3*c*d*e^2-2*I/f^2*b^
3*d^2*e^2*x
```

maxima [B] time = 3.98, size = 5411, normalized size = 12.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^3*c^2 + (f*x + e)^3*a^3*d^2/f^2 - 3*(f*x + e)^2*a^3*d^2*
e/f^2 + 3*(f*x + e)*a^3*d^2*e^2/f^2 + 3*(f*x + e)^2*a^3*c*d/f - 6*(f*x + e)
*a^3*c*d*e/f + 9*a^2*b*c^2*log(sin(f*x + e)) + 9*a^2*b*d^2*e^2*log(sin(f*x
+ e))/f^2 - 18*a^2*b*c*d*e*log(sin(f*x + e))/f + 3*(36*a*b^2*d^2*e^2 + 36*a
*b^2*c^2*f^2 - 2*(3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^3*d^2 - 12*b^3*d^2*e
 + 6*((3*a^2*b - 3*I*a*b^2 - b^3)*d^2*e - (3*a^2*b - 3*I*a*b^2 - b^3)*c*d*f
)*(f*x + e)^2 + 6*((3*I*a*b^2 + b^3)*d^2*e^2 + 2*(-3*I*a*b^2 - b^3)*c*d*e*f
 + (3*I*a*b^2 + b^3)*c^2*f^2)*(f*x + e) - 12*(6*a*b^2*c*d*e - b^3*c*d)*f -
(6*b^3*d^2*e^2 + 6*b^3*c^2*f^2 + 36*a*b^2*d^2*e - 6*(3*a^2*b - b^3)*(f*x +
e)^2*d^2 - 6*b^3*d^2 - 12*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b -
 b^3)*c*d*f)*(f*x + e) - 12*(b^3*c*d*e + 3*a*b^2*c*d)*f + 6*(b^3*d^2*e^2 +
b^3*c^2*f^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - b^3*d^2 - 2
*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*(f*x + e) -
2*(b^3*c*d*e + 3*a*b^2*c*d)*f)*cos(4*f*x + 4*e) - 12*(b^3*d^2*e^2 + b^3*c^2
*f^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - b^3*d^2 - 2*(3*a*b
^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*(f*x + e) - 2*(b^3*
c*d*e + 3*a*b^2*c*d)*f)*cos(2*f*x + 2*e) + (6*I*b^3*d^2*e^2 + 6*I*b^3*c^2*f
^2 + 36*I*a*b^2*d^2*e + (-18*I*a^2*b + 6*I*b^3)*(f*x + e)^2*d^2 - 6*I*b^3*d
^2 + (-36*I*a*b^2*d^2 + (36*I*a^2*b - 12*I*b^3)*d^2*e + (-36*I*a^2*b + 12*I
*b^3)*c*d*f)*(f*x + e) + (-12*I*b^3*c*d*e - 36*I*a*b^2*c*d)*f)*sin(4*f*x +
4*e) + (-12*I*b^3*d^2*e^2 - 12*I*b^3*c^2*f^2 - 72*I*a*b^2*d^2*e + (36*I*a^2
*b - 12*I*b^3)*(f*x + e)^2*d^2 + 12*I*b^3*d^2 + (72*I*a*b^2*d^2 + (-72*I*a^
2*b + 24*I*b^3)*d^2*e + (72*I*a^2*b - 24*I*b^3)*c*d*f)*(f*x + e) + (24*I*b^
3*c*d*e + 72*I*a*b^2*c*d)*f)*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*
x + e) + 1) - (6*b^3*d^2*e^2 + 6*b^3*c^2*f^2 + 36*a*b^2*d^2*e - 6*b^3*d^2 -
 12*(b^3*c*d*e + 3*a*b^2*c*d)*f + 6*(b^3*d^2*e^2 + b^3*c^2*f^2 + 6*a*b^2*d^
2*e - b^3*d^2 - 2*(b^3*c*d*e + 3*a*b^2*c*d)*f)*cos(4*f*x + 4*e) - 12*(b^3*d
^2*e^2 + b^3*c^2*f^2 + 6*a*b^2*d^2*e - b^3*d^2 - 2*(b^3*c*d*e + 3*a*b^2*c*d
)*f)*cos(2*f*x + 2*e) + (6*I*b^3*d^2*e^2 + 6*I*b^3*c^2*f^2 + 36*I*a*b^2*d^2
*e - 6*I*b^3*d^2 + (-12*I*b^3*c*d*e - 36*I*a*b^2*c*d)*f)*sin(4*f*x + 4*e) +
 (-12*I*b^3*d^2*e^2 - 12*I*b^3*c^2*f^2 - 72*I*a*b^2*d^2*e + 12*I*b^3*d^2 +
(24*I*b^3*c*d*e + 72*I*a*b^2*c*d)*f)*sin(2*f*x + 2*e))*arctan2(sin(f*x + e)
, cos(f*x + e) - 1) - (6*(3*a^2*b - b^3)*(f*x + e)^2*d^2 + 12*(3*a*b^2*d^2
- (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*(f*x + e) + 6*((3*a^2*b -
b^3)*(f*x + e)^2*d^2 + 2*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b -
b^3)*c*d*f)*(f*x + e))*cos(4*f*x + 4*e) - 12*((3*a^2*b - b^3)*(f*x + e)^2*d
^2 + 2*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*(f*x +
 e))*cos(2*f*x + 2*e) + ((18*I*a^2*b - 6*I*b^3)*(f*x + e)^2*d^2 + (36*I*a*b
^2*d^2 + (-36*I*a^2*b + 12*I*b^3)*d^2*e + (36*I*a^2*b - 12*I*b^3)*c*d*f)*(f
*x + e))*sin(4*f*x + 4*e) + ((-36*I*a^2*b + 12*I*b^3)*(f*x + e)^2*d^2 + (-7
2*I*a*b^2*d^2 + (72*I*a^2*b - 24*I*b^3)*d^2*e + (-72*I*a^2*b + 24*I*b^3)*c*
d*f)*(f*x + e))*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), -cos(f*x + e) + 1)
- 2*((3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^3*d^2 + 3*(6*a*b^2*d^2 - (3*a^2*
b - 3*I*a*b^2 - b^3)*d^2*e + (3*a^2*b - 3*I*a*b^2 - b^3)*c*d*f)*(f*x + e)^2
 - 3*(12*a*b^2*d^2*e - 2*b^3*d^2 + (3*I*a*b^2 + b^3)*d^2*e^2 + (3*I*a*b^2 +
 b^3)*c^2*f^2 - 2*(6*a*b^2*c*d - (-3*I*a*b^2 - b^3)*c*d*e)*f)*(f*x + e))*co
s(4*f*x + 4*e) + (4*(3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^3*d^2 + 12*b^3*d^
2*e - (36*a*b^2 + 12*I*b^3)*d^2*e^2 - (36*a*b^2 + 12*I*b^3)*c^2*f^2 - (12*(
3*a^2*b - 3*I*a*b^2 - b^3)*d^2*e - 12*(3*a^2*b - 3*I*a*b^2 - b^3)*c*d*f - (
```

```
36*a*b^2 - 12*I*b^3)*d^2)*(f*x + e)^2 + (12*b^3*d^2 + 12*(-3*I*a*b^2 - b^3)
*d^2*e^2 + 12*(-3*I*a*b^2 - b^3)*c^2*f^2 - (72*a*b^2 - 24*I*b^3)*d^2*e + (2
4*(3*I*a*b^2 + b^3)*c*d*e + (72*a*b^2 - 24*I*b^3)*c*d)*f)*(f*x + e) - (12*b
^3*c*d - (72*a*b^2 + 24*I*b^3)*c*d*e)*f)*cos(2*f*x + 2*e) - (36*a*b^2*d^2 +
 12*(3*a^2*b - b^3)*(f*x + e)*d^2 - 12*(3*a^2*b - b^3)*d^2*e + 12*(3*a^2*b
- b^3)*c*d*f + 12*(3*a*b^2*d^2 + (3*a^2*b - b^3)*(f*x + e)*d^2 - (3*a^2*b -
 b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*cos(4*f*x + 4*e) - 24*(3*a*b^2*d^2 + (
3*a^2*b - b^3)*(f*x + e)*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*
f)*cos(2*f*x + 2*e) + (36*I*a*b^2*d^2 + (36*I*a^2*b - 12*I*b^3)*(f*x + e)*d
^2 + (-36*I*a^2*b + 12*I*b^3)*d^2*e + (36*I*a^2*b - 12*I*b^3)*c*d*f)*sin(4*
f*x + 4*e) + (-72*I*a*b^2*d^2 + (-72*I*a^2*b + 24*I*b^3)*(f*x + e)*d^2 + (7
2*I*a^2*b - 24*I*b^3)*d^2*e + (-72*I*a^2*b + 24*I*b^3)*c*d*f)*sin(2*f*x + 2
*e))*dilog(-e^(I*f*x + I*e)) - (36*a*b^2*d^2 + 12*(3*a^2*b - b^3)*(f*x + e)
*d^2 - 12*(3*a^2*b - b^3)*d^2*e + 12*(3*a^2*b - b^3)*c*d*f + 12*(3*a*b^2*d^
2 + (3*a^2*b - b^3)*(f*x + e)*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)
*c*d*f)*cos(4*f*x + 4*e) - 24*(3*a*b^2*d^2 + (3*a^2*b - b^3)*(f*x + e)*d^2
- (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*cos(2*f*x + 2*e) + (36*I*a
*b^2*d^2 + (36*I*a^2*b - 12*I*b^3)*(f*x + e)*d^2 + (-36*I*a^2*b + 12*I*b^3)
*d^2*e + (36*I*a^2*b - 12*I*b^3)*c*d*f)*sin(4*f*x + 4*e) + (-72*I*a*b^2*d^2
 + (-72*I*a^2*b + 24*I*b^3)*(f*x + e)*d^2 + (72*I*a^2*b - 24*I*b^3)*d^2*e +
 (-72*I*a^2*b + 24*I*b^3)*c*d*f)*sin(2*f*x + 2*e))*dilog(e^(I*f*x + I*e)) -
 (-3*I*b^3*d^2*e^2 - 3*I*b^3*c^2*f^2 - 18*I*a*b^2*d^2*e + (9*I*a^2*b - 3*I*
b^3)*(f*x + e)^2*d^2 + 3*I*b^3*d^2 + (18*I*a*b^2*d^2 + (-18*I*a^2*b + 6*I*b
^3)*d^2*e + (18*I*a^2*b - 6*I*b^3)*c*d*f)*(f*x + e) + (6*I*b^3*c*d*e + 18*I
*a*b^2*c*d)*f + (-3*I*b^3*d^2*e^2 - 3*I*b^3*c^2*f^2 - 18*I*a*b^2*d^2*e + (9
*I*a^2*b - 3*I*b^3)*(f*x + e)^2*d^2 + 3*I*b^3*d^2 + (18*I*a*b^2*d^2 + (-18*
I*a^2*b + 6*I*b^3)*d^2*e + (18*I*a^2*b - 6*I*b^3)*c*d*f)*(f*x + e) + (6*I*b
^3*c*d*e + 18*I*a*b^2*c*d)*f)*cos(4*f*x + 4*e) + (6*I*b^3*d^2*e^2 + 6*I*b^3
*c^2*f^2 + 36*I*a*b^2*d^2*e + (-18*I*a^2*b + 6*I*b^3)*(f*x + e)^2*d^2 - 6*I
*b^3*d^2 + (-36*I*a*b^2*d^2 + (36*I*a^2*b - 12*I*b^3)*d^2*e + (-36*I*a^2*b
+ 12*I*b^3)*c*d*f)*(f*x + e) + (-12*I*b^3*c*d*e - 36*I*a*b^2*c*d)*f)*cos(2*
f*x + 2*e) + 3*(b^3*d^2*e^2 + b^3*c^2*f^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)
*(f*x + e)^2*d^2 - b^3*d^2 - 2*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^
2*b - b^3)*c*d*f)*(f*x + e) - 2*(b^3*c*d*e + 3*a*b^2*c*d)*f)*sin(4*f*x + 4*
e) - 6*(b^3*d^2*e^2 + b^3*c^2*f^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*(f*x +
e)^2*d^2 - b^3*d^2 - 2*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^
3)*c*d*f)*(f*x + e) - 2*(b^3*c*d*e + 3*a*b^2*c*d)*f)*sin(2*f*x + 2*e))*log(
cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) - (-3*I*b^3*d^2*e^2 -
 3*I*b^3*c^2*f^2 - 18*I*a*b^2*d^2*e + (9*I*a^2*b - 3*I*b^3)*(f*x + e)^2*d^2
 + 3*I*b^3*d^2 + (18*I*a*b^2*d^2 + (-18*I*a^2*b + 6*I*b^3)*d^2*e + (18*I*a^
2*b - 6*I*b^3)*c*d*f)*(f*x + e) + (6*I*b^3*c*d*e + 18*I*a*b^2*c*d)*f + (-3*
I*b^3*d^2*e^2 - 3*I*b^3*c^2*f^2 - 18*I*a*b^2*d^2*e + (9*I*a^2*b - 3*I*b^3)*
(f*x + e)^2*d^2 + 3*I*b^3*d^2 + (18*I*a*b^2*d^2 + (-18*I*a^2*b + 6*I*b^3)*d
^2*e + (18*I*a^2*b - 6*I*b^3)*c*d*f)*(f*x + e) + (6*I*b^3*c*d*e + 18*I*a*b^
2*c*d)*f)*cos(4*f*x + 4*e) + (6*I*b^3*d^2*e^2 + 6*I*b^3*c^2*f^2 + 36*I*a*b^
2*d^2*e + (-18*I*a^2*b + 6*I*b^3)*(f*x + e)^2*d^2 - 6*I*b^3*d^2 + (-36*I*a*
b^2*d^2 + (36*I*a^2*b - 12*I*b^3)*d^2*e + (-36*I*a^2*b + 12*I*b^3)*c*d*f)*(
f*x + e) + (-12*I*b^3*c*d*e - 36*I*a*b^2*c*d)*f)*cos(2*f*x + 2*e) + 3*(b^3*
d^2*e^2 + b^3*c^2*f^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - b
^3*d^2 - 2*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*(f
*x + e) - 2*(b^3*c*d*e + 3*a*b^2*c*d)*f)*sin(4*f*x + 4*e) - 6*(b^3*d^2*e^2
+ b^3*c^2*f^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - b^3*d^2 -
 2*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*(f*x + e)
- 2*(b^3*c*d*e + 3*a*b^2*c*d)*f)*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin
(f*x + e)^2 - 2*cos(f*x + e) + 1) - ((36*I*a^2*b - 12*I*b^3)*d^2*cos(4*f*x
+ 4*e) + (-72*I*a^2*b + 24*I*b^3)*d^2*cos(2*f*x + 2*e) - 12*(3*a^2*b - b^3)
*d^2*sin(4*f*x + 4*e) + 24*(3*a^2*b - b^3)*d^2*sin(2*f*x + 2*e) + (36*I*a^2
*b - 12*I*b^3)*d^2)*polylog(3, -e^(I*f*x + I*e)) - ((36*I*a^2*b - 12*I*b^3)
*d^2*cos(4*f*x + 4*e) + (-72*I*a^2*b + 24*I*b^3)*d^2*cos(2*f*x + 2*e) - 12*
```

```
(3*a^2*b - b^3)*d^2*sin(4*f*x + 4*e) + 24*(3*a^2*b - b^3)*d^2*sin(2*f*x + 2
*e) + (36*I*a^2*b - 12*I*b^3)*d^2)*polylog(3, e^(I*f*x + I*e)) - ((6*I*a^2*
b + 6*a*b^2 - 2*I*b^3)*(f*x + e)^3*d^2 + (36*I*a*b^2*d^2 + (-18*I*a^2*b - 1
8*a*b^2 + 6*I*b^3)*d^2*e + (18*I*a^2*b + 18*a*b^2 - 6*I*b^3)*c*d*f)*(f*x +
e)^2 + (-72*I*a*b^2*d^2*e + 12*I*b^3*d^2 + (18*a*b^2 - 6*I*b^3)*d^2*e^2 + (
18*a*b^2 - 6*I*b^3)*c^2*f^2 + (72*I*a*b^2*c*d - (36*a*b^2 - 12*I*b^3)*c*d*e
)*f)*(f*x + e))*sin(4*f*x + 4*e) - ((-12*I*a^2*b - 12*a*b^2 + 4*I*b^3)*(f*x
 + e)^3*d^2 - 12*I*b^3*d^2*e - 12*(-3*I*a*b^2 + b^3)*d^2*e^2 - 12*(-3*I*a*b
^2 + b^3)*c^2*f^2 + ((36*I*a^2*b + 36*a*b^2 - 12*I*b^3)*d^2*e + (-36*I*a^2*
b - 36*a*b^2 + 12*I*b^3)*c*d*f - 12*(3*I*a*b^2 + b^3)*d^2)*(f*x + e)^2 + (-
12*I*b^3*d^2 - (36*a*b^2 - 12*I*b^3)*d^2*e^2 - (36*a*b^2 - 12*I*b^3)*c^2*f^
2 - 24*(-3*I*a*b^2 - b^3)*d^2*e + ((72*a*b^2 - 24*I*b^3)*c*d*e - 24*(3*I*a*
b^2 + b^3)*c*d)*f)*(f*x + e) - 12*(-I*b^3*c*d + 2*(3*I*a*b^2 - b^3)*c*d*e)*
f)*sin(2*f*x + 2*e))/(-6*I*f^2*cos(4*f*x + 4*e) + 12*I*f^2*cos(2*f*x + 2*e)
 + 6*f^2*sin(4*f*x + 4*e) - 12*f^2*sin(2*f*x + 2*e) - 6*I*f^2))/f
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\cot\left(e + f\,x\right)\right)^3 (c + d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^3*(c + d*x)^2,x)
```

```
[Out] int((a + b*cot(e + f*x))^3*(c + d*x)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot\left(e + f\,x\right)\right)^3 (c + dx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*cot(f*x+e))**3,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**3*(c + d*x)**2, x)
```

3.49 $\int (c+dx)(a+b\cot(e+fx))^3\,dx$

Optimal. Leaf size=278

$$\frac{a^3(c+dx)^2}{2d}+\frac{3a^2b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{3ia^2b(c+dx)^2}{2d}-\frac{3ia^2bd\mathrm{Li}_2\left(e^{2i(e+fx)}\right)}{2f^2}-\frac{3ab^2(c+dx)\cot(e+fx)}{f}-$$

```
[Out] -3*a*b^2*c*x-1/2*b^3*d*x/f-3/2*a*b^2*d*x^2+1/2*a^3*(d*x+c)^2/d-3/2*I*a^2*b*
(d*x+c)^2/d+1/2*I*b^3*(d*x+c)^2/d-1/2*b^3*d*cot(f*x+e)/f^2-3*a*b^2*(d*x+c)*
cot(f*x+e)/f-1/2*b^3*(d*x+c)*cot(f*x+e)^2/f+3*a^2*b*(d*x+c)*ln(1-exp(2*I*(f
*x+e)))/f-b^3*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f+3*a*b^2*d*ln(sin(f*x+e))/f^2
-3/2*I*a^2*b*d*polylog(2,exp(2*I*(f*x+e)))/f^2+1/2*I*b^3*d*polylog(2,exp(2*
I*(f*x+e)))/f^2
```

Rubi [A] time = 0.32, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3722, 3717, 2190, 2279, 2391, 3720, 3475, 3473, 8}

$$-\frac{3ia^2bd\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{2f^2}+\frac{ib^3d\mathrm{PolyLog}\left(2,e^{2i(e+fx)}\right)}{2f^2}+\frac{3a^2b(c+dx)\log\left(1-e^{2i(e+fx)}\right)}{f}-\frac{3ia^2b(c+dx)^2}{2d}+\frac{a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Cot[e + f*x])^3,x]
```

```
[Out] -3*a*b^2*c*x - (b^3*d*x)/(2*f) - (3*a*b^2*d*x^2)/2 + (a^3*(c + d*x)^2)/(2*d
) - (((3*I)/2)*a^2*b*(c + d*x)^2)/d + ((I/2)*b^3*(c + d*x)^2)/d - (b^3*d*Co
t[e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)*Cot[e + f*x])/f - (b^3*(c + d*x)*C
ot[e + f*x]^2)/(2*f) + (3*a^2*b*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f -
 (b^3*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f + (3*a*b^2*d*Log[Sin[e + f*
x]])/f^2 - (((3*I)/2)*a^2*b*d*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + ((I/2)
*b^3*d*PolyLog[2, E^((2*I)*(e + f*x))])/f^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)(a+b\cot(e+fx))^3\,dx &= \int \left(a^3(c+dx)+3a^2b(c+dx)\cot(e+fx)+3ab^2(c+dx)\cot^2(e+fx)+b^3(c+\right.\\
&= \frac{a^3(c+dx)^2}{2d}+\left(3a^2b\right)\int (c+dx)\cot(e+fx)\,dx+\left(3ab^2\right)\int (c+dx)\cot^2(e+f\\
&= \frac{a^3(c+dx)^2}{2d}-\frac{3ia^2b(c+dx)^2}{2d}-\frac{3ab^2(c+dx)\cot(e+fx)}{f}-\frac{b^3(c+dx)\cot^2(e+\\
&= -3ab^2cx-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3ia^2b(c+dx)^2}{2d}+\frac{ib^3(c+dx)^2}{2d}-\frac{b^3d\cot(}{2f}\\
&= -3ab^2cx-\frac{b^3dx}{2f}-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3ia^2b(c+dx)^2}{2d}+\frac{ib^3(c+dx)^2}{2d}-\\
&= -3ab^2cx-\frac{b^3dx}{2f}-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3ia^2b(c+dx)^2}{2d}+\frac{ib^3(c+dx)^2}{2d}-\\
&= -3ab^2cx-\frac{b^3dx}{2f}-\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3ia^2b(c+dx)^2}{2d}+\frac{ib^3(c+dx)^2}{2d}-
\end{aligned}$$

Mathematica [A] time = 3.65, size = 276, normalized size = 0.99

$$\frac{\sin(e+fx)(a+b\cot(e+fx))^3\left(ibd\left(b^2-3a^2\right)\mathrm{Li}_2\left(e^{2i(e+fx)}\right)\sin^2(e+fx)+\sin^2(e+fx)\left(2b\log(\sin(e+fx))\left(a^2(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Cot[e + f*x])^3,x]
```

```
[Out] ((a + b*Cot[e + f*x])^3*Sin[e + f*x]*((-((e + f*x)*((3*I)*a^2*b*d*(e + f*x)
 - I*b^3*d*(e + f*x) + 3*a*b^2*(-(d*e) + 2*c*f + d*f*x) + a^3*(-2*c*f + d*(
e - f*x)))) - 2*b*(-3*a^2 + b^2)*d*(e + f*x)*Log[1 - E^((2*I)*(e + f*x))] +
 2*b*(3*a*b*d + b^2*(d*e - c*f) + a^2*(-3*d*e + 3*c*f))*Log[Sin[e + f*x]])*
Sin[e + f*x]^2 + I*b*(-3*a^2 + b^2)*d*PolyLog[2, E^((2*I)*(e + f*x))]*Sin[e
 + f*x]^2 - (b^2*(2*b*f*(c + d*x) + (b*d + 6*a*f*(c + d*x))*Sin[2*(e + f*x)
]))/2))/(2*f^2*(b*Cos[e + f*x] + a*Sin[e + f*x])^3)
```

fricas [B] time = 0.61, size = 721, normalized size = 2.59

$$-\frac{2\left(a^3-3ab^2\right)df^2x^2-4b^3cf-4\left(b^3df-\left(a^3-3ab^2\right)cf^2\right)x-2\left(\left(a^3-3ab^2\right)df^2x^2+2\left(a^3-3ab^2\right)cf^2x\right)\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^3 - 3*a*b^2)*d*f^2*x^2 - 4*b^3*c*f - 4*(b^3*d*f - (a^3 - 3*a*b^2
)*c*f^2)*x - 2*((a^3 - 3*a*b^2)*d*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*f^2*x)*cos(
2*f*x + 2*e) - (-I*(3*a^2*b - b^3)*d*cos(2*f*x + 2*e) + I*(3*a^2*b - b^3)*d
)*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) - (I*(3*a^2*b - b^3)*d*cos(2
*f*x + 2*e) - I*(3*a^2*b - b^3)*d)*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2
*e)) + 2*(3*a*b^2*d - (3*a^2*b - b^3)*d*e + (3*a^2*b - b^3)*c*f - (3*a*b^2*
d - (3*a^2*b - b^3)*d*e + (3*a^2*b - b^3)*c*f)*cos(2*f*x + 2*e))*log(-1/2*c
os(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2) + 2*(3*a*b^2*d - (3*a^2*b -
 b^3)*d*e + (3*a^2*b - b^3)*c*f - (3*a*b^2*d - (3*a^2*b - b^3)*d*e + (3*a^2
*b - b^3)*c*f)*cos(2*f*x + 2*e))*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*
x + 2*e) + 1/2) + 2*((3*a^2*b - b^3)*d*f*x + (3*a^2*b - b^3)*d*e - ((3*a^2*
b - b^3)*d*f*x + (3*a^2*b - b^3)*d*e)*cos(2*f*x + 2*e))*log(-cos(2*f*x + 2*
e) + I*sin(2*f*x + 2*e) + 1) + 2*((3*a^2*b - b^3)*d*f*x + (3*a^2*b - b^3)*d
*e - ((3*a^2*b - b^3)*d*f*x + (3*a^2*b - b^3)*d*e)*cos(2*f*x + 2*e))*log(-c
os(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1) - 2*(6*a*b^2*d*f*x + 6*a*b^2*c*f
+ b^3*d)*sin(2*f*x + 2*e))/(f^2*cos(2*f*x + 2*e) - f^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\left(b\cot\left(fx+e\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*cot(f*x + e) + a)^3, x)
```

maple [B] time = 1.81, size = 745, normalized size = 2.68

$$\frac{b^2\left(-6iadfx\,e^{2i\left(fx+e\right)}-6iacf\,e^{2i\left(fx+e\right)}+2bdfx\,e^{2i\left(fx+e\right)}+6iadfx-ibd\,e^{2i\left(fx+e\right)}+2bcf\,e^{2i\left(fx+e\right)}+6iacf+idb\right)}{f^2\left(e^{2i\left(fx+e\right)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*cot(f*x+e))^3,x)
```

```
[Out] -1/f*b^3*c*ln(exp(I*(f*x+e))-1)+2/f*b^3*c*ln(exp(I*(f*x+e)))-1/f*b^3*c*ln(e
xp(I*(f*x+e))+1)-6*I/f*b*a^2*d*e*x-I*b^3*c*x+b^2*(-6*I*a*d*f*x*exp(2*I*(f*x
+e))-6*I*a*c*f*exp(2*I*(f*x+e))+2*b*d*f*x*exp(2*I*(f*x+e))+6*I*a*d*f*x-I*b*
d*exp(2*I*(f*x+e))+2*b*c*f*exp(2*I*(f*x+e))+6*I*a*c*f+I*d*b)/f^2/(exp(2*I*(
```

```
f*x+e))-1)^2-3/2*I*a^2*b*d*x^2+1/2*a^3*d*x^2+a^3*c*x-3*a*b^2*c*x-3/2*a*b^2*
d*x^2+1/2*I*b^3*d*x^2+3/f*b*a^2*c*ln(exp(I*(f*x+e))+1)+3/f*b*a^2*c*ln(exp(I
*(f*x+e))-1)-6/f*b*a^2*c*ln(exp(I*(f*x+e)))+I/f^2*b^3*d*polylog(2,exp(I*(f*
x+e)))+1/f^2*b^3*d*e*ln(exp(I*(f*x+e))-1)+3/f^2*b^2*a*d*ln(exp(I*(f*x+e))+1
)+3/f^2*b^2*a*d*ln(exp(I*(f*x+e))-1)-6/f^2*b^2*a*d*ln(exp(I*(f*x+e)))-2/f^2
*b^3*d*e*ln(exp(I*(f*x+e)))-1/f^2*b^3*ln(1-exp(I*(f*x+e)))*d*e-1/f*b^3*ln(1
-exp(I*(f*x+e)))*d*x-1/f*b^3*ln(exp(I*(f*x+e))+1)*d*x+3*I*a^2*b*c*x+I/f^2*b
^3*d*e^2+I/f^2*b^3*d*polylog(2,-exp(I*(f*x+e)))+3/f^2*b*ln(1-exp(I*(f*x+e))
)*a^2*d*e+3/f*b*ln(exp(I*(f*x+e))+1)*a^2*d*x+3/f*b*ln(1-exp(I*(f*x+e)))*a^2
*d*x+6/f^2*b*a^2*d*e*ln(exp(I*(f*x+e)))-3/f^2*b*a^2*d*e*ln(exp(I*(f*x+e))-1
)-3*I/f^2*b*a^2*d*polylog(2,-exp(I*(f*x+e)))-3*I/f^2*b*a^2*d*e^2-3*I/f^2*b*
a^2*d*polylog(2,exp(I*(f*x+e)))+2*I/f*b^3*d*e*x
```

maxima [B] time = 1.21, size = 2017, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(f*x + e)*a^3*c + (f*x + e)^2*a^3*d/f - 2*(f*x + e)*a^3*d*e/f + 6*a^
2*b*c*log(sin(f*x + e)) - 6*a^2*b*d*e*log(sin(f*x + e))/f - 2*(12*a*b^2*d*e
 - 12*a*b^2*c*f + (3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^2*d - 2*b^3*d - 2*(
(-3*I*a*b^2 - b^3)*d*e + (3*I*a*b^2 + b^3)*c*f)*(f*x + e) - (2*b^3*d*e - 2*
b^3*c*f + 6*a*b^2*d + 2*(3*a^2*b - b^3)*(f*x + e)*d + 2*(b^3*d*e - b^3*c*f
+ 3*a*b^2*d + (3*a^2*b - b^3)*(f*x + e)*d)*cos(4*f*x + 4*e) - 4*(b^3*d*e -
b^3*c*f + 3*a*b^2*d + (3*a^2*b - b^3)*(f*x + e)*d)*cos(2*f*x + 2*e) - (-2*I
*b^3*d*e + 2*I*b^3*c*f - 6*I*a*b^2*d + (-6*I*a^2*b + 2*I*b^3)*(f*x + e)*d)*
sin(4*f*x + 4*e) - (4*I*b^3*d*e - 4*I*b^3*c*f + 12*I*a*b^2*d + (12*I*a^2*b
- 4*I*b^3)*(f*x + e)*d)*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e
) + 1) - (2*b^3*d*e - 2*b^3*c*f + 6*a*b^2*d + 2*(b^3*d*e - b^3*c*f + 3*a*b^
2*d)*cos(4*f*x + 4*e) - 4*(b^3*d*e - b^3*c*f + 3*a*b^2*d)*cos(2*f*x + 2*e)
- (-2*I*b^3*d*e + 2*I*b^3*c*f - 6*I*a*b^2*d)*sin(4*f*x + 4*e) - (4*I*b^3*d*
e - 4*I*b^3*c*f + 12*I*a*b^2*d)*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos
(f*x + e) - 1) + (2*(3*a^2*b - b^3)*(f*x + e)*d*cos(4*f*x + 4*e) - 4*(3*a^2
*b - b^3)*(f*x + e)*d*cos(2*f*x + 2*e) + (6*I*a^2*b - 2*I*b^3)*(f*x + e)*d*
sin(4*f*x + 4*e) + (-12*I*a^2*b + 4*I*b^3)*(f*x + e)*d*sin(2*f*x + 2*e) + 2
*(3*a^2*b - b^3)*(f*x + e)*d)*arctan2(sin(f*x + e), -cos(f*x + e) + 1) + ((
3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^2*d + 2*(6*a*b^2*d - (-3*I*a*b^2 - b^3
)*d*e - (3*I*a*b^2 + b^3)*c*f)*(f*x + e))*cos(4*f*x + 4*e) - (2*(3*a^2*b -
3*I*a*b^2 - b^3)*(f*x + e)^2*d - 2*b^3*d + (12*a*b^2 + 4*I*b^3)*d*e - (12*a
*b^2 + 4*I*b^3)*c*f + (4*(3*I*a*b^2 + b^3)*d*e + 4*(-3*I*a*b^2 - b^3)*c*f +
 (12*a*b^2 - 4*I*b^3)*d)*(f*x + e))*cos(2*f*x + 2*e) + (2*(3*a^2*b - b^3)*d
*cos(4*f*x + 4*e) - 4*(3*a^2*b - b^3)*d*cos(2*f*x + 2*e) + (6*I*a^2*b - 2*I
*b^3)*d*sin(4*f*x + 4*e) + (-12*I*a^2*b + 4*I*b^3)*d*sin(2*f*x + 2*e) + 2*(
3*a^2*b - b^3)*d)*dilog(-e^(I*f*x + I*e)) + (2*(3*a^2*b - b^3)*d*cos(4*f*x
+ 4*e) - 4*(3*a^2*b - b^3)*d*cos(2*f*x + 2*e) + (6*I*a^2*b - 2*I*b^3)*d*sin
(4*f*x + 4*e) + (-12*I*a^2*b + 4*I*b^3)*d*sin(2*f*x + 2*e) + 2*(3*a^2*b - b
^3)*d)*dilog(e^(I*f*x + I*e)) + (I*b^3*d*e - I*b^3*c*f + 3*I*a*b^2*d + (3*I
*a^2*b - I*b^3)*(f*x + e)*d + (I*b^3*d*e - I*b^3*c*f + 3*I*a*b^2*d + (3*I*a
^2*b - I*b^3)*(f*x + e)*d)*cos(4*f*x + 4*e) + (-2*I*b^3*d*e + 2*I*b^3*c*f -
 6*I*a*b^2*d + (-6*I*a^2*b + 2*I*b^3)*(f*x + e)*d)*cos(2*f*x + 2*e) - (b^3*
d*e - b^3*c*f + 3*a*b^2*d + (3*a^2*b - b^3)*(f*x + e)*d)*sin(4*f*x + 4*e) +
 2*(b^3*d*e - b^3*c*f + 3*a*b^2*d + (3*a^2*b - b^3)*(f*x + e)*d)*sin(2*f*x
+ 2*e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + (I*b^3*
d*e - I*b^3*c*f + 3*I*a*b^2*d + (3*I*a^2*b - I*b^3)*(f*x + e)*d + (I*b^3*d*
e - I*b^3*c*f + 3*I*a*b^2*d + (3*I*a^2*b - I*b^3)*(f*x + e)*d)*cos(4*f*x +
4*e) + (-2*I*b^3*d*e + 2*I*b^3*c*f - 6*I*a*b^2*d + (-6*I*a^2*b + 2*I*b^3)*(
f*x + e)*d)*cos(2*f*x + 2*e) - (b^3*d*e - b^3*c*f + 3*a*b^2*d + (3*a^2*b -
b^3)*(f*x + e)*d)*sin(4*f*x + 4*e) + 2*(b^3*d*e - b^3*c*f + 3*a*b^2*d + (3*
```

```
a^2*b - b^3)*(f*x + e)*d)*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f*x +
e)^2 - 2*cos(f*x + e) + 1) + ((3*I*a^2*b + 3*a*b^2 - I*b^3)*(f*x + e)^2*d +
 (12*I*a*b^2*d - (6*a*b^2 - 2*I*b^3)*d*e + (6*a*b^2 - 2*I*b^3)*c*f)*(f*x +
e))*sin(4*f*x + 4*e) + ((-6*I*a^2*b - 6*a*b^2 + 2*I*b^3)*(f*x + e)^2*d + 2*
I*b^3*d - 4*(3*I*a*b^2 - b^3)*d*e - 4*(-3*I*a*b^2 + b^3)*c*f + ((12*a*b^2 -
 4*I*b^3)*d*e - (12*a*b^2 - 4*I*b^3)*c*f - 4*(3*I*a*b^2 + b^3)*d)*(f*x + e)
)*sin(2*f*x + 2*e))/(-2*I*f*cos(4*f*x + 4*e) + 4*I*f*cos(2*f*x + 2*e) + 2*f
*sin(4*f*x + 4*e) - 4*f*sin(2*f*x + 2*e) - 2*I*f))/f
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\cot\left(e + f\,x\right)\right)^3\,(c + d\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^3*(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))^3*(c + d*x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot\left(e + f\,x\right)\right)^3\,(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))**3,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**3*(c + d*x), x)
```

3.50 $\int \frac{(a+b\cot(e+fx))^3}{c+dx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\cot(e+fx))^3}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))^3/(d*x+c),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cot(e+fx))^3}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])^3/(c + d*x),x]
```

```
[Out] Defer[Int][(a + b*Cot[e + f*x])^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{(a+b\cot(e+fx))^3}{c+dx}\,dx = \int \frac{(a+b\cot(e+fx))^3}{c+dx}\,dx$$

Mathematica [A] time = 9.59, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(e+fx))^3}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x),x]
```

```
[Out] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x), x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\cot\left(fx+e\right)^3+3\,ab^2\cot\left(fx+e\right)^2+3\,a^2b\cot\left(fx+e\right)+a^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b^3*cot(f*x + e)^3 + 3*a*b^2*cot(f*x + e)^2 + 3*a^2*b*cot(f*x + e
) + a^3)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\cot\left(fx+e\right)+a\right)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e) + a)^3/(d*x + c), x)
```

maple [A] time = 8.52, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\cot\left(fx + e\right)\right)^3}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))^3/(d*x+c),x)
```

```
[Out] int((a+b*cot(f*x+e))^3/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^
2)*c^2*f^2)*cos(4*f*x + 4*e)^2*log(d*x + c) + ((a^3 - 3*a*b^2)*d^2*f^2*x^2
+ 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c)*sin(4
*f*x + 4*e)^2 - 4*(b^3*d^2*f*x + b^3*c*d*f - ((a^3 - 3*a*b^2)*d^2*f^2*x^2 +
 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*cos(2
*f*x + 2*e)^2 - 4*(b^3*d^2*f*x + b^3*c*d*f - ((a^3 - 3*a*b^2)*d^2*f^2*x^2 +
 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*sin(2
*f*x + 2*e)^2 + (2*(b^3*d^2*f*x + b^3*c*d*f - 2*((a^3 - 3*a*b^2)*d^2*f^2*x^
2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*co
s(2*f*x + 2*e) + 2*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2
*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c) + (6*a*b^2*d^2*f*x + 6*a*b^2*c*d
*f - b^3*d^2)*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(b^3*d^2*f*x + b^3*c*d
*f - 2*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 -
3*a*b^2)*c^2*f^2)*log(d*x + c))*cos(2*f*x + 2*e) + (d^3*f^2*x^2 + 2*c*d^2*f
^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(4*f*x + 4*
e)^2 + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e)^2 + (d^
3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(4*f*x + 4*e)^2 - 4*(d^3*f^2*x^2
+ 2*c*d^2*f^2*x + c^2*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d^3*f^2
*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(2*f*x + 2*e)^2 + 2*(d^3*f^2*x^2 + 2*c
*d^2*f^2*x + c^2*d*f^2 - 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*
f*x + 2*e))*cos(4*f*x + 4*e) - 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*
cos(2*f*x + 2*e))*integrate(-((3*a^2*b - b^3)*d^2*f^2*x^2 - 3*a*b^2*c*d*f +
 b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 - (3*a*b^2*d^2*f - 2*(3*a^2*b - b^3)*c*d
*f^2)*x)*sin(f*x + e)/(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*
f^2 + (d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2)*cos(f*x + e
)^2 + (d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2)*sin(f*x + e
)^2 + 2*(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2)*cos(f*x +
 e)), x) - (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^
2*f^2*x + c^2*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x +
c^2*d*f^2)*cos(2*f*x + 2*e)^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*s
in(4*f*x + 4*e)^2 - 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(2*
f*x + 2*e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*(d^3*f^2*x^2
+ 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(d^3*f^
2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e))*integrate(-((3*a^2*b -
 b^3)*d^2*f^2*x^2 - 3*a*b^2*c*d*f + b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 - (3*
a*b^2*d^2*f - 2*(3*a^2*b - b^3)*c*d*f^2)*x)*sin(f*x + e)/(d^3*f^2*x^3 + 3*c
*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2 + (d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3
*c^2*d*f^2*x + c^3*f^2)*cos(f*x + e)^2 + (d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3
*c^2*d*f^2*x + c^3*f^2)*sin(f*x + e)^2 - 2*(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 +
 3*c^2*d*f^2*x + c^3*f^2)*cos(f*x + e)), x) + ((a^3 - 3*a*b^2)*d^2*f^2*x^2
+ 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c) + (6*
```

```
a*b^2*d^2*f*x + 6*a*b^2*c*d*f - b^3*d^2 - (6*a*b^2*d^2*f*x + 6*a*b^2*c*d*f
- b^3*d^2)*cos(2*f*x + 2*e) + 2*(b^3*d^2*f*x + b^3*c*d*f - 2*((a^3 - 3*a*b^
2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log
(d*x + c))*sin(2*f*x + 2*e))*sin(4*f*x + 4*e) - (6*a*b^2*d^2*f*x + 6*a*b^2*
c*d*f - b^3*d^2)*sin(2*f*x + 2*e))/(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2
 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^3*f^
2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e)^2 + (d^3*f^2*x^2 + 2*c*
d^2*f^2*x + c^2*d*f^2)*sin(4*f*x + 4*e)^2 - 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x
+ c^2*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d^3*f^2*x^2 + 2*c*d^2*f
^2*x + c^2*d*f^2)*sin(2*f*x + 2*e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2
*d*f^2 - 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) - 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e)
)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\cot\left(e + f\,x\right)\right)^3}{c + d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^3/(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))^3/(c + d*x), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\cot\left(e + f\,x\right)\right)^3}{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))**3/(d*x+c),x)
```

```
[Out] Integral((a + b*cot(e + f*x))**3/(c + d*x), x)
```

3.51 $\int \frac{(a+b\cot(e+fx))^3}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\cot(e+fx))^3}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*cot(f*x+e))^3/(d*x+c)^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cot(e+fx))^3}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Cot[e + f*x])^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(a + b*Cot[e + f*x])^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{(a+b\cot(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(a+b\cot(e+fx))^3}{(c+dx)^2}\,dx$$

Mathematica [A] time = 12.56, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot(e+fx))^3}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x)^2, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\cot\left(fx+e\right)^3+3\,ab^2\cot\left(fx+e\right)^2+3\,a^2b\cot\left(fx+e\right)+a^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*cot(f*x + e)^3 + 3*a*b^2*cot(f*x + e)^2 + 3*a^2*b*cot(f*x + e
) + a^3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\cot\left(fx+e\right)+a\right)^3}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e) + a)^3/(d*x + c)^2, x)
```

maple [A] time = 11.17, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\cot\left(fx + e\right)\right)^3}{(dx + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(f*x+e))^3/(d*x+c)^2,x)
```

```
[Out] int((a+b*cot(f*x+e))^3/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^
2)*c^2*f^2 + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (
a^3 - 3*a*b^2)*c^2*f^2)*cos(4*f*x + 4*e)^2 + 4*((a^3 - 3*a*b^2)*d^2*f^2*x^2
 + b^3*c*d*f + (a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 2*(a^3 - 3*a*b^2)*c*d
*f^2)*x)*cos(2*f*x + 2*e)^2 + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b
^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*sin(4*f*x + 4*e)^2 + 4*((a^3 - 3*a
*b^2)*d^2*f^2*x^2 + b^3*c*d*f + (a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 2*(a
^3 - 3*a*b^2)*c*d*f^2)*x)*sin(2*f*x + 2*e)^2 + 2*((a^3 - 3*a*b^2)*d^2*f^2*x
^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2 - (2*(a^3 - 3*a*
b^2)*d^2*f^2*x^2 + b^3*c*d*f + 2*(a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 4*(
a^3 - 3*a*b^2)*c*d*f^2)*x)*cos(2*f*x + 2*e) - (3*a*b^2*d^2*f*x + 3*a*b^2*c*
d*f - b^3*d^2)*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 2*(2*(a^3 - 3*a*b^2)*d^
2*f^2*x^2 + b^3*c*d*f + 2*(a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 4*(a^3 - 3
*a*b^2)*c*d*f^2)*x)*cos(2*f*x + 2*e) - (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c
^2*d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x
 + c^3*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2
*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e)^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2
 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)^2 - 4*(d^4*f^2*x^3 + 3*c*d
^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(2*f*
x + 2*e)^2 + 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2
 - 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*
x + 2*e))*cos(4*f*x + 4*e) - 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f
^2*x + c^3*d*f^2)*cos(2*f*x + 2*e))*integrate(-((3*a^2*b - b^3)*d^2*f^2*x^2
 - 6*a*b^2*c*d*f + 3*b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 - 2*(3*a*b^2*d^2*f -
 (3*a^2*b - b^3)*c*d*f^2)*x)*sin(f*x + e)/(d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 +
6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f^2*x^4 + 4*c*d^3*f^2*x^
3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2)*cos(f*x + e)^2 + (d^4*f^2*
x^4 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2)*sin(f*
x + e)^2 + 2*(d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f
^2*x + c^4*f^2)*cos(f*x + e)), x) + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*
d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x +
c^3*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^
2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e)^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 +
3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)^2 - 4*(d^4*f^2*x^3 + 3*c*d^3*
f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(2*f*x +
 2*e)^2 + 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 -
2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x +
 2*e))*cos(4*f*x + 4*e) - 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*
```

```
x + c^3*d*f^2)*cos(2*f*x + 2*e))*integrate(-((3*a^2*b - b^3)*d^2*f^2*x^2 -
6*a*b^2*c*d*f + 3*b^3*d^2 + (3*a^2*b - b^3)*c^2*f^2 - 2*(3*a*b^2*d^2*f - (3
*a^2*b - b^3)*c*d*f^2)*x)*sin(f*x + e)/(d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 + 6*c
^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 +
 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2)*cos(f*x + e)^2 + (d^4*f^2*x^4
 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2)*sin(f*x +
 e)^2 - 2*(d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*
x + c^4*f^2)*cos(f*x + e)), x) - 2*(3*a*b^2*d^2*f*x + 3*a*b^2*c*d*f - b^3*d
^2 - (3*a*b^2*d^2*f*x + 3*a*b^2*c*d*f - b^3*d^2)*cos(2*f*x + 2*e) + (2*(a^3
 - 3*a*b^2)*d^2*f^2*x^2 + b^3*c*d*f + 2*(a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*
f + 4*(a^3 - 3*a*b^2)*c*d*f^2)*x)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e) + 2*(3
*a*b^2*d^2*f*x + 3*a*b^2*c*d*f - b^3*d^2)*sin(2*f*x + 2*e))/(d^4*f^2*x^3 +
3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*
x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^4*f^2*x^3 + 3*
c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e)^2 + (d^4*f^2*
x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)^2 - 4
*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x
+ c^3*d*f^2)*sin(2*f*x + 2*e)^2 + 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*
d^2*f^2*x + c^3*d*f^2 - 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x
+ c^3*d*f^2)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(d^4*f^2*x^3 + 3*c*d^3*
f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\cot\left(e + f\,x\right)\right)^3}{\left(c + d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(e + f*x))^3/(c + d*x)^2,x)
```

```
[Out] int((a + b*cot(e + f*x))^3/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\cot\left(e + fx\right)\right)^3}{\left(c + dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(f*x+e))**3/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**3/(c + d*x)**2, x)
```

3.52 $\int \frac{(c+dx)^3}{a+b\cot(e+fx)}\,dx$

Optimal. Leaf size=242

$$-\frac{3bd^2(c+dx)\mathrm{Li}_3\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^3\left(a^2+b^2\right)}+\frac{3ibd(c+dx)^2\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^2\left(a^2+b^2\right)}-\frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f\left(a^2+b^2\right)}-\frac{3ibd^3\mathrm{Li}_4\left(\frac{(a+i}{4f^4\left(a^2+\right.}$$

```
[Out] 1/4*(d*x+c)^4/(a-I*b)/d-b*(d*x+c)^3*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/
(a^2+b^2)/f+3/2*I*b*d*(d*x+c)^2*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))
/(a^2+b^2)/f^2-3/2*b*d^2*(d*x+c)*polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b)
)/(a^2+b^2)/f^3-3/4*I*b*d^3*polylog(4,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^
2+b^2)/f^4
```

Rubi [A] time = 0.34, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3731, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3bd^2(c+dx)\mathrm{PolyLog}\left(3,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^3\left(a^2+b^2\right)}+\frac{3ibd(c+dx)^2\mathrm{PolyLog}\left(2,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^2\left(a^2+b^2\right)}-\frac{3ibd^3\mathrm{PolyLog}\left(4,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{4f^4\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Cot[e + f*x]),x]
```

```
[Out] (c + d*x)^4/(4*(a - I*b)*d) - (b*(c + d*x)^3*Log[1 - ((a + I*b)*E^((2*I)*(e
 + f*x)))/(a - I*b)])/((a^2 + b^2)*f) + (((3*I)/2)*b*d*(c + d*x)^2*PolyLog[
2, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)*f^2) - (3*b*d^2
*(c + d*x)*PolyLog[3, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/(2*(a^2 +
 b^2)*f^3) - (((3*I)/4)*b*d^3*PolyLog[4, ((a + I*b)*E^((2*I)*(e + f*x)))/(a
 - I*b)])/((a^2 + b^2)*f^4)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3731

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist
[2*I*b, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])/((a + I*b)^
2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x]), x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+b\cot(e+fx)}\,dx = \frac{(c+dx)^4}{4(a-ib)d} + (2ib)\int \frac{e^{2i(e+fx)}(c+dx)^3}{(a-ib)^2+\left(-a^2-b^2\right)e^{2i(e+fx)}}\,dx$$

$$= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{(3bd)\int(c+dx)^2\log\left(1+\frac{\left(-a^2-b^2\right)e^{2i(}}{(a-ib)^2}\right.}{\left(a^2+b^2\right)f}$$

$$= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2\left(a^2+b^2\right)f^2} - \frac{(3}{}$$

$$= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2\left(a^2+b^2\right)f^2} - \frac{3b}{}$$

$$= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2\left(a^2+b^2\right)f^2} - \frac{3b}{}$$

$$= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{3ibd(c+dx)^2\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2\left(a^2+b^2\right)f^2} - \frac{3b}{}$$

Mathematica [A] time = 2.14, size = 345, normalized size = 1.43

$$\frac{x\sin(e)\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)}{4(a\sin(e)+b\cos(e))} + \frac{b\left(\frac{3d\left(b\left(1+e^{2ie}\right)-ia\left(-1+e^{2ie}\right)\right)\left(2f^2(c+dx)^2\mathrm{Li}_2\left(\frac{(a-ib)e^{-2i(e+fx)}}{a+ib}\right)+d\left(-2if(c+dx)\mathrm{Li}_3\left(\frac{(a-ib)e^{-2i}}{a+ib}\right.\right.\right.}{f^4\left(a^2+b^2\right)}}{4\left(a\left(-1\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Cot[e + f*x]),x]
```

```
[Out] (b*(((2*I)*(c + d*x)^4)/((a + I*b)*d) - (4*(a*(-1 + E^((2*I)*e)) + I*b*(1 +
 E^((2*I)*e)))*(c + d*x)^3*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x)
))])/((a^2 + b^2)*f) + (3*d*((-I)*a*(-1 + E^((2*I)*e)) + b*(1 + E^((2*I)*e)
))*(2*f^2*(c + d*x)^2*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]
 + d*((-2*I)*f*(c + d*x)*PolyLog[3, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)
))] - d*PolyLog[4, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]))))/((a^2 + b^
2)*f^4)))/(4*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))) + (x*(4*c^3 +
6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Sin[e])/(4*(b*Cos[e] + a*Sin[e]))
```

fricas [C] time = 0.59, size = 1029, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*d^3*f^4*x^4 + 8*a*c*d^2*f^4*x^3 + 12*a*c^2*d*f^4*x^2 + 8*a*c^3*f^4
*x - 3*I*b*d^3*polylog(4, ((a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2
- 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) + 3*I*b*d^3*polylog(4, ((a^
2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x +
2*e))/(a^2 + b^2)) + (6*I*b*d^3*f^2*x^2 + 12*I*b*c*d^2*f^2*x + 6*I*b*c^2*d*
f^2)*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 +
 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) + (-6*I*b*d^3*f^2*x^2 -
12*I*b*c*d^2*f^2*x - 6*I*b*c^2*d*f^2)*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b -
b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^
2) + 1) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log
(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2
+ I*b^2)*sin(2*f*x + 2*e)) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f
^2 - b*c^3*f^3)*log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x
+ 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 4*(b*d^3*f^3*x^3 + 3*b*c*d
^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^
2)*log((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*
b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^
3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*lo
g((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*
b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 6*(b*d^3*f*x + b*c*d^2*f)*polylog(3,
((a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 - 2*a*b - I*b^2)*sin(2*f*x
 + 2*e))/(a^2 + b^2)) - 6*(b*d^3*f*x + b*c*d^2*f)*polylog(3, ((a^2 - 2*I*a*
b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2
 + b^2)))/((a^2 + b^2)*f^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{b\cot\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*cot(f*x + e) + a), x)
```

maple [B] time = 2.56, size = 1386, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*cot(f*x+e)),x)
```

```
[Out] 3/2*I/f^3/(b-I*a)*b*d^3/(a-I*b)*polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))
*x+I/f^4/(b-I*a)*b*d^3*e^3/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))+I
/f/(b-I*a)*b*d^3/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x^3+3/2*I/f
^3/(b-I*a)*b*c*d^2/(a-I*b)*polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))-2*I/
f^4/(b-I*a)*b*d^3*e^3/(I*b-a)*ln(exp(I*(f*x+e)))+3/f^2/(b-I*a)*b*c*d^2/(a-I
*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x-6/f^2/(b-I*a)*b*c*d^2/(a-
I*b)*e^2*x+6/f/(b-I*a)*b*c^2*d/(a-I*b)*e*x+I/f^4/(b-I*a)*b*d^3*e^3/(I*b-a)*
ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+3*I/f/(b-I*a)*b*c*d^2/(a-
I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x^2+1/2/(b-I*a)*b*d^3/(a-I*b)*x
^4+1/4/(a+I*b)*d^3*x^4+1/(a+I*b)*c^3*x+1/(a+I*b)*c*d^2*x^3+3/2/(a+I*b)*c^2*
d*x^2+3/2/f^2/(b-I*a)*b*d^3/(a-I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I
*b))*x^2+3/2/f^2/(b-I*a)*b*c^2*d/(a-I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))
/(a-I*b))-4/f^3/(b-I*a)*b*c*d^2/(a-I*b)*e^3+3/f^2/(b-I*a)*b*c^2*d/(a-I*b)*e
^2+2/f^3/(b-I*a)*b*d^3/(a-I*b)*e^3*x+2*I/f/(b-I*a)*b*c^3/(I*b-a)*ln(exp(I*(
f*x+e)))-I/f/(b-I*a)*b*c^3/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))
*b-a+I*b)+3*I/f^2/(b-I*a)*b*c^2*d/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-
I*b))*e+3*I/f^2/(b-I*a)*b*c^2*d*e/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(
f*x+e))*b-a+I*b)-3*I/f^3/(b-I*a)*b*c*d^2*e^2/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(
f*x+e))/(a-I*b))-3*I/f^3/(b-I*a)*b*c*d^2*e^2/(I*b-a)*ln(a*exp(2*I*(f*x+e))+
I*exp(2*I*(f*x+e))*b-a+I*b)+3*I/f/(b-I*a)*b*c^2*d/(a-I*b)*ln(1-(a+I*b)*exp(
2*I*(f*x+e))/(a-I*b))*x+6*I/f^3/(b-I*a)*b*c*d^2*e^2/(I*b-a)*ln(exp(I*(f*x+e
)))-6*I/f^2/(b-I*a)*b*c^2*d*e/(I*b-a)*ln(exp(I*(f*x+e)))+2/(b-I*a)*b*c*d^2/
(a-I*b)*x^3+3/(b-I*a)*b*c^2*d/(a-I*b)*x^2-3/4/f^4/(b-I*a)*b*d^3/(a-I*b)*pol
ylog(4,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))+3/2/f^4/(b-I*a)*b*d^3/(a-I*b)*e^4
```

maxima [B] time = 0.85, size = 987, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/12*(18*c^2*d*e*(2*(f*x + e)*a/((a^2 + b^2)*f) - 2*b*log(a*tan(f*x + e) +
 b)/((a^2 + b^2)*f) + b*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*f)) - 6*(2*(f*
x + e)*a/(a^2 + b^2) - 2*b*log(a*tan(f*x + e) + b)/(a^2 + b^2) + b*log(tan(
f*x + e)^2 + 1)/(a^2 + b^2))*c^3 - (3*(f*x + e)^4*(a + I*b)*d^3 - 12*I*b*d^
3*polylog(4, (I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - 12*((a + I*b)*d^3*e
 - (a + I*b)*c*d^2*f)*(f*x + e)^3 + 18*((a + I*b)*d^3*e^2 - 2*(a + I*b)*c*d
^2*e*f + (a + I*b)*c^2*d*f^2)*(f*x + e)^2 - 12*((a + I*b)*d^3*e^3 - 3*(a +
I*b)*c*d^2*e^2*f)*(f*x + e) - (-12*I*b*d^3*e^3 + 36*I*b*c*d^2*e^2*f)*arctan
2(b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*cos(2*f*x + 2*e) - b*sin(2
*f*x + 2*e) - a) - (16*I*(f*x + e)^3*b*d^3 + (-36*I*b*d^3*e + 36*I*b*c*d^2*
f)*(f*x + e)^2 + (36*I*b*d^3*e^2 - 72*I*b*c*d^2*e*f + 36*I*b*c^2*d*f^2)*(f*
x + e))*arctan2(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/(a
^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*cos(2*f*x + 2*
e))/(a^2 + b^2)) - (-24*I*(f*x + e)^2*b*d^3 - 18*I*b*d^3*e^2 + 36*I*b*c*d^2
*e*f - 18*I*b*c^2*d*f^2 + (36*I*b*d^3*e - 36*I*b*c*d^2*f)*(f*x + e))*dilog(
(I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) + 6*(b*d^3*e^3 - 3*b*c*d^2*e^2*f)*
log((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*s
in(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e)) - 2*(4*(f*x
 + e)^3*b*d^3 - 9*(b*d^3*e - b*c*d^2*f)*(f*x + e)^2 + 9*(b*d^3*e^2 - 2*b*c*
d^2*e*f + b*c^2*d*f^2)*(f*x + e))*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a
*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 -
 b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) - 6*(4*(f*x + e)*b*d^3 - 3*b*d^3*e + 3
*b*c*d^2*f)*polylog(3, (I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)))/((a^2 + b^
2)*f^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{a+b\cot\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*cot(e + f*x)),x)
```

```
[Out] int((c + d*x)^3/(a + b*cot(e + f*x)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{a+b\cot\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*cot(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**3/(a + b*cot(e + f*x)), x)
```

3.53 $\int \frac{(c+dx)^2}{a+b\cot(e+fx)}\,dx$

Optimal. Leaf size=181

$$\frac{ibd(c+dx)\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f^2\left(a^2+b^2\right)}-\frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f\left(a^2+b^2\right)}-\frac{bd^2\mathrm{Li}_3\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^3\left(a^2+b^2\right)}+\frac{(c+dx)^3}{3d(a-ib)}$$

```
[Out] 1/3*(d*x+c)^3/(a-I*b)/d-b*(d*x+c)^2*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/
(a^2+b^2)/f+I*b*d*(d*x+c)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+
b^2)/f^2-1/2*b*d^2*polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f^
3
```

Rubi [A] time = 0.29, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3731, 2190, 2531, 2282, 6589}

$$\frac{ibd(c+dx)\mathrm{PolyLog}\left(2,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f^2\left(a^2+b^2\right)}-\frac{bd^2\mathrm{PolyLog}\left(3,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^3\left(a^2+b^2\right)}-\frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f\left(a^2+b^2\right)}+\frac{(c+dx}{3d(a-}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Cot[e + f*x]),x]
```

```
[Out] (c + d*x)^3/(3*(a - I*b)*d) - (b*(c + d*x)^2*Log[1 - ((a + I*b)*E^((2*I)*(e
 + f*x)))/(a - I*b)])/((a^2 + b^2)*f) + (I*b*d*(c + d*x)*PolyLog[2, ((a + I
*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)*f^2) - (b*d^2*PolyLog[3,
((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/(2*(a^2 + b^2)*f^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3731

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist
[2*I*b, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])/((a + I*b)^
2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x]), x], x] /; FreeQ[{a,
```

```
b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+b\cot(e+fx)}\,dx = \frac{(c+dx)^3}{3(a-ib)d} + (2ib)\int \frac{e^{2i(e+fx)}(c+dx)^2}{(a-ib)^2+\left(-a^2-b^2\right)e^{2i(e+fx)}}\,dx$$

$$= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{(2bd)\int(c+dx)\log\left(1+\frac{\left(-a^2-b^2\right)e^{2i(e+fx)}}{(a-ib)^2}\right.}{\left(a^2+b^2\right)f}$$

$$= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{ibd(c+dx)\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f^2} - \frac{\left(ibd^2\right)\int}{}$$

$$= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{ibd(c+dx)\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f^2} - \frac{\left(bd^2\right)\mathrm{S}}{}$$

$$= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{ibd(c+dx)\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f^2} - \frac{bd^2\mathrm{Li}_3}{2\left(a\right.}$$

Mathematica [A] time = 1.55, size = 289, normalized size = 1.60

$$\frac{x\sin(e)\left(3c^2+3cdx+d^2x^2\right)}{3(a\sin(e)+b\cos(e))} + \frac{b\left(\frac{3d\left(b\left(1+e^{2ie}\right)-ia\left(-1+e^{2ie}\right)\right)\left(2f(c+dx)\mathrm{Li}_2\left(\frac{(a-ib)e^{-2i(e+fx)}}{a+ib}\right)-id\mathrm{Li}_3\left(\frac{(a-ib)e^{-2i(e+fx)}}{a+ib}\right)\right)}{f^3\left(a^2+b^2\right)} - \frac{6\left(a\left(-1+e^{2ie}\right)+ib\left(1+\right.\right.}{6\left(a\left(-1+e^{2ie}\right)+ib\left(1+e^{2ie}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Cot[e + f*x]),x]
```

```
[Out] (b*(((4*I)*(c + d*x)^3)/((a + I*b)*d) - (6*(a*(-1 + E^((2*I)*e)) + I*b*(1 +
 E^((2*I)*e)))*(c + d*x)^2*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x)
))])/((a^2 + b^2)*f) + (3*d*((-I)*a*(-1 + E^((2*I)*e)) + b*(1 + E^((2*I)*e)
))*(2*f*(c + d*x)*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))] - I
*d*PolyLog[3, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]))/((a^2 + b^2)*f^3
)))/(6*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))) + (x*(3*c^2 + 3*c*d*
x + d^2*x^2)*Sin[e])/(3*(b*Cos[e] + a*Sin[e]))
```

fricas [C] time = 0.75, size = 730, normalized size = 4.03

$$\frac{4\,ad^2f^3x^3 + 12\,acdf^3x^2 + 12\,ac^2f^3x - 3\,bd^2\mathrm{polylog}\left(3, \frac{\left(a^2+2i\,ab-b^2\right)\cos\left(2\,fx+2\,e\right)+\left(i\,a^2-2\,ab-i\,b^2\right)\sin\left(2\,fx+2\,e\right)}{a^2+b^2}\right) - 3\,bd^2\mathrm{po}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*d^2*f^3*x^3 + 12*a*c*d*f^3*x^2 + 12*a*c^2*f^3*x - 3*b*d^2*polylog
(3, ((a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 - 2*a*b - I*b^2)*sin(2
*f*x + 2*e))/(a^2 + b^2)) - 3*b*d^2*polylog(3, ((a^2 - 2*I*a*b - b^2)*cos(2
*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) + (6*
I*b*d^2*f*x + 6*I*b*c*d*f)*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*
f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) +
(-6*I*b*d^2*f*x - 6*I*b*c*d*f)*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*co
s(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1)
 - 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(1/2*a^2 + I*a*b - 1/2*b^2 -
1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) -
6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/
2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 6*
(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log((a^2 + b^2 -
(a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x
 + 2*e))/(a^2 + b^2)) - 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*
c*d*e*f)*log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 +
 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)))/((a^2 + b^2)*f^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{b\cot\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*cot(f*x + e) + a), x)
```

maple [B] time = 2.27, size = 881, normalized size = 4.87

$$\frac{d^2x^3}{3ib+3a}+\frac{cd\,x^2}{ib+a}+\frac{c^2x}{ib+a}-\frac{ib\,c^2\ln\left(a\,e^{2i\left(fx+e\right)}+ie^{2i\left(fx+e\right)}b-a+ib\right)}{f\left(-ia+b\right)\left(ib-a\right)}+\frac{ib\,d^2\ln\left(1-\frac{(ib+a)e^{2i\left(fx+e\right)}}{-ib+a}\right)x^2}{f\left(-ia+b\right)\left(-ib+a\right)}+\frac{ib\,d^2\,\text{polyl}}{2f^3\left(-i\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*cot(f*x+e)),x)
```

```
[Out] 1/3/(a+I*b)*d^2*x^3+1/(a+I*b)*c*d*x^2+1/(a+I*b)*c^2*x-I/f/(b-I*a)*b*c^2/(I*
b-a)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+I/f/(b-I*a)*b*d^2/(a
-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x^2+2*I/f^2/(b-I*a)*b*c*d*e/(I
*b-a)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+1/2*I/f^3/(b-I*a)*b
*d^2/(a-I*b)*polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))+2/3/(b-I*a)*b*d^2/
(a-I*b)*x^3-2/f^2/(b-I*a)*b*d^2/(a-I*b)*e^2*x-4/3/f^3/(b-I*a)*b*d^2/(a-I*b)
*e^3-4*I/f^2/(b-I*a)*b*c*d*e/(I*b-a)*ln(exp(I*(f*x+e)))+2*I/f/(b-I*a)*b*c*d
/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x+2*I/f^3/(b-I*a)*b*d^2*e^2
/(I*b-a)*ln(exp(I*(f*x+e)))+2*I/f/(b-I*a)*b*c^2/(I*b-a)*ln(exp(I*(f*x+e)))-
I/f^3/(b-I*a)*b*d^2*e^2/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-
a+I*b)+1/f^2/(b-I*a)*b*d^2/(a-I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*
b))*x-I/f^3/(b-I*a)*b*d^2/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*e^
2+2*I/f^2/(b-I*a)*b*c*d/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*e+2/
(b-I*a)*b*c*d/(a-I*b)*x^2+4/f/(b-I*a)*b*c*d/(a-I*b)*e*x+2/f^2/(b-I*a)*b*c*d
/(a-I*b)*e^2+1/f^2/(b-I*a)*b*c*d/(a-I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))
/(a-I*b))
```

maxima [B] time = 0.82, size = 719, normalized size = 3.97

$$-\frac{6\,cde\left(\frac{2\left(fx+e\right)a}{\left(a^2+b^2\right)f}-\frac{2\,b\log\left(a\tan\left(fx+e\right)+b\right)}{\left(a^2+b^2\right)f}+\frac{b\log\left(\tan\left(fx+e\right)^2+1\right)}{\left(a^2+b^2\right)f}\right)-3\left(\frac{2\left(fx+e\right)a}{a^2+b^2}-\frac{2\,b\log\left(a\tan\left(fx+e\right)+b\right)}{a^2+b^2}+\frac{b\log\left(\tan\left(fx+e\right)^2+1\right)}{a^2+b^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/6*(6*c*d*e*(2*(f*x + e)*a/((a^2 + b^2)*f) - 2*b*log(a*tan(f*x + e) + b)/
((a^2 + b^2)*f) + b*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*f)) - 3*(2*(f*x +
e)*a/(a^2 + b^2) - 2*b*log(a*tan(f*x + e) + b)/(a^2 + b^2) + b*log(tan(f*x
+ e)^2 + 1)/(a^2 + b^2))*c^2 - (2*(f*x + e)^3*(a + I*b)*d^2 + 6*(f*x + e)*(
a + I*b)*d^2*e^2 - 6*I*b*d^2*e^2*arctan2(b*cos(2*f*x + 2*e) + a*sin(2*f*x +
 2*e) + b, a*cos(2*f*x + 2*e) - b*sin(2*f*x + 2*e) - a) - 3*b*d^2*e^2*log((
a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*
f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e)) - 3*b*d^2*polylo
g(3, (I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - 6*((a + I*b)*d^2*e - (a + I
*b)*c*d*f)*(f*x + e)^2 - (6*I*(f*x + e)^2*b*d^2 + (-12*I*b*d^2*e + 12*I*b*c
*d*f)*(f*x + e))*arctan2(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x +
 2*e))/(a^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*cos(2
*f*x + 2*e))/(a^2 + b^2)) - (-6*I*(f*x + e)*b*d^2 + 6*I*b*d^2*e - 6*I*b*c*d
*f)*dilog((I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - 3*((f*x + e)^2*b*d^2 -
 2*(b*d^2*e - b*c*d*f)*(f*x + e))*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a
*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 -
 b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)))/((a^2 + b^2)*f^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{a+b\cot\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*cot(e + f*x)),x)
```

```
[Out] int((c + d*x)^2/(a + b*cot(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{a+b\cot\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*cot(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**2/(a + b*cot(e + f*x)), x)
```

3.54 $\int \frac{c+dx}{a+b\cot(e+fx)}\,dx$

Optimal. Leaf size=126

$$-\frac{b(c+dx)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f\left(a^2+b^2\right)}+\frac{ibd\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^2\left(a^2+b^2\right)}+\frac{(c+dx)^2}{2d(a-ib)}$$

```
[Out] 1/2*(d*x+c)^2/(a-I*b)/d-b*(d*x+c)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a
^2+b^2)/f+1/2*I*b*d*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f
^2
```

Rubi [A] time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3731, 2190, 2279, 2391}

$$\frac{ibd\,\mathrm{PolyLog}\left(2,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^2\left(a^2+b^2\right)}-\frac{b(c+dx)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f\left(a^2+b^2\right)}+\frac{(c+dx)^2}{2d(a-ib)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Cot[e + f*x]),x]
```

```
[Out] (c + d*x)^2/(2*(a - I*b)*d) - (b*(c + d*x)*Log[1 - ((a + I*b)*E^((2*I)*(e +
 f*x)))/(a - I*b)])/((a^2 + b^2)*f) + ((I/2)*b*d*PolyLog[2, ((a + I*b)*E^((
2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)*f^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3731

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist
[2*I*b, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])/((a + I*b)^
2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x]), x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{a+b\cot(e+fx)}\,dx = \frac{(c+dx)^2}{2(a-ib)d} + (2ib)\int \frac{e^{2i(e+fx)}(c+dx)}{(a-ib)^2+\left(-a^2-b^2\right)e^{2i(e+fx)}}\,dx$$

$$= \frac{(c+dx)^2}{2(a-ib)d} - \frac{b(c+dx)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{(bd)\int \log\left(1+\frac{\left(-a^2-b^2\right)e^{2i(e+fx)}}{(a-ib)^2}\right)dx}{\left(a^2+b^2\right)f}$$

$$= \frac{(c+dx)^2}{2(a-ib)d} - \frac{b(c+dx)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} - \frac{(ibd)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{\left(-a^2-b^2\right)x}{(a-ib)^2}\right)}{x}\,dx, x, e^{2i(e+}\right.}{2\left(a^2+b^2\right)f^2}$$

$$= \frac{(c+dx)^2}{2(a-ib)d} - \frac{b(c+dx)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{\left(a^2+b^2\right)f} + \frac{ibd\,\text{Li}_2\left(\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2\left(a^2+b^2\right)f^2}$$

Mathematica [A] time = 1.86, size = 182, normalized size = 1.44

$$\frac{x\sin(e)(2c+dx)}{2(a\sin(e)+b\cos(e))} + \frac{1}{2}b\left(-\frac{2(c+dx)\log\left(1+\frac{(-a+ib)e^{-2i(e+fx)}}{a+ib}\right)}{f\left(a^2+b^2\right)} - \frac{id\,\text{Li}_2\left(\frac{(a-ib)e^{-2i(e+fx)}}{a+ib}\right)}{f^2\left(a^2+b^2\right)} + \frac{2i(c+dx)^2}{d(a+ib)\left(a\left(-1+e^{2ie}\right)+\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Cot[e + f*x]),x]
```

```
[Out] (b*(((2*I)*(c + d*x)^2)/((a + I*b)*d*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2
*I)*e)))) - (2*(c + d*x)*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x)))
])/((a^2 + b^2)*f) - (I*d*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x
)))])/((a^2 + b^2)*f^2)))/2 + (x*(2*c + d*x)*Sin[e])/(2*(b*Cos[e] + a*Sin[e
]))
```

fricas [B] time = 0.66, size = 475, normalized size = 3.77

$$\frac{2\,adf^2x^2 + 4\,acf^2x + i\,bd\,\text{Li}_2\left(-\frac{a^2+b^2-\left(a^2+2i\,ab-b^2\right)\cos\left(2\,fx+2\,e\right)+\left(-i\,a^2+2\,ab+i\,b^2\right)\sin\left(2\,fx+2\,e\right)}{a^2+b^2}+1\right) - i\,bd\,\text{Li}_2\left(-\frac{a^2+b^2-\left(a^2-2i\,a\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d*f^2*x^2 + 4*a*c*f^2*x + I*b*d*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b
 - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2
+ b^2) + 1) - I*b*d*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2
*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) + 2*(b*d*e
 - b*c*f)*log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*cos(2*f*x + 2*e)
+ 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) + 2*(b*d*e - b*c*f)*log(-1/2*a^2 +
I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*si
n(2*f*x + 2*e)) - 2*(b*d*f*x + b*d*e)*log((a^2 + b^2 - (a^2 + 2*I*a*b - b^2
)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)
) - 2*(b*d*f*x + b*d*e)*log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x +
2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)))/((a^2 + b^2)
*f^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx+c}{b\cot\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*cot(f*x + e) + a), x)
```

maple [B] time = 2.25, size = 445, normalized size = 3.53

$$\frac{d\,x^2}{2ib+2a}+\frac{cx}{ib+a}-\frac{2bc\ln\left(\mathrm{e}^{i\left(fx+e\right)}\right)}{f\left(ib+a\right)\left(ib-a\right)}+\frac{bc\ln\left(a\,\mathrm{e}^{2i\left(fx+e\right)}+i\mathrm{e}^{2i\left(fx+e\right)}b-a+ib\right)}{f\left(ib+a\right)\left(ib-a\right)}-\frac{bd\ln\left(1-\frac{(ib+a)\mathrm{e}^{2i\left(fx+e\right)}}{-ib+a}\right)x}{f\left(ib+a\right)\left(-ib+a\right)}-\frac{bd\ln}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*cot(f*x+e)),x)
```

```
[Out] 1/2/(a+I*b)*d*x^2+1/(a+I*b)*c*x-2/f*b/(a+I*b)*c/(I*b-a)*ln(exp(I*(f*x+e)))+
1/f*b/(a+I*b)*c/(I*b-a)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)-1
/f*b/(a+I*b)*d/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x-1/f^2*b/(a+
I*b)*d/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*e+I*b/(a+I*b)*d/(a-I*
b)*x^2+2*I/f*b/(a+I*b)*d/(a-I*b)*e*x+I/f^2*b/(a+I*b)*d/(a-I*b)*e^2+1/2*I/f^
2*b/(a+I*b)*d/(a-I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))+2/f^2*b/(
a+I*b)*d*e/(I*b-a)*ln(exp(I*(f*x+e)))-1/f^2*b/(a+I*b)*d*e/(I*b-a)*ln(a*exp(
2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)
```

maxima [B] time = 0.67, size = 406, normalized size = 3.22

$$\frac{(a+i\,b)df^2x^2+2\,(a+i\,b)cf^2x-2i\,bdfx\arctan\left(-\frac{2\,ab\cos\left(2\,fx+2\,e\right)+\left(a^2-b^2\right)\sin\left(2\,fx+2\,e\right)}{a^2+b^2},\frac{2\,ab\sin\left(2\,fx+2\,e\right)+a^2+b^2-\left(a^2-b^2\right.}{a^2+b^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((a + I*b)*d*f^2*x^2 + 2*(a + I*b)*c*f^2*x - 2*I*b*d*f*x*arctan2(-(2*a*
b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*sin(
2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) - b*d
*f*x*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 +
b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e))/(a^2
+ b^2)) - 2*I*b*c*f*arctan2(b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*
cos(2*f*x + 2*e) - b*sin(2*f*x + 2*e) - a) - b*c*f*log((a^2 + b^2)*cos(2*f*
x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2
+ b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e)) + I*b*d*dilog((I*a - b)*e^(2*I*f*x
+ 2*I*e)/(I*a + b)))/((a^2 + b^2)*f^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{a+b\cot\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*cot(e + f*x)),x)
```

```
[Out] int((c + d*x)/(a + b*cot(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{a+b\cot\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x)
[Out] Integral((c + d*x)/(a + b*cot(e + f*x)), x)
```

3.55 $\int \frac{1}{(c+dx)(a+b\cot(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\cot(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*cot(f*x+e)),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Cot[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Cot[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\cot(e+fx))}\,dx$$

Mathematica [A] time = 1.97, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{adx+ac+(bdx+bc)\cot\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*cot(f*x + e)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(b\cot\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*cot(f*x + e) + a)), x)
```

maple [A] time = 3.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+b\cot\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*cot(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+b*cot(f*x+e)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-2\left(a^2b+b^3\right)d\int \frac{2\,ab\cos\left(2\,fx+2\,e\right)+\left(a^2-b^2\right)\sin(}{\left(a^4+2\,a^2b^2+b^4\right)dx+\left(\left(a^4+2\,a^2b^2+b^4\right)dx+\left(a^4+2\,a^2b^2+b^4\right)c\right)\cos\left(2\,fx+2\,e\right)^2+\left(\left(a^4+2\,a^2b^2+b^4\right)dx+\left(a^4+2\,a^2b^2+b^4\right)c\right)\sin\left(2\,fx+2\,e\right.}$$
$$\left(a^2+b^2\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] (2*(a^2*b + b^3)*d*integrate(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f
*x + 2*e))/((a^4 + 2*a^2*b^2 + b^4)*d*x + ((a^4 + 2*a^2*b^2 + b^4)*d*x + (a
^4 + 2*a^2*b^2 + b^4)*c)*cos(2*f*x + 2*e)^2 + ((a^4 + 2*a^2*b^2 + b^4)*d*x
+ (a^4 + 2*a^2*b^2 + b^4)*c)*sin(2*f*x + 2*e)^2 + (a^4 + 2*a^2*b^2 + b^4)*c
 - 2*((a^4 - b^4)*d*x + (a^4 - b^4)*c)*cos(2*f*x + 2*e) + 4*((a^3*b + a*b^3
)*d*x + (a^3*b + a*b^3)*c)*sin(2*f*x + 2*e)), x) + a*log(d*x + c))/((a^2 +
b^2)*d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\cot\left(e+f\,x\right)\right)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cot(e + f*x))*(c + d*x)),x)
```

```
[Out] int(1/((a + b*cot(e + f*x))*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\cot\left(e+fx\right)\right)\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x)
```

```
[Out] Integral(1/((a + b*cot(e + f*x))*(c + d*x)), x)
```

3.56 $\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\cot(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*cot(f*x+e)),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Cot[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Cot[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+b\cot(e+fx))}\,dx$$

Mathematica [A] time = 3.94, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])), x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{ad^2x^2+2\,acdx+ac^2+\left(bd^2x^2+2\,bcdx+bc^2\right)\cot\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*cot(f*x + e)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(b\cot\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*cot(f*x + e) + a)), x)
```

maple [A] time = 5.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(a+b\cot\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*cot(f*x+e)),x)
[Out] int(1/(d*x+c)^2/(a+b*cot(f*x+e)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-2\left(\left(a^2b+b^3\right)d^2x+\left(a^2b+b^3\right)cd\right)\int \frac{}{\left(a^4+2\,a^2b^2+b^4\right)d^2x^2+2\left(a^4+2\,a^2b^2+b^4\right)cdx+\left(a^4+2\,a^2b^2+b^4\right)c^2+\left(\left(a^4+2\,a^2b^2+b^4\right)d^2x^2+2\left(a^4+2\,a^2b^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="maxima")
[Out] (2*((a^2*b + b^3)*d^2*x + (a^2*b + b^3)*c*d)*integrate(-(2*a*b*cos(2*f*x +
2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/((a^4 + 2*a^2*b^2 + b^4)*d^2*x^2 + 2*(
a^4 + 2*a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b^4)*c^2 + ((a^4 + 2*a^2*
b^2 + b^4)*d^2*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b
^4)*c^2)*cos(2*f*x + 2*e)^2 + ((a^4 + 2*a^2*b^2 + b^4)*d^2*x^2 + 2*(a^4 + 2
*a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b^4)*c^2)*sin(2*f*x + 2*e)^2 - 2
*((a^4 - b^4)*d^2*x^2 + 2*(a^4 - b^4)*c*d*x + (a^4 - b^4)*c^2)*cos(2*f*x +
2*e) + 4*((a^3*b + a*b^3)*d^2*x^2 + 2*(a^3*b + a*b^3)*c*d*x + (a^3*b + a*b^
3)*c^2)*sin(2*f*x + 2*e)), x) - a)/((a^2 + b^2)*d^2*x + (a^2 + b^2)*c*d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\cot\left(e+f\,x\right)\right)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cot(e + f*x))*(c + d*x)^2),x)
[Out] int(1/((a + b*cot(e + f*x))*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\cot\left(e+fx\right)\right)(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*cot(f*x+e)),x)
[Out] Integral(1/((a + b*cot(e + f*x))*(c + d*x)**2), x)
```

3.57 $\int \frac{(c+dx)^3}{(a+b\cot(e+fx))^2}\,dx$

Optimal. Leaf size=839

$$-\frac{b(c+dx)^4}{(a+ib)^2(ia+b)d}+\frac{(c+dx)^4}{4(a+ib)^2d}-\frac{b^2(c+dx)^4}{\left(a^2+b^2\right)^2 d}-\frac{2b\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^3}{(a-ib)(a+ib)^2f}-\frac{2ib^2\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(}{\left(a^2+b^2\right)^2 f}$$

```
[Out] -3*I*b^2*d^2*(d*x+c)*polylog(3,(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2
)^2/f^3-2*b^2*(d*x+c)^3/(a-I*b)/(a+I*b)^2/(I*a+b-(I*a-b)*exp(2*I*e+2*I*f*x)
)/f+1/4*(d*x+c)^4/(a+I*b)^2/d-b*(d*x+c)^4/(a+I*b)^2/(I*a+b)/d-b^2*(d*x+c)^4
/(a^2+b^2)^2/d+3*b^2*d*(d*x+c)^2*ln(1-(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(
a^2+b^2)^2/f^2-2*b*(d*x+c)^3*ln(1-(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a-I*
b)/(a+I*b)^2/f-3*I*b^2*d^2*(d*x+c)*polylog(2,(a+I*b)*exp(2*I*e+2*I*f*x)/(a-
I*b))/(a^2+b^2)^2/f^3-2*I*b^2*(d*x+c)^3*ln(1-(a+I*b)*exp(2*I*e+2*I*f*x)/(a-
I*b))/(a^2+b^2)^2/f-3*b*d*(d*x+c)^2*polylog(2,(a+I*b)*exp(2*I*e+2*I*f*x)/(a
-I*b))/(a+I*b)^2/(I*a+b)/f^2-3*b^2*d*(d*x+c)^2*polylog(2,(a+I*b)*exp(2*I*e+
2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^2+3/2*b^2*d^3*polylog(3,(a+I*b)*exp(2*I*e+2
*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^4-3*b*d^2*(d*x+c)*polylog(3,(a+I*b)*exp(2*I*
e+2*I*f*x)/(a-I*b))/(a-I*b)/(a+I*b)^2/f^3-2*I*b^2*(d*x+c)^3/(a^2+b^2)^2/f+3
/2*b*d^3*polylog(4,(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a+I*b)^2/(I*a+b)/f^
4+3/2*b^2*d^3*polylog(4,(a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^4
```

Rubi [A] time = 1.97, antiderivative size = 839, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3734, 2185, 2184, 2190, 2531, 6609, 2282, 6589, 2191}

$$-\frac{b(c+dx)^4}{(a+ib)^2(ia+b)d}+\frac{(c+dx)^4}{4(a+ib)^2d}-\frac{b^2(c+dx)^4}{\left(a^2+b^2\right)^2 d}-\frac{2b\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^3}{(a-ib)(a+ib)^2f}-\frac{2ib^2\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(}{\left(a^2+b^2\right)^2 f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((-2*I)*b^2*(c + d*x)^3)/((a^2 + b^2)^2*f) - (2*b^2*(c + d*x)^3)/((a - I*b)
*(a + I*b)^2*(I*a + b - (I*a - b)*E^((2*I)*e + (2*I)*f*x))*f) + (c + d*x)^4
/(4*(a + I*b)^2*d) - (b*(c + d*x)^4)/((a + I*b)^2*(I*a + b)*d) - (b^2*(c +
d*x)^4)/((a^2 + b^2)^2*d) + (3*b^2*d*(c + d*x)^2*Log[1 - ((a + I*b)*E^((2*I
)*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^2) - (2*b*(c + d*x)^3*Log[1
- ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a - I*b)*(a + I*b)^2*f)
 - ((2*I)*b^2*(c + d*x)^3*Log[1 - ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a -
I*b)])/((a^2 + b^2)^2*f) - ((3*I)*b^2*d^2*(c + d*x)*PolyLog[2, ((a + I*b)*E
^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^3) - (3*b*d*(c + d*x)^
2*PolyLog[2, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a + I*b)^2*(
I*a + b)*f^2) - (3*b^2*d*(c + d*x)^2*PolyLog[2, ((a + I*b)*E^((2*I)*e + (2*
I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^2) + (3*b^2*d^3*PolyLog[3, ((a + I*b)
*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/(2*(a^2 + b^2)^2*f^4) - (3*b*d^2*(c +
 d*x)*PolyLog[3, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a - I*b)
*(a + I*b)^2*f^3) - ((3*I)*b^2*d^2*(c + d*x)*PolyLog[3, ((a + I*b)*E^((2*I)
*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^3) + (3*b*d^3*PolyLog[4, ((a
+ I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/(2*(a + I*b)^2*(I*a + b)*f^4) +
 (3*b^2*d^3*PolyLog[4, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/(2*(
a^2 + b^2)^2*f^4)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
```

```
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2185

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2191

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3734

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - (2*I*b)/(a^2
+ b^2 + (a - I*b)^2*E^(2*I*(e + f*x))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+b\cot(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^3}{(a+ib)^2} - \frac{4b^2(c+dx)^3}{(ia-b)^2\left(ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}\right)^2} + \frac{4b}{(a+ib)^2\left(-ia\left(1-\right.\right.} \right.$$

$$= \frac{(c+dx)^4}{4(a+ib)^2 d} + \frac{(4b)\int \frac{(c+dx)^3}{-ia\left(1-\frac{ib}{a}\right)+ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}\,dx}{(a+ib)^2} - \frac{\left(4b^2\right)\int \frac{(c+dx)^3}{\left(ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}\right)^2}}{(ia-b)^2}$$

$$= \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{b(c+dx)^4}{(a+ib)^2(ia+b)d} + \frac{\left(4b^2\right)\int \frac{(c+dx)^3}{ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}\,dx}{(a+ib)^2(ia+b)} + \frac{(4b)\int \frac{}{-ia\left(1\right.}}$$

$$= -\frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{b(c+dx)^4}{(a+ib)^2(ia+b)d}$$

$$= -\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2 f} - \frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{}{(}$$

$$= -\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2 f} - \frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{}{(}$$

$$= -\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2 f} - \frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{}{(}$$

$$= -\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2 f} - \frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{}{(}$$

$$= -\frac{2ib^2(c+dx)^3}{\left(a^2+b^2\right)^2 f} - \frac{2b^2(c+dx)^3}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{}{(}$$

Mathematica [B] time = 10.66, size = 1682, normalized size = 2.00

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] (b*((-4*b*(c + d*x)^3)/(a + I*b) + (2*a*f*(c + d*x)^4)/((a + I*b)*d) + (12*
c*d*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(-(b*d) + a*c*f)*x*Log[1
```

```
 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x)))])/((a - I*b)*((-I)*a + b)*f)
- (6*d^2*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(b*d - 2*a*c*f)*x^2
*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x)))])/((a - I*b)*((-I)*a +
b)*f) + (4*a*d^3*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*x^3*Log[1 +
 (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x)))])/((a - I*b)*((-I)*a + b)) + (2
*c^2*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(-3*b*d + 2*a*c*f)*(2*f
*x + I*Log[a - I*b - (a + I*b)*E^((2*I)*(e + f*x))]))/((a^2 + b^2)*f) - (6*
c*d*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(-(b*d) + a*c*f)*PolyLog
[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))])/((a^2 + b^2)*f^2) + (3*d^2*
(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(b*d - 2*a*c*f)*(2*f*x*PolyL
og[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))] - I*PolyLog[3, (a - I*b)/(
(a + I*b)*E^((2*I)*(e + f*x)))]))/((a^2 + b^2)*f^3) - (3*a*d^3*(a*(-1 + E^(
(2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(2*f^2*x^2*PolyLog[2, (a - I*b)/((a + I*
b)*E^((2*I)*(e + f*x)))] - (2*I)*f*x*PolyLog[3, (a - I*b)/((a + I*b)*E^((2*
I)*(e + f*x)))] - PolyLog[4, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]))/(
(a^2 + b^2)*f^3)))/(2*(a - I*b)*(a + I*b)*((-I)*a*(-1 + E^((2*I)*e)) + b*(1
 + E^((2*I)*e)))*f) + (3*x^2*(-(a*c^2*d) - I*b*c^2*d + a*c^2*d*Cos[2*e] - I
*b*c^2*d*Cos[2*e] + I*a*c^2*d*Sin[2*e] + b*c^2*d*Sin[2*e]))/(2*(a - I*b)*(a
 + I*b)*(-a + I*b + a*Cos[2*e] + I*b*Cos[2*e] + I*a*Sin[2*e] - b*Sin[2*e]))
 + (x^3*(-(a*c*d^2) - I*b*c*d^2 + a*c*d^2*Cos[2*e] - I*b*c*d^2*Cos[2*e] + I
*a*c*d^2*Sin[2*e] + b*c*d^2*Sin[2*e]))/((a - I*b)*(a + I*b)*(-a + I*b + a*C
os[2*e] + I*b*Cos[2*e] + I*a*Sin[2*e] - b*Sin[2*e])) + (x^4*(-(a*d^3) - I*b
*d^3 + a*d^3*Cos[2*e] - I*b*d^3*Cos[2*e] + I*a*d^3*Sin[2*e] + b*d^3*Sin[2*e
]))/(4*(a - I*b)*(a + I*b)*(-a + I*b + a*Cos[2*e] + I*b*Cos[2*e] + I*a*Sin[
2*e] - b*Sin[2*e])) + x*(c^3/(a^2 - (2*I)*a*b - b^2 + a^2*Cos[4*e] + (2*I)*
a*b*Cos[4*e] - b^2*Cos[4*e] + I*a^2*Sin[4*e] - 2*a*b*Sin[4*e] - I*b^2*Sin[4
*e]) + (((-I)*a - b - I*a*Cos[2*e] + b*Cos[2*e] + a*Sin[2*e] + I*b*Sin[2*e]
)*(4*a*b*c^3*Cos[2*e] + (4*I)*a*b*c^3*Sin[2*e]))/((a - I*b)*(a + I*b)*(-a +
 I*b + a*Cos[2*e] + I*b*Cos[2*e] + I*a*Sin[2*e] - b*Sin[2*e])*(a^2 - (2*I)*
a*b - b^2 + a^2*Cos[4*e] + (2*I)*a*b*Cos[4*e] - b^2*Cos[4*e] + I*a^2*Sin[4*
e] - 2*a*b*Sin[4*e] - I*b^2*Sin[4*e])) + (c^3*Cos[4*e] + I*c^3*Sin[4*e])/(a
^2 - (2*I)*a*b - b^2 + a^2*Cos[4*e] + (2*I)*a*b*Cos[4*e] - b^2*Cos[4*e] + I
*a^2*Sin[4*e] - 2*a*b*Sin[4*e] - I*b^2*Sin[4*e])) + (b^2*c^3*Sin[f*x] + 3*b
^2*c^2*d*x*Sin[f*x] + 3*b^2*c*d^2*x^2*Sin[f*x] + b^2*d^3*x^3*Sin[f*x])/((a
- I*b)*(a + I*b)*f*(b*Cos[e] + a*Sin[e])*(b*Cos[e + f*x] + a*Sin[e + f*x]))
```

fricas [C] time = 0.87, size = 3359, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2*b - b^3)*d^3*f^4*x^4 - 4*a*b^2*c^3*f^3 - 4*(a*b^2*d^3*f^3 - (a^2*
b - b^3)*c*d^2*f^4)*x^3 - 6*(2*a*b^2*c*d^2*f^3 - (a^2*b - b^3)*c^2*d*f^4)*x
^2 - 4*(3*a*b^2*c^2*d*f^3 - (a^2*b - b^3)*c^3*f^4)*x + ((a^2*b - b^3)*d^3*f
^4*x^4 - 4*a*b^2*c^3*f^3 - 4*(a*b^2*d^3*f^3 - (a^2*b - b^3)*c*d^2*f^4)*x^3
- 6*(2*a*b^2*c*d^2*f^3 - (a^2*b - b^3)*c^2*d*f^4)*x^2 - 4*(3*a*b^2*c^2*d*f^
3 - (a^2*b - b^3)*c^3*f^4)*x)*cos(2*f*x + 2*e) + (6*I*a*b^2*d^3*f^2*x^2 + 6
*I*a*b^2*c^2*d*f^2 - 6*I*b^3*c*d^2*f + 6*I*(2*a*b^2*c*d^2*f^2 - b^3*d^3*f)*
x + (6*I*a*b^2*d^3*f^2*x^2 + 6*I*a*b^2*c^2*d*f^2 - 6*I*b^3*c*d^2*f + 6*I*(2
*a*b^2*c*d^2*f^2 - b^3*d^3*f)*x)*cos(2*f*x + 2*e) + (6*I*a^2*b*d^3*f^2*x^2
+ 6*I*a^2*b*c^2*d*f^2 - 6*I*a*b^2*c*d^2*f + 6*I*(2*a^2*b*c*d^2*f^2 - a*b^2*
d^3*f)*x)*sin(2*f*x + 2*e))*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2
*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) +
 (-6*I*a*b^2*d^3*f^2*x^2 - 6*I*a*b^2*c^2*d*f^2 + 6*I*b^3*c*d^2*f - 6*I*(2*a
*b^2*c*d^2*f^2 - b^3*d^3*f)*x + (-6*I*a*b^2*d^3*f^2*x^2 - 6*I*a*b^2*c^2*d*f
^2 + 6*I*b^3*c*d^2*f - 6*I*(2*a*b^2*c*d^2*f^2 - b^3*d^3*f)*x)*cos(2*f*x + 2
*e) + (-6*I*a^2*b*d^3*f^2*x^2 - 6*I*a^2*b*c^2*d*f^2 + 6*I*a*b^2*c*d^2*f - 6
*I*(2*a^2*b*c*d^2*f^2 - a*b^2*d^3*f)*x)*sin(2*f*x + 2*e))*dilog(-(a^2 + b^2
```

```
 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f
*x + 2*e))/(a^2 + b^2) + 1) + 2*(2*a*b^2*d^3*e^3 - 2*a*b^2*c^3*f^3 + 3*b^3*
d^3*e^2 + 3*(2*a*b^2*c^2*d*e + b^3*c^2*d)*f^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*
d^2*e)*f + (2*a*b^2*d^3*e^3 - 2*a*b^2*c^3*f^3 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*
c^2*d*e + b^3*c^2*d)*f^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f)*cos(2*f*x +
 2*e) + (2*a^2*b*d^3*e^3 - 2*a^2*b*c^3*f^3 + 3*a*b^2*d^3*e^2 + 3*(2*a^2*b*c
^2*d*e + a*b^2*c^2*d)*f^2 - 6*(a^2*b*c*d^2*e^2 + a*b^2*c*d^2*e)*f)*sin(2*f*
x + 2*e))*log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*cos(2*f*x + 2*e)
 + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) + 2*(2*a*b^2*d^3*e^3 - 2*a*b^2*c^3*
f^3 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*c^2*d*e + b^3*c^2*d)*f^2 - 6*(a*b^2*c*d^2*
e^2 + b^3*c*d^2*e)*f + (2*a*b^2*d^3*e^3 - 2*a*b^2*c^3*f^3 + 3*b^3*d^3*e^2 +
 3*(2*a*b^2*c^2*d*e + b^3*c^2*d)*f^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f)
*cos(2*f*x + 2*e) + (2*a^2*b*d^3*e^3 - 2*a^2*b*c^3*f^3 + 3*a*b^2*d^3*e^2 +
3*(2*a^2*b*c^2*d*e + a*b^2*c^2*d)*f^2 - 6*(a^2*b*c*d^2*e^2 + a*b^2*c*d^2*e)
*f)*sin(2*f*x + 2*e))*log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(
2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 2*(2*a*b^2*d^3*f^3*x
^3 + 2*a*b^2*d^3*e^3 + 6*a*b^2*c^2*d*e*f^2 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*c*d
^2*f^3 - b^3*d^3*f^2)*x^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f + 6*(a*b^2*
c^2*d*f^3 - b^3*c*d^2*f^2)*x + (2*a*b^2*d^3*f^3*x^3 + 2*a*b^2*d^3*e^3 + 6*a
*b^2*c^2*d*e*f^2 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*c*d^2*f^3 - b^3*d^3*f^2)*x^2
 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f + 6*(a*b^2*c^2*d*f^3 - b^3*c*d^2*f^2)
*x)*cos(2*f*x + 2*e) + (2*a^2*b*d^3*f^3*x^3 + 2*a^2*b*d^3*e^3 + 6*a^2*b*c^2
*d*e*f^2 + 3*a*b^2*d^3*e^2 + 3*(2*a^2*b*c*d^2*f^3 - a*b^2*d^3*f^2)*x^2 - 6*
(a^2*b*c*d^2*e^2 + a*b^2*c*d^2*e)*f + 6*(a^2*b*c^2*d*f^3 - a*b^2*c*d^2*f^2)
*x)*sin(2*f*x + 2*e))*log((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*
e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 2*(2*a*b^2*d
^3*f^3*x^3 + 2*a*b^2*d^3*e^3 + 6*a*b^2*c^2*d*e*f^2 + 3*b^3*d^3*e^2 + 3*(2*a
*b^2*c*d^2*f^3 - b^3*d^3*f^2)*x^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f + 6
*(a*b^2*c^2*d*f^3 - b^3*c*d^2*f^2)*x + (2*a*b^2*d^3*f^3*x^3 + 2*a*b^2*d^3*e
^3 + 6*a*b^2*c^2*d*e*f^2 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*c*d^2*f^3 - b^3*d^3*f
^2)*x^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f + 6*(a*b^2*c^2*d*f^3 - b^3*c*
d^2*f^2)*x)*cos(2*f*x + 2*e) + (2*a^2*b*d^3*f^3*x^3 + 2*a^2*b*d^3*e^3 + 6*a
^2*b*c^2*d*e*f^2 + 3*a*b^2*d^3*e^2 + 3*(2*a^2*b*c*d^2*f^3 - a*b^2*d^3*f^2)*
x^2 - 6*(a^2*b*c*d^2*e^2 + a*b^2*c*d^2*e)*f + 6*(a^2*b*c^2*d*f^3 - a*b^2*c*
d^2*f^2)*x)*sin(2*f*x + 2*e))*log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*
f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) + (-3*I
*a*b^2*d^3*cos(2*f*x + 2*e) - 3*I*a^2*b*d^3*sin(2*f*x + 2*e) - 3*I*a*b^2*d^
3)*polylog(4, ((a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 - 2*a*b - I*
b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) + (3*I*a*b^2*d^3*cos(2*f*x + 2*e) + 3*I
*a^2*b*d^3*sin(2*f*x + 2*e) + 3*I*a*b^2*d^3)*polylog(4, ((a^2 - 2*I*a*b - b
^2)*cos(2*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^
2)) - 3*(2*a*b^2*d^3*f*x + 2*a*b^2*c*d^2*f - b^3*d^3 + (2*a*b^2*d^3*f*x + 2
*a*b^2*c*d^2*f - b^3*d^3)*cos(2*f*x + 2*e) + (2*a^2*b*d^3*f*x + 2*a^2*b*c*d
^2*f - a*b^2*d^3)*sin(2*f*x + 2*e))*polylog(3, ((a^2 + 2*I*a*b - b^2)*cos(2
*f*x + 2*e) + (I*a^2 - 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 3*(2
*a*b^2*d^3*f*x + 2*a*b^2*c*d^2*f - b^3*d^3 + (2*a*b^2*d^3*f*x + 2*a*b^2*c*d
^2*f - b^3*d^3)*cos(2*f*x + 2*e) + (2*a^2*b*d^3*f*x + 2*a^2*b*c*d^2*f - a*b
^2*d^3)*sin(2*f*x + 2*e))*polylog(3, ((a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e
) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) + ((a^3 - a*b^2
)*d^3*f^4*x^4 + 4*b^3*c^3*f^3 + 4*(b^3*d^3*f^3 + (a^3 - a*b^2)*c*d^2*f^4)*x
^3 + 6*(2*b^3*c*d^2*f^3 + (a^3 - a*b^2)*c^2*d*f^4)*x^2 + 4*(3*b^3*c^2*d*f^3
 + (a^3 - a*b^2)*c^3*f^4)*x)*sin(2*f*x + 2*e))/((a^4*b + 2*a^2*b^3 + b^5)*f
^4*cos(2*f*x + 2*e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^4*sin(2*f*x + 2*e) + (a^4
*b + 2*a^2*b^3 + b^5)*f^4)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{\left(b\cot\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*cot(f*x + e) + a)^2, x)
```

maple [B] time = 2.92, size = 3446, normalized size = 4.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*cot(f*x+e))^2,x)
```

```
[Out] -8/(b-I*a)^2/f^3/(I*a+b)*b*a*c*d^2/(a-I*b)*e^3-12/(b-I*a)^2/f^2/(I*a+b)*b^2
*c*d^2/(a-I*b)*e*x+4/(b-I*a)^2/f^3/(I*a+b)*b*a*d^3*e^3/(a-I*b)*x+3/(b-I*a)^
2/f^2/(I*a+b)*b*a*c^2*d/(a-I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))
+3/(b-I*a)^2/f^2/(I*a+b)*b*a*d^3/(a-I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))
/(a-I*b))*x^2+4*I/(b-I*a)^2/f/(I*a+b)*b*a*c^3/(I*b-a)*ln(exp(I*(f*x+e)))-6/
(b-I*a)^2/f^3/(I*a+b)*b^2*c*d^2/(a-I*b)*e^2-6/(b-I*a)^2/f/(I*a+b)*b^2*c*d^2
/(a-I*b)*x^2-3/2/(b-I*a)^2/f^4/(I*a+b)*b*a*d^3/(a-I*b)*polylog(4,(a+I*b)*ex
p(2*I*(f*x+e))/(a-I*b))-3/(b-I*a)^2/f^3/(I*a+b)*b^2*c*d^2/(a-I*b)*polylog(2
,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))+3/(b-I*a)^2/f^4/(I*a+b)*b*a*d^3*e^4/(a-I
*b)+6/(b-I*a)^2/f^3/(I*a+b)*b^2*d^3/(a-I*b)*e^2*x-3/(b-I*a)^2/f^3/(I*a+b)*b
^2*d^3/(a-I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x+4/(b-I*a)^2/(I
*a+b)*b*a*c*d^2/(a-I*b)*x^3+6/(b-I*a)^2/(I*a+b)*b*a*c^2*d/(a-I*b)*x^2-3/2*I
/(b-I*a)^2/f^4/(I*a+b)*b^2*d^3/(a-I*b)*polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(
a-I*b))+2*I*b^2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(I*a+b)/f/(b-I*a)^2/(ex
p(2*I*(f*x+e))*b-I*exp(2*I*(f*x+e))*a+b+I*a)-6*I/(b-I*a)^2/f^2/(I*a+b)*b^2*
c^2*d/(I*b-a)*ln(exp(I*(f*x+e)))-3*I/(b-I*a)^2/f^2/(I*a+b)*b^2*d^3/(a-I*b)*
ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x^2-6*I/(b-I*a)^2/f^4/(I*a+b)*b^2*d^
3*e^2/(I*b-a)*ln(exp(I*(f*x+e)))+3*I/(b-I*a)^2/f^4/(I*a+b)*b^2*d^3/(a-I*b)*
ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*e^2+6*I/(b-I*a)^2/f^2/(I*a+b)*b*a^2*
c^2*d*e/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)-6
*I/(b-I*a)^2/f^3/(I*a+b)*b*a^2*c*d^2*e^2/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+
e))+I*exp(2*I*(f*x+e))*b-a+I*b)-6*I/(b-I*a)^2/f^3/(I*a+b)*b^2*c*d^2*e/(I*b-
a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)*a+1/(2*I*a*b+a
^2-b^2)*c*d^2*x^3+3/2/(2*I*a*b+a^2-b^2)*c^2*d*x^2+1/(b-I*a)^2/(I*a+b)*b*a*d
^3/(a-I*b)*x^4-2/(b-I*a)^2/f/(I*a+b)*b^2*d^3/(a-I*b)*x^3+4/(b-I*a)^2/f^4/(I
*a+b)*b^2*d^3/(a-I*b)*e^3+1/4/(2*I*a*b+a^2-b^2)*d^3*x^4+1/(2*I*a*b+a^2-b^2)
*c^3*x-3/(b-I*a)^2/f^2/(I*a+b)*b^3*c^2*d/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+
e))+I*exp(2*I*(f*x+e))*b-a+I*b)+2/(b-I*a)^2/f/(I*a+b)*b^2*a*c^3/(I*b-a)/(a+
I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+6/(b-I*a)^2/f^2/(I*a
+b)*b*a*c^2*d/(a-I*b)*e^2-3/(b-I*a)^2/f^4/(I*a+b)*b^3*d^3*e^2/(I*b-a)/(a+I*
b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)-6/(b-I*a)^2/f^2/(I*a+b
)*b^2*a*c^2*d*e/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-
a+I*b)+6/(b-I*a)^2/f^3/(I*a+b)*b^2*a*c*d^2*e^2/(I*b-a)/(a+I*b)*ln(a*exp(2*I
*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+6*I/(b-I*a)^2/f/(I*a+b)*b*a*c*d^2/(a-
I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x^2+2*I/(b-I*a)^2/f^4/(I*a+b)*b
*a^2*d^3*e^3/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I
*b)+12*I/(b-I*a)^2/f^3/(I*a+b)*b*a*c*d^2*e^2/(I*b-a)*ln(exp(I*(f*x+e)))+6*I
/(b-I*a)^2/f/(I*a+b)*b*a*c^2*d/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b
))*x+3*I/(b-I*a)^2/f^2/(I*a+b)*b^2*c^2*d/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+
e))+I*exp(2*I*(f*x+e))*b-a+I*b)*a+6*I/(b-I*a)^2/f^2/(I*a+b)*b*a*c^2*d/(a-I*
b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*e-12*I/(b-I*a)^2/f^2/(I*a+b)*b*a*
c^2*d*e/(I*b-a)*ln(exp(I*(f*x+e)))+3*I/(b-I*a)^2/f^4/(I*a+b)*b^2*d^3*e^2/(I
*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)*a-6*I/(b-I*
a)^2/f^3/(I*a+b)*b*a*c*d^2*e^2/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b
))-2/(b-I*a)^2/f^4/(I*a+b)*b^2*a*d^3*e^3/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+
e))+I*exp(2*I*(f*x+e))*b-a+I*b)+2*I/(b-I*a)^2/f^4/(I*a+b)*b*a*d^3*e^3/(a-I*
b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))+6/(b-I*a)^2/f^2/(I*a+b)*b*a*c*d^2
```

```
/(a-I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x-12/(b-I*a)^2/f^2/(I*
a+b)*b*a*c*d^2/(a-I*b)*e^2*x+12/(b-I*a)^2/f/(I*a+b)*b*a*c^2*d/(a-I*b)*e*x-6
*I/(b-I*a)^2/f^2/(I*a+b)*b^2*c*d^2/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a
-I*b))*x-6*I/(b-I*a)^2/f^3/(I*a+b)*b^2*c*d^2/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(
f*x+e))/(a-I*b))*e+3*I/(b-I*a)^2/f^3/(I*a+b)*b*a*d^3/(a-I*b)*polylog(3,(a+I
*b)*exp(2*I*(f*x+e))/(a-I*b))*x+2*I/(b-I*a)^2/f/(I*a+b)*b*a*d^3/(a-I*b)*ln(
1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x^3+12*I/(b-I*a)^2/f^3/(I*a+b)*b^2*c*d^
2*e/(I*b-a)*ln(exp(I*(f*x+e)))+3*I/(b-I*a)^2/f^3/(I*a+b)*b*a*c*d^2/(a-I*b)*
polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))-4*I/(b-I*a)^2/f^4/(I*a+b)*b*a*d
^3*e^3/(I*b-a)*ln(exp(I*(f*x+e)))-2*I/(b-I*a)^2/f/(I*a+b)*b*a^2*c^3/(I*b-a)
/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+6/(b-I*a)^2/f^3/
(I*a+b)*b^3*c*d^2*e/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e)
)*b-a+I*b)
```

maxima [B] time = 4.72, size = 4699, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (3*(b^2/((a^4 + a^2*b^2)*f*tan(f*x + e) + (a^3*b + a*b^3)*f) + 2*a*b*log(a*
tan(f*x + e) + b)/((a^4 + 2*a^2*b^2 + b^4)*f) - a*b*log(tan(f*x + e)^2 + 1)
/((a^4 + 2*a^2*b^2 + b^4)*f) - (a^2 - b^2)*(f*x + e)/((a^4 + 2*a^2*b^2 + b^
4)*f))*c^2*d*e - (2*a*b*log(a*tan(f*x + e) + b)/(a^4 + 2*a^2*b^2 + b^4) - a
*b*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*(f*x + e)/
(a^4 + 2*a^2*b^2 + b^4) + b^2/(a^3*b + a*b^3 + (a^4 + a^2*b^2)*tan(f*x + e)
))*c^3 - ((3*a^3 + 3*I*a^2*b + 3*a*b^2 + 3*I*b^3)*(f*x + e)^4*d^3 + 24*(-I*
a*b^2 - b^3)*d^3*e^3 + 72*(I*a*b^2 + b^3)*c*d^2*e^2*f - ((12*a^3 + 12*I*a^2
*b + 12*a*b^2 + 12*I*b^3)*d^3*e - (12*a^3 + 12*I*a^2*b + 12*a*b^2 + 12*I*b^
3)*c*d^2*f)*(f*x + e)^3 + ((18*a^3 + 18*I*a^2*b + 18*a*b^2 + 18*I*b^3)*d^3*
e^2 - (36*a^3 + 36*I*a^2*b + 36*a*b^2 + 36*I*b^3)*c*d^2*e*f + (18*a^3 + 18*
I*a^2*b + 18*a*b^2 + 18*I*b^3)*c^2*d*f^2)*(f*x + e)^2 - ((12*a^3 + 12*I*a^2
*b + 12*a*b^2 + 12*I*b^3)*d^3*e^3 - (36*a^3 + 36*I*a^2*b + 36*a*b^2 + 36*I*
b^3)*c*d^2*e^2*f)*(f*x + e) - ((-24*I*a^2*b - 24*a*b^2)*d^3*e^3 - 36*(I*a*b
^2 + b^3)*d^3*e^2 - 36*(I*a*b^2 + b^3)*c^2*d*f^2 + ((72*I*a^2*b + 72*a*b^2)
*c*d^2*e^2 - 72*(-I*a*b^2 - b^3)*c*d^2*e)*f + ((24*I*a^2*b - 24*a*b^2)*d^3*
e^3 - 36*(-I*a*b^2 + b^3)*d^3*e^2 - 36*(-I*a*b^2 + b^3)*c^2*d*f^2 + ((-72*I
*a^2*b + 72*a*b^2)*c*d^2*e^2 - 72*(I*a*b^2 - b^3)*c*d^2*e)*f)*cos(2*f*x + 2
*e) - (24*(a^2*b + I*a*b^2)*d^3*e^3 + (36*a*b^2 + 36*I*b^3)*d^3*e^2 + (36*a
*b^2 + 36*I*b^3)*c^2*d*f^2 - (72*(a^2*b + I*a*b^2)*c*d^2*e^2 + (72*a*b^2 +
72*I*b^3)*c*d^2*e)*f)*sin(2*f*x + 2*e))*arctan2(b*cos(2*f*x + 2*e) + a*sin(
2*f*x + 2*e) + b, a*cos(2*f*x + 2*e) - b*sin(2*f*x + 2*e) - a) - ((32*I*a^2
*b + 32*a*b^2)*(f*x + e)^3*d^3 + ((-72*I*a^2*b - 72*a*b^2)*d^3*e + (72*I*a^
2*b + 72*a*b^2)*c*d^2*f - 36*(I*a*b^2 + b^3)*d^3)*(f*x + e)^2 + ((72*I*a^2*
b + 72*a*b^2)*d^3*e^2 + (72*I*a^2*b + 72*a*b^2)*c^2*d*f^2 - 72*(-I*a*b^2 -
b^3)*d^3*e + ((-144*I*a^2*b - 144*a*b^2)*c*d^2*e - 72*(I*a*b^2 + b^3)*c*d^2
)*f)*(f*x + e) + ((-32*I*a^2*b + 32*a*b^2)*(f*x + e)^3*d^3 + ((72*I*a^2*b -
 72*a*b^2)*d^3*e + (-72*I*a^2*b + 72*a*b^2)*c*d^2*f - 36*(-I*a*b^2 + b^3)*d
^3)*(f*x + e)^2 + ((-72*I*a^2*b + 72*a*b^2)*d^3*e^2 + (-72*I*a^2*b + 72*a*b
^2)*c^2*d*f^2 - 72*(I*a*b^2 - b^3)*d^3*e + ((144*I*a^2*b - 144*a*b^2)*c*d^2
*e - 72*(-I*a*b^2 + b^3)*c*d^2)*f)*(f*x + e))*cos(2*f*x + 2*e) + (32*(a^2*b
 + I*a*b^2)*(f*x + e)^3*d^3 - (72*(a^2*b + I*a*b^2)*d^3*e - 72*(a^2*b + I*a
*b^2)*c*d^2*f + (36*a*b^2 + 36*I*b^3)*d^3)*(f*x + e)^2 + (72*(a^2*b + I*a*b
^2)*d^3*e^2 + 72*(a^2*b + I*a*b^2)*c^2*d*f^2 + (72*a*b^2 + 72*I*b^3)*d^3*e
- (144*(a^2*b + I*a*b^2)*c*d^2*e + (72*a*b^2 + 72*I*b^3)*c*d^2)*f)*(f*x + e
))*sin(2*f*x + 2*e))*arctan2(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f
*x + 2*e))/(a^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*c
os(2*f*x + 2*e))/(a^2 + b^2)) - ((3*a^3 + 9*I*a^2*b - 9*a*b^2 - 3*I*b^3)*(f
*x + e)^4*d^3 - ((12*a^3 + 36*I*a^2*b - 36*a*b^2 - 12*I*b^3)*d^3*e - (12*a^
```

```
3 + 36*I*a^2*b - 36*a*b^2 - 12*I*b^3)*c*d^2*f + 24*(I*a*b^2 - b^3)*d^3)*(f*
x + e)^3 + ((18*a^3 + 54*I*a^2*b - 54*a*b^2 - 18*I*b^3)*d^3*e^2 + (18*a^3 +
 54*I*a^2*b - 54*a*b^2 - 18*I*b^3)*c^2*d*f^2 - 72*(-I*a*b^2 + b^3)*d^3*e -
((36*a^3 + 108*I*a^2*b - 108*a*b^2 - 36*I*b^3)*c*d^2*e + 72*(I*a*b^2 - b^3)
*c*d^2)*f)*(f*x + e)^2 - ((12*a^3 + 36*I*a^2*b - 36*a*b^2 - 12*I*b^3)*d^3*e
^3 + 72*(I*a*b^2 - b^3)*d^3*e^2 + 72*(I*a*b^2 - b^3)*c^2*d*f^2 - ((36*a^3 +
 108*I*a^2*b - 108*a*b^2 - 36*I*b^3)*c*d^2*e^2 - 144*(-I*a*b^2 + b^3)*c*d^2
*e)*f)*(f*x + e))*cos(2*f*x + 2*e) - ((-48*I*a^2*b - 48*a*b^2)*(f*x + e)^2*
d^3 + (-36*I*a^2*b - 36*a*b^2)*d^3*e^2 + (-36*I*a^2*b - 36*a*b^2)*c^2*d*f^2
 - 36*(I*a*b^2 + b^3)*d^3*e + ((72*I*a^2*b + 72*a*b^2)*d^3*e + (-72*I*a^2*b
 - 72*a*b^2)*c*d^2*f - 36*(-I*a*b^2 - b^3)*d^3)*(f*x + e) + ((72*I*a^2*b +
72*a*b^2)*c*d^2*e - 36*(-I*a*b^2 - b^3)*c*d^2)*f + ((48*I*a^2*b - 48*a*b^2)
*(f*x + e)^2*d^3 + (36*I*a^2*b - 36*a*b^2)*d^3*e^2 + (36*I*a^2*b - 36*a*b^2
)*c^2*d*f^2 - 36*(-I*a*b^2 + b^3)*d^3*e + ((-72*I*a^2*b + 72*a*b^2)*d^3*e +
 (72*I*a^2*b - 72*a*b^2)*c*d^2*f - 36*(I*a*b^2 - b^3)*d^3)*(f*x + e) + ((-7
2*I*a^2*b + 72*a*b^2)*c*d^2*e - 36*(I*a*b^2 - b^3)*c*d^2)*f)*cos(2*f*x + 2*
e) - (48*(a^2*b + I*a*b^2)*(f*x + e)^2*d^3 + 36*(a^2*b + I*a*b^2)*d^3*e^2 +
 36*(a^2*b + I*a*b^2)*c^2*d*f^2 + (36*a*b^2 + 36*I*b^3)*d^3*e - (72*(a^2*b
+ I*a*b^2)*d^3*e - 72*(a^2*b + I*a*b^2)*c*d^2*f + (36*a*b^2 + 36*I*b^3)*d^3
)*(f*x + e) - (72*(a^2*b + I*a*b^2)*c*d^2*e + (36*a*b^2 + 36*I*b^3)*c*d^2)*
f)*sin(2*f*x + 2*e))*dilog((I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) + (12*(
a^2*b - I*a*b^2)*d^3*e^3 + (18*a*b^2 - 18*I*b^3)*d^3*e^2 + (18*a*b^2 - 18*I
*b^3)*c^2*d*f^2 - (36*(a^2*b - I*a*b^2)*c*d^2*e^2 + (36*a*b^2 - 36*I*b^3)*c
*d^2*e)*f - (12*(a^2*b + I*a*b^2)*d^3*e^3 + (18*a*b^2 + 18*I*b^3)*d^3*e^2 +
 (18*a*b^2 + 18*I*b^3)*c^2*d*f^2 - (36*(a^2*b + I*a*b^2)*c*d^2*e^2 + (36*a*
b^2 + 36*I*b^3)*c*d^2*e)*f)*cos(2*f*x + 2*e) - ((12*I*a^2*b - 12*a*b^2)*d^3
*e^3 - 18*(-I*a*b^2 + b^3)*d^3*e^2 - 18*(-I*a*b^2 + b^3)*c^2*d*f^2 + ((-36*
I*a^2*b + 36*a*b^2)*c*d^2*e^2 - 36*(I*a*b^2 - b^3)*c*d^2*e)*f)*sin(2*f*x +
2*e))*log((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 +
b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e)) - (16
*(a^2*b - I*a*b^2)*(f*x + e)^3*d^3 - (36*(a^2*b - I*a*b^2)*d^3*e - 36*(a^2*
b - I*a*b^2)*c*d^2*f + (18*a*b^2 - 18*I*b^3)*d^3)*(f*x + e)^2 + (36*(a^2*b
- I*a*b^2)*d^3*e^2 + 36*(a^2*b - I*a*b^2)*c^2*d*f^2 + (36*a*b^2 - 36*I*b^3)
*d^3*e - (72*(a^2*b - I*a*b^2)*c*d^2*e + (36*a*b^2 - 36*I*b^3)*c*d^2)*f)*(f
*x + e) - (16*(a^2*b + I*a*b^2)*(f*x + e)^3*d^3 - (36*(a^2*b + I*a*b^2)*d^3
*e - 36*(a^2*b + I*a*b^2)*c*d^2*f + (18*a*b^2 + 18*I*b^3)*d^3)*(f*x + e)^2
+ (36*(a^2*b + I*a*b^2)*d^3*e^2 + 36*(a^2*b + I*a*b^2)*c^2*d*f^2 + (36*a*b^
2 + 36*I*b^3)*d^3*e - (72*(a^2*b + I*a*b^2)*c*d^2*e + (36*a*b^2 + 36*I*b^3)
*c*d^2)*f)*(f*x + e))*cos(2*f*x + 2*e) + ((-16*I*a^2*b + 16*a*b^2)*(f*x + e
)^3*d^3 + ((36*I*a^2*b - 36*a*b^2)*d^3*e + (-36*I*a^2*b + 36*a*b^2)*c*d^2*f
 - 18*(-I*a*b^2 + b^3)*d^3)*(f*x + e)^2 + ((-36*I*a^2*b + 36*a*b^2)*d^3*e^2
 + (-36*I*a^2*b + 36*a*b^2)*c^2*d*f^2 - 36*(I*a*b^2 - b^3)*d^3*e + ((72*I*a
^2*b - 72*a*b^2)*c*d^2*e - 36*(-I*a*b^2 + b^3)*c*d^2)*f)*(f*x + e))*sin(2*f
*x + 2*e))*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (
a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e))
/(a^2 + b^2)) - ((-24*I*a^2*b + 24*a*b^2)*d^3*cos(2*f*x + 2*e) + 24*(a^2*b
+ I*a*b^2)*d^3*sin(2*f*x + 2*e) + (24*I*a^2*b + 24*a*b^2)*d^3)*polylog(4, (
I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - (48*(a^2*b - I*a*b^2)*(f*x + e)*d
^3 - 36*(a^2*b - I*a*b^2)*d^3*e + 36*(a^2*b - I*a*b^2)*c*d^2*f - (18*a*b^2
- 18*I*b^3)*d^3 - (48*(a^2*b + I*a*b^2)*(f*x + e)*d^3 - 36*(a^2*b + I*a*b^2
)*d^3*e + 36*(a^2*b + I*a*b^2)*c*d^2*f - (18*a*b^2 + 18*I*b^3)*d^3)*cos(2*f
*x + 2*e) + ((-48*I*a^2*b + 48*a*b^2)*(f*x + e)*d^3 + (36*I*a^2*b - 36*a*b^
2)*d^3*e + (-36*I*a^2*b + 36*a*b^2)*c*d^2*f - 18*(-I*a*b^2 + b^3)*d^3)*sin(
2*f*x + 2*e))*polylog(3, (I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - ((3*I*a
^3 - 9*a^2*b - 9*I*a*b^2 + 3*b^3)*(f*x + e)^4*d^3 + ((-12*I*a^3 + 36*a^2*b
+ 36*I*a*b^2 - 12*b^3)*d^3*e + (12*I*a^3 - 36*a^2*b - 36*I*a*b^2 + 12*b^3)*
c*d^2*f + (24*a*b^2 + 24*I*b^3)*d^3)*(f*x + e)^3 + ((18*I*a^3 - 54*a^2*b -
54*I*a*b^2 + 18*b^3)*d^3*e^2 + (18*I*a^3 - 54*a^2*b - 54*I*a*b^2 + 18*b^3)*
c^2*d*f^2 - (72*a*b^2 + 72*I*b^3)*d^3*e + ((-36*I*a^3 + 108*a^2*b + 108*I*a
```

```
*b^2 - 36*b^3)*c*d^2*e + (72*a*b^2 + 72*I*b^3)*c*d^2)*f)*(f*x + e)^2 + ((-1
2*I*a^3 + 36*a^2*b + 36*I*a*b^2 - 12*b^3)*d^3*e^3 + (72*a*b^2 + 72*I*b^3)*d
^3*e^2 + (72*a*b^2 + 72*I*b^3)*c^2*d*f^2 + ((36*I*a^3 - 108*a^2*b - 108*I*a
*b^2 + 36*b^3)*c*d^2*e^2 - (144*a*b^2 + 144*I*b^3)*c*d^2*e)*f)*(f*x + e))*s
in(2*f*x + 2*e))/((12*a^5 + 12*I*a^4*b + 24*a^3*b^2 + 24*I*a^2*b^3 + 12*a*b
^4 + 12*I*b^5)*f^3*cos(2*f*x + 2*e) - (-12*I*a^5 + 12*a^4*b - 24*I*a^3*b^2
+ 24*a^2*b^3 - 12*I*a*b^4 + 12*b^5)*f^3*sin(2*f*x + 2*e) - (12*a^5 - 12*I*a
^4*b + 24*a^3*b^2 - 24*I*a^2*b^3 + 12*a*b^4 - 12*I*b^5)*f^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{\left(a+b\cot\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*cot(e + f*x))^2,x)
```

```
[Out] int((c + d*x)^3/(a + b*cot(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{\left(a+b\cot\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**3/(a + b*cot(e + f*x))**2, x)
```

3.58 $\int \frac{(c+dx)^2}{(a+b\cot(e+fx))^2}\,dx$

Optimal. Leaf size=650

$$-\frac{2b^2d(c+dx)\mathrm{Li}_2\left(\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^2\left(a^2+b^2\right)^2}+\frac{2b^2d(c+dx)\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^2\left(a^2+b^2\right)^2}-\frac{2ib^2(c+dx)^2\log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f\left(a^2+b^2\right)^2}-\frac{2ib^2(c$$

```
[Out] -2*I*b^2*(d*x+c)^2/(a^2+b^2)^2/f-2*b^2*(d*x+c)^2/(a-I*b)/(a+I*b)^2/(I*a+b-(
I*a-b)*exp(2*I*e+2*I*f*x))/f+1/3*(d*x+c)^3/(a+I*b)^2/d-4/3*b*(d*x+c)^3/(a+I
*b)^2/(I*a+b)/d-4/3*b^2*(d*x+c)^3/(a^2+b^2)^2/d+2*b^2*d*(d*x+c)*ln(1-(a+I*b
)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^2-2*b*(d*x+c)^2*ln(1-(a+I*b)*ex
p(2*I*e+2*I*f*x)/(a-I*b))/(a-I*b)/(a+I*b)^2/f-2*I*b^2*(d*x+c)^2*ln(1-(a+I*b
)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f-I*b^2*d^2*polylog(2,(a+I*b)*exp
(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^3-2*b*d*(d*x+c)*polylog(2,(a+I*b)*ex
p(2*I*e+2*I*f*x)/(a-I*b))/(a+I*b)^2/(I*a+b)/f^2-2*b^2*d*(d*x+c)*polylog(2,(
a+I*b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^2-b*d^2*polylog(3,(a+I*b)*
exp(2*I*e+2*I*f*x)/(a-I*b))/(a-I*b)/(a+I*b)^2/f^3-I*b^2*d^2*polylog(3,(a+I*
b)*exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^3
```

Rubi [A] time = 1.48, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3734, 2185, 2184, 2190, 2531, 2282, 6589, 2191, 2279, 2391}

$$-\frac{2b^2d(c+dx)\mathrm{PolyLog}\left(2,\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^2\left(a^2+b^2\right)^2}-\frac{ib^2d^2\mathrm{PolyLog}\left(2,\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^3\left(a^2+b^2\right)^2}-\frac{ib^2d^2\mathrm{PolyLog}\left(3,\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^3\left(a^2+b^2\right)^2}-\frac{2bd(c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((-2*I)*b^2*(c + d*x)^2)/((a^2 + b^2)^2*f) - (2*b^2*(c + d*x)^2)/((a - I*b)
*(a + I*b)^2*(I*a + b - (I*a - b)*E^((2*I)*e + (2*I)*f*x))*f) + (c + d*x)^3
/(3*(a + I*b)^2*d) - (4*b*(c + d*x)^3)/(3*(a + I*b)^2*(I*a + b)*d) - (4*b^2
*(c + d*x)^3)/(3*(a^2 + b^2)^2*d) + (2*b^2*d*(c + d*x)*Log[1 - ((a + I*b)*E
^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^2) - (2*b*(c + d*x)^2*
Log[1 - ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a - I*b)*(a + I*b
)^2*f) - ((2*I)*b^2*(c + d*x)^2*Log[1 - ((a + I*b)*E^((2*I)*e + (2*I)*f*x))
/(a - I*b)])/((a^2 + b^2)^2*f) - (I*b^2*d^2*PolyLog[2, ((a + I*b)*E^((2*I)*
e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^3) - (2*b*d*(c + d*x)*PolyLog[
2, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a + I*b)^2*(I*a + b)*f
^2) - (2*b^2*d*(c + d*x)*PolyLog[2, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a
- I*b)])/((a^2 + b^2)^2*f^2) - (b*d^2*PolyLog[3, ((a + I*b)*E^((2*I)*e + (2
*I)*f*x))/(a - I*b)])/((a - I*b)*(a + I*b)^2*f^3) - (I*b^2*d^2*PolyLog[3, (
(a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^3)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2185

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
```

```
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2191

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3734

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - (2*I*b)/(a^2
+ b^2 + (a - I*b)^2*E^(2*I*(e + f*x))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+b\cot(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^2}{(a+ib)^2} - \frac{4b^2(c+dx)^2}{(ia-b)^2\left(ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}\right)^2} + \frac{4b(c+$$

$$= \frac{(c+dx)^3}{3(a+ib)^2d} + \frac{(4b)\int \frac{(c+dx)^2}{-ia\left(1-\frac{ib}{a}\right)+ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}\,dx}{(a+ib)^2} - \frac{\left(4b^2\right)\int \frac{(c+dx)^2}{\left(ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}\right)^2}\,dx}{(ia-b)^2}$$

$$= \frac{(c+dx)^3}{3(a+ib)^2d} - \frac{4b(c+dx)^3}{3(a+ib)^2(ia+b)d} + \frac{\left(4b^2\right)\int \frac{(c+dx)^2}{ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}\,dx}{(a+ib)^2(ia+b)} + \frac{(4b)\int \frac{e^{}}{-ia\left(1-\right.}}{}$$

$$= -\frac{2b^2(c+dx)^2}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a+ib)^2d} - \frac{4b(c+dx)^3}{3(a+ib)^2(ia+b)d} -$$

$$= -\frac{2ib^2(c+dx)^2}{\left(a^2+b^2\right)^2 f} - \frac{2b^2(c+dx)^2}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a+ib)^2d} - \frac{}{3(a}$$

$$= -\frac{2ib^2(c+dx)^2}{\left(a^2+b^2\right)^2 f} - \frac{2b^2(c+dx)^2}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a+ib)^2d} - \frac{}{3(a}$$

$$= -\frac{2ib^2(c+dx)^2}{\left(a^2+b^2\right)^2 f} - \frac{2b^2(c+dx)^2}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a+ib)^2d} - \frac{}{3(a}$$

$$= -\frac{2ib^2(c+dx)^2}{\left(a^2+b^2\right)^2 f} - \frac{2b^2(c+dx)^2}{(a+ib)\left(a^2+b^2\right)\left(ia+b-(ia-b)e^{2ie+2ifx}\right)f} + \frac{(c+dx)^3}{3(a+ib)^2d} - \frac{}{3(a}$$

Mathematica [A] time = 9.21, size = 718, normalized size = 1.10

$$\frac{-fx\left(a^2+b^2\right)\left(3c^2+3cdx+d^2x^2\right)\cos(2e+fx)+fx\left(a^2-b^2\right)\left(3c^2+3cdx+d^2x^2\right)\cos(fx)+2b\sin(fx)\left(3b(c+dx)^2-afx\left(3c^2+3cdx+d^2x^2\right)\right)}{(a\sin(e)+b\cos(e))(a\sin(e+fx)+b\cos(e+fx))} + \frac{2b\left(\frac{3d\left(a\left(-1+e^{2ie}\right)+ib\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] ((2*b*((12*c*(-(b*d) + a*c*f)*x)/(a + I*b) + (6*d*(-(b*d) + 2*a*c*f)*x^2)/(
a + I*b) + (4*a*d^2*f*x^3)/(a + I*b) - (6*d*(a*(-1 + E^((2*I)*e)) + I*b*(1
+ E^((2*I)*e)))*(b*d - 2*a*c*f)*x*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e
 + f*x)))])/((a - I*b)*((-I)*a + b)*f) + (6*a*d^2*(a*(-1 + E^((2*I)*e)) + I
```

```
*b*(1 + E^((2*I)*e)))*x^2*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x))
)])/((a - I*b)*((-I)*a + b)) + (6*c*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*
I)*e)))*(-(b*d) + a*c*f)*(2*f*x + I*Log[a - I*b - (a + I*b)*E^((2*I)*(e + f
*x))]))/((a^2 + b^2)*f) + (3*d*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)
))*(b*d - 2*a*c*f)*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))])/(
(a^2 + b^2)*f^2) - (3*a*d^2*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*
(2*f*x*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))] - I*PolyLog[3,
 (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]))/((a^2 + b^2)*f^2)))/((-I)*a*(
-1 + E^((2*I)*e)) + b*(1 + E^((2*I)*e))) + ((a^2 - b^2)*f*x*(3*c^2 + 3*c*d*
x + d^2*x^2)*Cos[f*x] - (a^2 + b^2)*f*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cos[2*e
 + f*x] + 2*b*(3*b*(c + d*x)^2 - a*f*x*(3*c^2 + 3*c*d*x + d^2*x^2))*Sin[f*x
])/((b*Cos[e] + a*Sin[e])*(b*Cos[e + f*x] + a*Sin[e + f*x])))/(6*(a^2 + b^2
)*f)
```

fricas [C] time = 0.60, size = 2036, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^2*b - b^3)*d^2*f^3*x^3 - 6*a*b^2*c^2*f^2 - 6*(a*b^2*d^2*f^2 - (a^
2*b - b^3)*c*d*f^3)*x^2 - 6*(2*a*b^2*c*d*f^2 - (a^2*b - b^3)*c^2*f^3)*x + 2
*((a^2*b - b^3)*d^2*f^3*x^3 - 3*a*b^2*c^2*f^2 - 3*(a*b^2*d^2*f^2 - (a^2*b -
 b^3)*c*d*f^3)*x^2 - 3*(2*a*b^2*c*d*f^2 - (a^2*b - b^3)*c^2*f^3)*x)*cos(2*f
*x + 2*e) + (6*I*a*b^2*d^2*f*x + 6*I*a*b^2*c*d*f - 3*I*b^3*d^2 + (6*I*a*b^2
*d^2*f*x + 6*I*a*b^2*c*d*f - 3*I*b^3*d^2)*cos(2*f*x + 2*e) + (6*I*a^2*b*d^2
*f*x + 6*I*a^2*b*c*d*f - 3*I*a*b^2*d^2)*sin(2*f*x + 2*e))*dilog(-(a^2 + b^2
 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*
f*x + 2*e))/(a^2 + b^2) + 1) + (-6*I*a*b^2*d^2*f*x - 6*I*a*b^2*c*d*f + 3*I*
b^3*d^2 + (-6*I*a*b^2*d^2*f*x - 6*I*a*b^2*c*d*f + 3*I*b^3*d^2)*cos(2*f*x +
2*e) + (-6*I*a^2*b*d^2*f*x - 6*I*a^2*b*c*d*f + 3*I*a*b^2*d^2)*sin(2*f*x + 2
*e))*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 +
2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 6*(a*b^2*d^2*e^2 + a*b^
2*c^2*f^2 + b^3*d^2*e - (2*a*b^2*c*d*e + b^3*c*d)*f + (a*b^2*d^2*e^2 + a*b^
2*c^2*f^2 + b^3*d^2*e - (2*a*b^2*c*d*e + b^3*c*d)*f)*cos(2*f*x + 2*e) + (a^
2*b*d^2*e^2 + a^2*b*c^2*f^2 + a*b^2*d^2*e - (2*a^2*b*c*d*e + a*b^2*c*d)*f)*
sin(2*f*x + 2*e))*log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*cos(2*f*x
 + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 6*(a*b^2*d^2*e^2 + a*b^2*
c^2*f^2 + b^3*d^2*e - (2*a*b^2*c*d*e + b^3*c*d)*f + (a*b^2*d^2*e^2 + a*b^2*
c^2*f^2 + b^3*d^2*e - (2*a*b^2*c*d*e + b^3*c*d)*f)*cos(2*f*x + 2*e) + (a^2*
b*d^2*e^2 + a^2*b*c^2*f^2 + a*b^2*d^2*e - (2*a^2*b*c*d*e + a*b^2*c*d)*f)*si
n(2*f*x + 2*e))*log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x
+ 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 6*(a*b^2*d^2*f^2*x^2 - a*b
^2*d^2*e^2 + 2*a*b^2*c*d*e*f - b^3*d^2*e + (2*a*b^2*c*d*f^2 - b^3*d^2*f)*x
+ (a*b^2*d^2*f^2*x^2 - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f - b^3*d^2*e + (2*a*b
^2*c*d*f^2 - b^3*d^2*f)*x)*cos(2*f*x + 2*e) + (a^2*b*d^2*f^2*x^2 - a^2*b*d^
2*e^2 + 2*a^2*b*c*d*e*f - a*b^2*d^2*e + (2*a^2*b*c*d*f^2 - a*b^2*d^2*f)*x)*
sin(2*f*x + 2*e))*log((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) +
 (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 6*(a*b^2*d^2*f^2
*x^2 - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f - b^3*d^2*e + (2*a*b^2*c*d*f^2 - b^3
*d^2*f)*x + (a*b^2*d^2*f^2*x^2 - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f - b^3*d^2*
e + (2*a*b^2*c*d*f^2 - b^3*d^2*f)*x)*cos(2*f*x + 2*e) + (a^2*b*d^2*f^2*x^2
- a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f - a*b^2*d^2*e + (2*a^2*b*c*d*f^2 - a*b^2*
d^2*f)*x)*sin(2*f*x + 2*e))*log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*
x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 3*(a*b^
2*d^2*cos(2*f*x + 2*e) + a^2*b*d^2*sin(2*f*x + 2*e) + a*b^2*d^2)*polylog(3,
 ((a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 - 2*a*b - I*b^2)*sin(2*f*
x + 2*e))/(a^2 + b^2)) - 3*(a*b^2*d^2*cos(2*f*x + 2*e) + a^2*b*d^2*sin(2*f*
x + 2*e) + a*b^2*d^2)*polylog(3, ((a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) +
```

```
(-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) + 2*((a^3 - a*b^2)*
d^2*f^3*x^3 + 3*b^3*c^2*f^2 + 3*(b^3*d^2*f^2 + (a^3 - a*b^2)*c*d*f^3)*x^2 +
 3*(2*b^3*c*d*f^2 + (a^3 - a*b^2)*c^2*f^3)*x)*sin(2*f*x + 2*e))/((a^4*b + 2
*a^2*b^3 + b^5)*f^3*cos(2*f*x + 2*e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^3*sin(2*
f*x + 2*e) + (a^4*b + 2*a^2*b^3 + b^5)*f^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{\left(b\cot\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*cot(f*x + e) + a)^2, x)
```

maple [B] time = 2.72, size = 2137, normalized size = 3.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*cot(f*x+e))^2,x)
```

```
[Out] -4/(b-I*a)^2/f^2/(I*a+b)*b^2*a*c*d*e/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+
I*exp(2*I*(f*x+e))*b-a+I*b)-8*I/(b-I*a)^2/f^2/(I*a+b)*b*a*c*d*e/(I*b-a)*ln(
exp(I*(f*x+e)))+4*I/(b-I*a)^2/f^2/(I*a+b)*b*a*c*d/(a-I*b)*ln(1-(a+I*b)*exp(
2*I*(f*x+e))/(a-I*b))*e-2*I/(b-I*a)^2/f^3/(I*a+b)*b^2*d^2*e/(I*b-a)/(a+I*b)
*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)*a+1/(2*I*a*b+a^2-b^2)*c*
d*x^2+4/(b-I*a)^2/(I*a+b)*b*a*c*d/(a-I*b)*x^2-4/(b-I*a)^2/f^2/(I*a+b)*b^2*d
^2/(a-I*b)*e*x-8/3/(b-I*a)^2/f^3/(I*a+b)*b*a*d^2*e^3/(a-I*b)+4/3/(b-I*a)^2/
(I*a+b)*b*a*d^2/(a-I*b)*x^3-1/(b-I*a)^2/f^3/(I*a+b)*b^2*d^2/(a-I*b)*polylog
(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))-2/(b-I*a)^2/f/(I*a+b)*b^2*d^2/(a-I*b)*
x^2-2/(b-I*a)^2/f^3/(I*a+b)*b^2*d^2/(a-I*b)*e^2+2*I*b^2*(d^2*x^2+2*c*d*x+c^
2)/(I*a+b)/f/(b-I*a)^2/(exp(2*I*(f*x+e))*b-I*exp(2*I*(f*x+e))*a+b+I*a)+4/(b
-I*a)^2/f^2/(I*a+b)*b*a*c*d/(a-I*b)*e^2+2/(b-I*a)^2/f^2/(I*a+b)*b*a*d^2/(a-
I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x-4*I/(b-I*a)^2/f^2/(I*a+b
)*b^2*c*d/(I*b-a)*ln(exp(I*(f*x+e)))+4*I/(b-I*a)^2/f/(I*a+b)*b*a*c^2/(I*b-a
)*ln(exp(I*(f*x+e)))+4*I/(b-I*a)^2/f^3/(I*a+b)*b^2*d^2*e/(I*b-a)*ln(exp(I*(
f*x+e)))-2*I/(b-I*a)^2/f^2/(I*a+b)*b^2*d^2/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*
x+e))/(a-I*b))*x-2*I/(b-I*a)^2/f^3/(I*a+b)*b^2*d^2/(a-I*b)*ln(1-(a+I*b)*exp
(2*I*(f*x+e))/(a-I*b))*e+2*I/(b-I*a)^2/f^2/(I*a+b)*b^2*c*d/(I*b-a)/(a+I*b)*
ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)*a-2*I/(b-I*a)^2/f^3/(I*a+
b)*b*a^2*d^2*e^2/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b
-a+I*b)+4*I/(b-I*a)^2/f/(I*a+b)*b*a*c*d/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e
))/(a-I*b))*x-2/(b-I*a)^2/f^2/(I*a+b)*b^3*c*d/(I*b-a)/(a+I*b)*ln(a*exp(2*I*
(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+I/(b-I*a)^2/f^3/(I*a+b)*b*a*d^2/(a-I*b
)*polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))+2/(b-I*a)^2/f^2/(I*a+b)*b*a*c
*d/(a-I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))-4/(b-I*a)^2/f^2/(I*a
+b)*b*a*d^2*e^2/(a-I*b)*x+2/(b-I*a)^2/f/(I*a+b)*b^2*a*c^2/(I*b-a)/(a+I*b)*l
n(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+2/(b-I*a)^2/f^3/(I*a+b)*b^
3*d^2*e/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+4
*I/(b-I*a)^2/f^2/(I*a+b)*b*a^2*c*d*e/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+
I*exp(2*I*(f*x+e))*b-a+I*b)+1/3/(2*I*a*b+a^2-b^2)*d^2*x^3+1/(2*I*a*b+a^2-b^
2)*c^2*x-2*I/(b-I*a)^2/f^3/(I*a+b)*b*a*d^2/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*
x+e))/(a-I*b))*e^2+2/(b-I*a)^2/f^3/(I*a+b)*b^2*a*d^2*e^2/(I*b-a)/(a+I*b)*ln
(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+8/(b-I*a)^2/f/(I*a+b)*b*a*c
*d/(a-I*b)*e*x-2*I/(b-I*a)^2/f/(I*a+b)*b*a^2*c^2/(I*b-a)/(a+I*b)*ln(a*exp(2
*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+4*I/(b-I*a)^2/f^3/(I*a+b)*b*a*d^2*e
```

```
^2/(I*b-a)*ln(exp(I*(f*x+e)))+2*I/(b-I*a)^2/f/(I*a+b)*b*a*d^2/(a-I*b)*ln(1-
(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x^2
```

maxima [B] time = 2.21, size = 2592, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (2*(b^2/((a^4 + a^2*b^2)*f*tan(f*x + e) + (a^3*b + a*b^3)*f) + 2*a*b*log(a*
tan(f*x + e) + b)/((a^4 + 2*a^2*b^2 + b^4)*f) - a*b*log(tan(f*x + e)^2 + 1)
/((a^4 + 2*a^2*b^2 + b^4)*f) - (a^2 - b^2)*(f*x + e)/((a^4 + 2*a^2*b^2 + b^
4)*f))*c*d*e - (2*a*b*log(a*tan(f*x + e) + b)/(a^4 + 2*a^2*b^2 + b^4) - a*b
*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*(f*x + e)/(a
^4 + 2*a^2*b^2 + b^4) + b^2/(a^3*b + a*b^3 + (a^4 + a^2*b^2)*tan(f*x + e)))
*c^2 - ((a^3 + I*a^2*b + a*b^2 + I*b^3)*(f*x + e)^3*d^2 + (3*a^3 + 3*I*a^2*
b + 3*a*b^2 + 3*I*b^3)*(f*x + e)*d^2*e^2 + 6*(I*a*b^2 + b^3)*d^2*e^2 - ((3*
a^3 + 3*I*a^2*b + 3*a*b^2 + 3*I*b^3)*d^2*e - (3*a^3 + 3*I*a^2*b + 3*a*b^2 +
 3*I*b^3)*c*d*f)*(f*x + e)^2 - ((6*I*a^2*b + 6*a*b^2)*d^2*e^2 - 6*(-I*a*b^2
 - b^3)*d^2*e - 6*(I*a*b^2 + b^3)*c*d*f + ((-6*I*a^2*b + 6*a*b^2)*d^2*e^2 -
 6*(I*a*b^2 - b^3)*d^2*e - 6*(-I*a*b^2 + b^3)*c*d*f)*cos(2*f*x + 2*e) + (6*
(a^2*b + I*a*b^2)*d^2*e^2 + (6*a*b^2 + 6*I*b^3)*d^2*e - (6*a*b^2 + 6*I*b^3)
*c*d*f)*sin(2*f*x + 2*e))*arctan2(b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) +
 b, a*cos(2*f*x + 2*e) - b*sin(2*f*x + 2*e) - a) - ((6*I*a^2*b + 6*a*b^2)*(
f*x + e)^2*d^2 + ((-12*I*a^2*b - 12*a*b^2)*d^2*e + (12*I*a^2*b + 12*a*b^2)*
c*d*f - 6*(I*a*b^2 + b^3)*d^2)*(f*x + e) + ((-6*I*a^2*b + 6*a*b^2)*(f*x + e
)^2*d^2 + ((12*I*a^2*b - 12*a*b^2)*d^2*e + (-12*I*a^2*b + 12*a*b^2)*c*d*f -
 6*(-I*a*b^2 + b^3)*d^2)*(f*x + e))*cos(2*f*x + 2*e) + (6*(a^2*b + I*a*b^2)
*(f*x + e)^2*d^2 - (12*(a^2*b + I*a*b^2)*d^2*e - 12*(a^2*b + I*a*b^2)*c*d*f
 + (6*a*b^2 + 6*I*b^3)*d^2)*(f*x + e))*sin(2*f*x + 2*e))*arctan2(-(2*a*b*co
s(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*sin(2*f*
x + 2*e) + a^2 + b^2 - (a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) - ((a^3 +
 3*I*a^2*b - 3*a*b^2 - I*b^3)*(f*x + e)^3*d^2 - ((3*a^3 + 9*I*a^2*b - 9*a*b
^2 - 3*I*b^3)*d^2*e - (3*a^3 + 9*I*a^2*b - 9*a*b^2 - 3*I*b^3)*c*d*f + 6*(I*
a*b^2 - b^3)*d^2)*(f*x + e)^2 + ((3*a^3 + 9*I*a^2*b - 9*a*b^2 - 3*I*b^3)*d^
2*e^2 - 12*(-I*a*b^2 + b^3)*d^2*e - 12*(I*a*b^2 - b^3)*c*d*f)*(f*x + e))*co
s(2*f*x + 2*e) - ((-6*I*a^2*b - 6*a*b^2)*(f*x + e)*d^2 + (6*I*a^2*b + 6*a*b
^2)*d^2*e + (-6*I*a^2*b - 6*a*b^2)*c*d*f - 3*(-I*a*b^2 - b^3)*d^2 + ((6*I*a
^2*b - 6*a*b^2)*(f*x + e)*d^2 + (-6*I*a^2*b + 6*a*b^2)*d^2*e + (6*I*a^2*b -
 6*a*b^2)*c*d*f - 3*(I*a*b^2 - b^3)*d^2)*cos(2*f*x + 2*e) - (6*(a^2*b + I*a
*b^2)*(f*x + e)*d^2 - 6*(a^2*b + I*a*b^2)*d^2*e + 6*(a^2*b + I*a*b^2)*c*d*f
 - (3*a*b^2 + 3*I*b^3)*d^2)*sin(2*f*x + 2*e))*dilog((I*a - b)*e^(2*I*f*x +
2*I*e)/(I*a + b)) - (3*(a^2*b - I*a*b^2)*d^2*e^2 + (3*a*b^2 - 3*I*b^3)*d^2*
e - (3*a*b^2 - 3*I*b^3)*c*d*f - (3*(a^2*b + I*a*b^2)*d^2*e^2 + (3*a*b^2 + 3
*I*b^3)*d^2*e - (3*a*b^2 + 3*I*b^3)*c*d*f)*cos(2*f*x + 2*e) + ((-3*I*a^2*b
+ 3*a*b^2)*d^2*e^2 - 3*(I*a*b^2 - b^3)*d^2*e - 3*(-I*a*b^2 + b^3)*c*d*f)*si
n(2*f*x + 2*e))*log((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e)
 + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2
*e)) - (3*(a^2*b - I*a*b^2)*(f*x + e)^2*d^2 - (6*(a^2*b - I*a*b^2)*d^2*e -
6*(a^2*b - I*a*b^2)*c*d*f + (3*a*b^2 - 3*I*b^3)*d^2)*(f*x + e) - (3*(a^2*b
+ I*a*b^2)*(f*x + e)^2*d^2 - (6*(a^2*b + I*a*b^2)*d^2*e - 6*(a^2*b + I*a*b^
2)*c*d*f + (3*a*b^2 + 3*I*b^3)*d^2)*(f*x + e))*cos(2*f*x + 2*e) + ((-3*I*a^
2*b + 3*a*b^2)*(f*x + e)^2*d^2 + ((6*I*a^2*b - 6*a*b^2)*d^2*e + (-6*I*a^2*b
 + 6*a*b^2)*c*d*f - 3*(-I*a*b^2 + b^3)*d^2)*(f*x + e))*sin(2*f*x + 2*e))*lo
g(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*si
n(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2))
 + (3*(a^2*b + I*a*b^2)*d^2*cos(2*f*x + 2*e) - (-3*I*a^2*b + 3*a*b^2)*d^2*s
in(2*f*x + 2*e) - 3*(a^2*b - I*a*b^2)*d^2)*polylog(3, (I*a - b)*e^(2*I*f*x
+ 2*I*e)/(I*a + b)) - ((I*a^3 - 3*a^2*b - 3*I*a*b^2 + b^3)*(f*x + e)^3*d^2
```

```
+ ((-3*I*a^3 + 9*a^2*b + 9*I*a*b^2 - 3*b^3)*d^2*e + (3*I*a^3 - 9*a^2*b - 9*
I*a*b^2 + 3*b^3)*c*d*f + (6*a*b^2 + 6*I*b^3)*d^2)*(f*x + e)^2 + ((3*I*a^3 -
 9*a^2*b - 9*I*a*b^2 + 3*b^3)*d^2*e^2 - (12*a*b^2 + 12*I*b^3)*d^2*e + (12*a
*b^2 + 12*I*b^3)*c*d*f)*(f*x + e))*sin(2*f*x + 2*e))/((3*a^5 + 3*I*a^4*b +
6*a^3*b^2 + 6*I*a^2*b^3 + 3*a*b^4 + 3*I*b^5)*f^2*cos(2*f*x + 2*e) - (-3*I*a
^5 + 3*a^4*b - 6*I*a^3*b^2 + 6*a^2*b^3 - 3*I*a*b^4 + 3*b^5)*f^2*sin(2*f*x +
 2*e) - (3*a^5 - 3*I*a^4*b + 6*a^3*b^2 - 6*I*a^2*b^3 + 3*a*b^4 - 3*I*b^5)*f
^2))/f
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^2}{\left(a+b\cot\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*cot(e + f*x))^2,x)
```

```
[Out] int((c + d*x)^2/(a + b*cot(e + f*x))^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{\left(a+b\cot\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**2/(a + b*cot(e + f*x))**2, x)
```

3.59 $\int \frac{c+dx}{(a+b\cot(e+fx))^2}\,dx$

Optimal. Leaf size=213

$$\frac{b(-2acf-2adfx+bd)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f^2\left(a^2+b^2\right)^2}+\frac{b(c+dx)}{f\left(a^2+b^2\right)(a+b\cot(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2+b^2\right)}+\frac{iabd\mathrm{Li}_2\left(\frac{(a+ib)e^{2i(}}{a-ib}\right.}{f^2\left(a^2+b^2\right)^2}$$

```
[Out] -1/2*(d*x+c)^2/(a^2+b^2)/d+1/4*(-2*a*d*f*x-2*a*c*f+b*d)^2/a/(a-I*b)^2/(a+I*
b)/d/f^2+b*(d*x+c)/(a^2+b^2)/f/(a+b*cot(f*x+e))+b*(-2*a*d*f*x-2*a*c*f+b*d)*
ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)^2/f^2+I*a*b*d*polylog(2,(a
+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)^2/f^2
```

Rubi [A] time = 0.29, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3733, 3731, 2190, 2279, 2391}

$$\frac{iabd\mathrm{PolyLog}\left(2,\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f^2\left(a^2+b^2\right)^2}+\frac{b(-2acf-2adfx+bd)\log\left(1-\frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f^2\left(a^2+b^2\right)^2}+\frac{b(c+dx)}{f\left(a^2+b^2\right)(a+b\cot(e+fx))}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] -(c + d*x)^2/(2*(a^2 + b^2)*d) + (b*d - 2*a*c*f - 2*a*d*f*x)^2/(4*a*(a - I*
b)^2*(a + I*b)*d*f^2) + (b*(c + d*x))/((a^2 + b^2)*f*(a + b*Cot[e + f*x]))
+ (b*(b*d - 2*a*c*f - 2*a*d*f*x)*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a
 - I*b)])/((a^2 + b^2)^2*f^2) + (I*a*b*d*PolyLog[2, ((a + I*b)*E^((2*I)*(e
+ f*x)))/(a - I*b)])/((a^2 + b^2)^2*f^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3731

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist
[2*I*b, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])/((a + I*b)^
2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x]), x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3733

```
Int[((c_.) + (d_.)*(x_))/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol
] :> -Simp[(c + d*x)^2/(2*d*(a^2 + b^2)), x] + (Dist[1/(f*(a^2 + b^2)), Int
[(b*d + 2*a*c*f + 2*a*d*f*x)/(a + b*Tan[e + f*x]), x], x] - Simp[(b*(c + d*
x))/(f*(a^2 + b^2)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+b\cot(e+fx))^2}\,dx = -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d} + \frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\cot(e+fx))} + \frac{\int \frac{-bd+2acf+2adfx}{a+b\cot(e+fx)}\,dx}{\left(a^2+b^2\right)f}$$

$$= -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d} + \frac{(bd-2acf-2adfx)^2}{4a(a-ib)^2(a+ib)df^2} + \frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\cot(e+fx))} + \frac{(2ib)\int \frac{e^{}}{(}}{}$$

$$= -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d} + \frac{(bd-2acf-2adfx)^2}{4a(a-ib)^2(a+ib)df^2} + \frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\cot(e+fx))} + \frac{b(bd-2}{}$$

$$= -\frac{(c+dx)^2}{2\left(a^2+b^2\right)d} + \frac{(bd-2acf-2adfx)^2}{4a(a-ib)^2(a+ib)df^2} + \frac{b(c+dx)}{\left(a^2+b^2\right)f(a+b\cot(e+fx))} + \frac{b(bd-2}{}$$

Mathematica [B] time = 7.10, size = 730, normalized size = 3.43

$$-\frac{2ac\csc^2(e+fx)(a\sin(e+fx)+b\cos(e+fx))^2(b\log(a\sin(e+fx)+b\cos(e+fx))-a(e+fx))}{f(b-ia)(b+ia)\left(a^2+b^2\right)(a+b\cot(e+fx))^2} + \frac{d\csc^2(e+f}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] -1/2*((e + f*x)*(-2*d*e + 2*c*f + d*(e + f*x))*Csc[e + f*x]^2*(b*Cos[e + f*
x] + a*Sin[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*f^2*(a + b*Cot[e + f*x])^2)
 + (b*d*Csc[e + f*x]^2*(-(a*(e + f*x)) + b*Log[b*Cos[e + f*x] + a*Sin[e + f
*x]])*(b*Cos[e + f*x] + a*Sin[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*(a^2 + b
^2)*f^2*(a + b*Cot[e + f*x])^2) + (2*a*d*e*Csc[e + f*x]^2*(-(a*(e + f*x)) +
 b*Log[b*Cos[e + f*x] + a*Sin[e + f*x]])*(b*Cos[e + f*x] + a*Sin[e + f*x])^
2)/(((-I)*a + b)*(I*a + b)*(a^2 + b^2)*f^2*(a + b*Cot[e + f*x])^2) - (2*a*c
*Csc[e + f*x]^2*(-(a*(e + f*x)) + b*Log[b*Cos[e + f*x] + a*Sin[e + f*x]])*(
b*Cos[e + f*x] + a*Sin[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*(a^2 + b^2)*f*(
a + b*Cot[e + f*x])^2) + (d*Csc[e + f*x]^2*(E^(I*ArcTan[b/a])*(e + f*x)^2 +
 (b*(I*(e + f*x)*(-Pi + 2*ArcTan[b/a]) - Pi*Log[1 + E^((-2*I)*(e + f*x))] -
 2*(e + f*x + ArcTan[b/a])*Log[1 - E^((2*I)*(e + f*x + ArcTan[b/a]))] + Pi*
Log[Cos[e + f*x]] + 2*ArcTan[b/a]*Log[Sin[e + f*x + ArcTan[b/a]]] + I*PolyL
og[2, E^((2*I)*(e + f*x + ArcTan[b/a]))]))/(a*Sqrt[1 + b^2/a^2]))*(b*Cos[e
 + f*x] + a*Sin[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*Sqrt[(a^2 + b^2)/a^2]*f
^2*(a + b*Cot[e + f*x])^2) + (Csc[e + f*x]^2*(b*Cos[e + f*x] + a*Sin[e + f*
```

```
x])*(-(b*d*e*Sin[e + f*x]) + b*c*f*Sin[e + f*x] + b*d*(e + f*x)*Sin[e + f*x
]))/(((-I)*a + b)*(I*a + b)*f^2*(a + b*Cot[e + f*x])^2)
```

fricas [B] time = 0.65, size = 1053, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*b - b^3)*d*f^2*x^2 - 2*a*b^2*c*f - 2*(a*b^2*d*f - (a^2*b - b^3)*c
*f^2)*x + ((a^2*b - b^3)*d*f^2*x^2 - 2*a*b^2*c*f - 2*(a*b^2*d*f - (a^2*b -
b^3)*c*f^2)*x)*cos(2*f*x + 2*e) + (I*a*b^2*d*cos(2*f*x + 2*e) + I*a^2*b*d*s
in(2*f*x + 2*e) + I*a*b^2*d)*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(
2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1)
+ (-I*a*b^2*d*cos(2*f*x + 2*e) - I*a^2*b*d*sin(2*f*x + 2*e) - I*a*b^2*d)*di
log(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b -
 I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) + (2*a*b^2*d*e - 2*a*b^2*c*f + b
^3*d + (2*a*b^2*d*e - 2*a*b^2*c*f + b^3*d)*cos(2*f*x + 2*e) + (2*a^2*b*d*e
- 2*a^2*b*c*f + a*b^2*d)*sin(2*f*x + 2*e))*log(1/2*a^2 + I*a*b - 1/2*b^2 -
1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) +
(2*a*b^2*d*e - 2*a*b^2*c*f + b^3*d + (2*a*b^2*d*e - 2*a*b^2*c*f + b^3*d)*co
s(2*f*x + 2*e) + (2*a^2*b*d*e - 2*a^2*b*c*f + a*b^2*d)*sin(2*f*x + 2*e))*lo
g(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^
2 + I*b^2)*sin(2*f*x + 2*e)) - 2*(a*b^2*d*f*x + a*b^2*d*e + (a*b^2*d*f*x +
a*b^2*d*e)*cos(2*f*x + 2*e) + (a^2*b*d*f*x + a^2*b*d*e)*sin(2*f*x + 2*e))*l
og((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b +
I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 2*(a*b^2*d*f*x + a*b^2*d*e + (a*b^2
*d*f*x + a*b^2*d*e)*cos(2*f*x + 2*e) + (a^2*b*d*f*x + a^2*b*d*e)*sin(2*f*x
+ 2*e))*log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 +
2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) + ((a^3 - a*b^2)*d*f^2*x^2 +
2*b^3*c*f + 2*(b^3*d*f + (a^3 - a*b^2)*c*f^2)*x)*sin(2*f*x + 2*e))/((a^4*b
+ 2*a^2*b^3 + b^5)*f^2*cos(2*f*x + 2*e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^2*sin
(2*f*x + 2*e) + (a^4*b + 2*a^2*b^3 + b^5)*f^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{\left(b \cot\left(fx + e\right) + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*cot(f*x + e) + a)^2, x)
```

maple [B] time = 2.56, size = 990, normalized size = 4.65

$$\frac{d\,x^2}{4iab + 2a^2 - 2b^2} + \frac{cx}{2iab + a^2 - b^2} + \frac{2ibad \ln\left(1 - \frac{(ib+a)e^{2i\left(fx+e\right)}}{-ib+a}\right) x}{(-ia+b)^2 f\,(ia+b)\,(-ib+a)} - \frac{2ib\,a^2 c \ln\left(a\,e^{2i\left(fx+e\right)} + ie^{2i\left(fx+e\right)} b - a + ib\right)}{(-ia+b)^2 f\,(ia+b)\,(ib-a)\,(ib+a)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*cot(f*x+e))^2,x)
```

```
[Out] 1/2/(2*I*a*b+a^2-b^2)*d*x^2+1/(2*I*a*b+a^2-b^2)*c*x+2*I/(b-I*a)^2/f/(I*a+b)
*b*a*d/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x-2*I/(b-I*a)^2/f/(I*
a+b)*b*a^2*c/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I
```

```
*b)-1/(b-I*a)^2/f^2/(I*a+b)*b^3*d/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*e
xp(2*I*(f*x+e))*b-a+I*b)+2*I/(b-I*a)^2/f^2/(I*a+b)*b*a^2*d*e/(I*b-a)/(a+I*b
)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)-4*I/(b-I*a)^2/f^2/(I*a+
b)*b*a*d*e/(I*b-a)*ln(exp(I*(f*x+e)))+2/(b-I*a)^2/f/(I*a+b)*b^2*a*c/(I*b-a)
/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)+4*I/(b-I*a)^2/f/
(I*a+b)*b*a*c/(I*b-a)*ln(exp(I*(f*x+e)))+I/(b-I*a)^2/f^2/(I*a+b)*b^2*d/(I*b
-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x+e))*b-a+I*b)*a-2/(b-I*a)^2
/f^2/(I*a+b)*b^2*a*d*e/(I*b-a)/(a+I*b)*ln(a*exp(2*I*(f*x+e))+I*exp(2*I*(f*x
+e))*b-a+I*b)+2*I*b^2*(d*x+c)/(I*a+b)/f/(b-I*a)^2/(exp(2*I*(f*x+e))*b-I*exp
(2*I*(f*x+e))*a+b+I*a)-2*I/(b-I*a)^2/f^2/(I*a+b)*b^2*d/(I*b-a)*ln(exp(I*(f*
x+e)))+2*I/(b-I*a)^2/f^2/(I*a+b)*b*a*d/(a-I*b)*ln(1-(a+I*b)*exp(2*I*(f*x+e)
)/(a-I*b))*e+2/(b-I*a)^2/(I*a+b)*b*a*d/(a-I*b)*x^2+4/(b-I*a)^2/f/(I*a+b)*b*
a*d/(a-I*b)*e*x+2/(b-I*a)^2/f^2/(I*a+b)*b*a*d/(a-I*b)*e^2+1/(b-I*a)^2/f^2/(
I*a+b)*b*a*d/(a-I*b)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))
```

maxima [B] time = 2.03, size = 1181, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -((a^3 + I*a^2*b + a*b^2 + I*b^3)*d*f^2*x^2 + (2*a^3 + 2*I*a^2*b + 2*a*b^2
+ 2*I*b^3)*c*f^2*x + 4*(I*a*b^2 + b^3)*c*f - ((4*I*a^2*b + 4*a*b^2)*c*f - 2
*(I*a*b^2 + b^3)*d + ((-4*I*a^2*b + 4*a*b^2)*c*f - 2*(-I*a*b^2 + b^3)*d)*co
s(2*f*x + 2*e) + (4*(a^2*b + I*a*b^2)*c*f - (2*a*b^2 + 2*I*b^3)*d)*sin(2*f*
x + 2*e))*arctan2(b*cos(2*f*x + 2*e) + a*sin(2*f*x + 2*e) + b, a*cos(2*f*x
+ 2*e) - b*sin(2*f*x + 2*e) - a) - ((-4*I*a^2*b + 4*a*b^2)*d*f*x*cos(2*f*x
+ 2*e) + 4*(a^2*b + I*a*b^2)*d*f*x*sin(2*f*x + 2*e) + (4*I*a^2*b + 4*a*b^2)
*d*f*x)*arctan2(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/(a
^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*cos(2*f*x + 2*
e))/(a^2 + b^2)) - ((a^3 + 3*I*a^2*b - 3*a*b^2 - I*b^3)*d*f^2*x^2 + ((2*a^3
 + 6*I*a^2*b - 6*a*b^2 - 2*I*b^3)*c*f^2 - 4*(I*a*b^2 - b^3)*d*f)*x)*cos(2*f
*x + 2*e) - ((2*I*a^2*b - 2*a*b^2)*d*cos(2*f*x + 2*e) - 2*(a^2*b + I*a*b^2)
*d*sin(2*f*x + 2*e) + (-2*I*a^2*b - 2*a*b^2)*d)*dilog((I*a - b)*e^(2*I*f*x
+ 2*I*e)/(I*a + b)) - (2*(a^2*b - I*a*b^2)*c*f - (a*b^2 - I*b^3)*d - (2*(a^
2*b + I*a*b^2)*c*f - (a*b^2 + I*b^3)*d)*cos(2*f*x + 2*e) + ((-2*I*a^2*b + 2
*a*b^2)*c*f + (I*a*b^2 - b^3)*d)*sin(2*f*x + 2*e))*log((a^2 + b^2)*cos(2*f*
x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2
+ b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e)) + (2*(a^2*b + I*a*b^2)*d*f*x*cos(2*
f*x + 2*e) - (-2*I*a^2*b + 2*a*b^2)*d*f*x*sin(2*f*x + 2*e) - 2*(a^2*b - I*a
*b^2)*d*f*x)*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) +
 (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e
))/(a^2 + b^2)) - ((I*a^3 - 3*a^2*b - 3*I*a*b^2 + b^3)*d*f^2*x^2 + ((2*I*a^
3 - 6*a^2*b - 6*I*a*b^2 + 2*b^3)*c*f^2 + (4*a*b^2 + 4*I*b^3)*d*f)*x)*sin(2*
f*x + 2*e))/((2*a^5 + 2*I*a^4*b + 4*a^3*b^2 + 4*I*a^2*b^3 + 2*a*b^4 + 2*I*b
^5)*f^2*cos(2*f*x + 2*e) - (-2*I*a^5 + 2*a^4*b - 4*I*a^3*b^2 + 4*a^2*b^3 -
2*I*a*b^4 + 2*b^5)*f^2*sin(2*f*x + 2*e) - (2*a^5 - 2*I*a^4*b + 4*a^3*b^2 -
4*I*a^2*b^3 + 2*a*b^4 - 2*I*b^5)*f^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c + d\,x}{\left(a + b\cot\left(e + f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*cot(e + f*x))^2,x)
```

```
[Out] int((c + d*x)/(a + b*cot(e + f*x))^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx}{\left(a + b\cot\left(e + fx\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e))**2,x)
[Out] Integral((c + d*x)/(a + b*cot(e + f*x))**2, x)
```

3.60 $\int \frac{1}{(c+dx)(a+b\cot(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\cot(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*cot(f*x+e))^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Cot[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Cot[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\cot(e+fx))^2}\,dx$$

Mathematica [A] time = 19.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])^2), x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2dx + a^2c + \left(b^2dx + b^2c\right)\cot\left(fx+e\right)^2 + 2\left(abdx + abc\right)\cot\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*cot(f*x + e)^2 + 2*(a*b*d*x
 + a*b*c)*cot(f*x + e)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(b\cot\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*cot(f*x + e) + a)^2), x)
```

maple [A] time = 13.35, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+b\cot\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*cot(f*x+e))^2,x)
[Out] int(1/(d*x+c)/(a+b*cot(f*x+e))^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\cot\left(e+f\,x\right)\right)^2\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cot(e + f*x))^2*(c + d*x)),x)
[Out] int(1/((a + b*cot(e + f*x))^2*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\cot\left(e+fx\right)\right)^2\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))**2,x)
[Out] Integral(1/((a + b*cot(e + f*x))**2*(c + d*x)), x)
```

3.61 $\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\cot(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Cot[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Cot[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2}\,dx$$

Mathematica [A] time = 17.17, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])^2), x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2d^2x^2 + 2\,a^2cdx + a^2c^2 + \left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\cot\left(fx+e\right)^2 + 2\left(abd^2x^2 + 2\,abcdx + abc^2\right)\cot\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cot(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*cot(f
*x + e)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(b\cot\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*cot(f*x + e) + a)^2), x)
```

maple [A] time = 21.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(a+b\cot\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x)
[Out] int(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\cot\left(e+f\,x\right)\right)^2(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cot(e + f*x))^2*(c + d*x)^2),x)
[Out] int(1/((a + b*cot(e + f*x))^2*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\cot\left(e+fx\right)\right)^2(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*cot(f*x+e))**2,x)
[Out] Integral(1/((a + b*cot(e + f*x))**2*(c + d*x)**2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#               Port of original Maple grading function by
#               Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#               added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
    )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
    type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```